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    HYDRAULICS

THE SOURCE OF WATER DERIVED FROM WELLS
ESSENTIAL FACTORS CONTROLLING THE RESPONSE
OF AN AQUIFER TO DEVELOPMENT

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The paper presented herewith, as a Ground Water Note, was
first published by the American Society of Civil Engineers in its
Civil Engineering magazine (p. 277-280) for May 1940. A number
of years ago the Ground Water Branch reprinted the paper in
limited quantity for distribution to the field offices and some field
personnel. The original paper is faithfully reproduced herein,
except for minor changes in notation to conform with current
Branch usage.

Theis' extremely lucid discussion of the source of water
discharged by wells should be clearly understood by all who would
attempt to analyze how a preexisting balanced hydraulic system
will adjust to newly imposed well discharge.

*Foreword by editor of civil engineering.- - Continued increase in the use of ground
water for municipal and industrial purposes, