Algorithms Used in SMBA

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Vector Dot Product Algorithm

Careful analysis of false moving-bed ship tracks, such as Figure 1, indicate that ship movement is typically perpendicular to the water velocity. When the StreamPro rotates both the boat and water velocities are rotated. Thus, the boat velocity and water velocities remain in the correct orientation relative to each other. This characteristic is what permits the discharge to be computed without a compass. This characteristic can also be used to determine the velocity of the boat in the upstream direction. The boat velocity in the upstream direction measured during a stationary moving-bed test can be computed using the vector dot product of the boat velocity and water velocity. The use of the dot product for analysis of a moving bed test was originally suggested by Randy Marsden, (Teledyne RD Instruments, oral communication, 2006). This approach has been subsequently developed and applied to the false moving bed problem associated with the StreamPro.

The vector dot product for two two-dimensional vectors A and B is defined as:

$$\vec{A} \bullet \vec{B} = \left|\vec{A}\right|\left|\vec{B}\right| cos(\theta) = A_x B_x + A_y B_y \tag{1}$$

where

$\vec{A}$ is a vector with components, $A_x\hat{i} + A_y\hat{j}$;
$\vec{B}$ is a vector with components, $B_x\hat{i} + B_y\hat{j}$;
$\left|\vec{X}\right|$ is the magnitude of the vector X; and
θ is the smaller angle between the two vectors.

If one of the vectors is a unit vector, say vector B, then the vector dot product computes the magnitude of vector A in the direction of vector B. The boat velocity in the upstream direction for each ensemble can be computed as the dot product of the boat velocity and the unit vector of the depth-averaged water velocity for each ensemble. If the water velocity vector is defined as,

$$\vec{V}_{(i,j)} = V_{x(i,j)}\hat{i} + V_{y(i,j)}\hat{j} \tag{2}$$

where

$\vec{V}_{(i)}$ is the water velocity vector at ensemble i, in depth cell j;
$V_{x(i,j)}$ is the x-component of the water velocity vector at ensemble i, in depth cell j; and
$V_{y(i,j)}$ is the y-component of the water velocity vector at ensemble i, in depth cell j.

The depth-averaged water velocity vector is computed by averaging the water velocity components for each ensemble.

$$\vec{W}_{(i)} = \frac{\sum_{j=1}^{m} V_{x(i,j)}}{m}\hat{i} + \frac{\sum_{j=1}^{m} V_{y(i,j)}}{m}\hat{j} = W_{x(i)}\hat{i} + W_{y(i)}\hat{j} \tag{3}$$

where

$\vec{W}_{(i)}$ is the depth-averaged water velocity vector at ensemble i;
$W_{x(i)}$ is the x-component of the depth-averaged water velocity vector at ensemble i;
$W_{y(i)}$ is the y-component of the depth-averaged water velocity vector at ensemble i; and
m is the number of valid depth cells.

The unit vector is computed by dividing the depth-averaged water velocity vector by its magnitude.

$$\vec{W}_{u(i)} = \frac{W_{x(i)}}{\left|\vec{W}_{(i)}\right|}\hat{i} + \frac{W_{y(i)}}{\left|\vec{W}_{(i)}\right|}\hat{j} \tag{4}$$

where

$\vec{W}_{u(i)}$ is the depth-averaged water velocity unit vector at ensemble i.

If we define the boat velocity vector as,

$$\vec{B}_{(i)} = B_{x(i)}\hat{i} + B_{y(i)}\hat{j} \tag{5}$$

where

$\vec{B}_{(i)}$ is the boat velocity vector at ensemble i;
$B_{x(i)}$ is the x-component of the boat velocity vector at ensemble i; and
$B_{y(i)}$ is the y-component of the boat velocity vector at ensemble i.

The boat velocity in the upstream direction (moving-bed velocity) can be computed as,

$$\left|\vec{V}_{MB(i)}\right| = -\left(\vec{B}_{(i)} \bullet \vec{W}_{u(i)}\right) \qquad (6)$$

where

$\left|\vec{V}_{MB(i)}\right|$ is the magnitude of the boat velocity vector in the upstream direction (moving-bed velocity).

Computing the average of the moving-bed velocities for each ensemble yields the mean moving-bed velocity that can be compared with the mean water velocity to assess the potential bias.

$$\overline{V}_{MB} = \frac{\sum_{i=1}^{n}\left|V_{MB(i)}\right|}{n} \qquad (7)$$

where

$\overline{V}_{MB}$ is the mean moving bed velocity; and
n is the number of valid ensembles.

Because the boat velocity and water velocity are in the correct orientation to each other, this approach is unaffected by the rotation and lateral movement of the StreamPro as it swims or kites at the end of a tether.

The validity of the proposed approach was evaluated using pairs of stationary measurements at three sites. At each site data were collected with the StreamPro swimming or kiting on the end of the tether and with StreamPro fixed at the approximately the same location by either a 3-point tether or a rod placed through float but outside of the beams. The moving-bed velocity was computed using the manual technique of dividing the distance made good by the duration for each of the measurements (Table 1). The data in Table 1 clearly show that the measured moving-bed velocities were significantly influenced by the swimming motion of the StreamPro and when the rotational and lateral motion of the StreamPro was restricted the moving-bed velocities were near zero. Software, discussed in the section titled, “Stationary Moving-Bed Test Analysis Software (SMBA) was written to apply the dot product approach. For all 6 stationary moving bed tests the dot product approach produce a moving bed velocity that was very near zero, consistent with the results obtained for the 3 tests where the rotation and lateral motion was restricted. Therefore, the dot product approach is a more accurate assessment of stationary moving-bed tests collected with a StreamPro ADCP than the manual computation.

Algorithm to Correct and Compute the Mean Water Velocity

The water velocity measured by a StreamPro is dependent on the boat velocity. The water velocity measured at a vertical and is thus, affected by the false upstream boat velocity induced by the rotation and lateral motion of StreamPro as it is held "approximately" on station. The depth-averaged water velocity can be corrected by adding the moving-bed velocity computed from the dot product method.

$$\vec{W}_{c(i)} = \vec{W}_{(i)} + \vec{W}_{u(i)} \left| \vec{V}_{MB(i)} \right| \tag{8}$$

The direction of the water velocity is difficult to assess because without a compass there is no reference for the water direction from one ensemble to the next. Therefore, only the magnitude of the depth-averaged water velocity can be used to compute a mean water velocity for the measurement. Although a stationary moving-bed test or velocity profile measurement should have nearly constant depths, the lateral motion of the StreamPro could result in changes in depth from one ensemble to the next. The mean depth for each ensemble is computed as the weighted depth of the valid beams.

$$D_{w(i)} = \sum_{k=1}^{nv(i)} \left[\frac{D_{k(i)} \omega_{k(i)}}{\sum_{k=1}^{nv} \omega_{k(i)}} \right] \tag{9}$$

$$\omega_{k(i)} = 1 - \frac{D_{k(i)}}{\sum_{k=1}^{nv(i)} D_{k(i)}} \tag{10}$$

$$\sum_{k=1}^{nv(i)} \omega_{k(i)} = nv(i) - 1 \tag{11}$$

where

$D_{w(i)}$ is the weight mean depth for ensemble i;
$nv(i)$ is the number of valid beam depths for ensemble i;
$D_{k(i)}$ is the depth in beam k for ensemble i; and
$\omega_{k(i)}$ is the depth weight for k for ensemble i.

The mean velocity magnitude is computed as the depth-weighted mean of the magnitudes of the depth-averaged velocities.

$$\overline{W} = \sum_{i=1}^{n} \left[\frac{W_{c(i)} D_{w(i)}}{\sum_{i=1}^{n} D_{w(i)}} \right] \tag{12}$$

Where

$\overline{W}$ is the mean water velocity magnitude for the stationary measurement.

Algorithm to Correct the Discharge

The algorithm used to correct the discharge based on the stationary moving-bed tests is similar to the distributed approach described by Mueller and Wagner (2006). The modifications to the approach for application in SMBA include:

a) The near-bed velocity is computed as 10 percent of the average depth for each ensemble.
b) The near-bed velocity for each stationary moving-bed test is the average of the near-bed velocity computed for each ensemble in the test.
c) The moving-bed velocity and the associated near-bed velocity are computed from the average of all stationary moving-bed tests processed in SMBA for that measurement.

References

Mueller, D.S., and Wagner, C.R., 2006, Application of the loop method for correcting acoustic Doppler current profiler discharge measurements biased by sediment transport: U.S. Geological Survey Scientific Investigations Report 2006-5079.

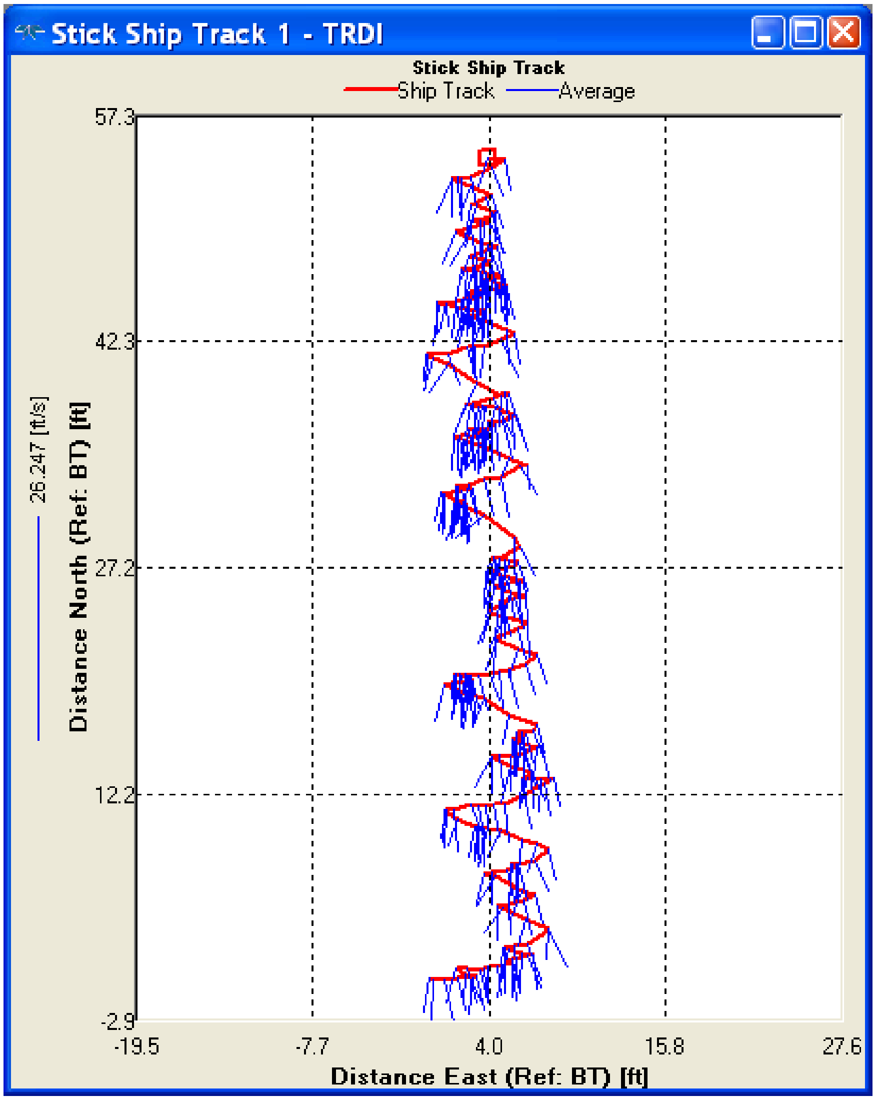

Figure 1. Ship track from WinRiver II of upstream movement caused by rotational and lateral motion of a StreamPro on a tether held at a fixed location.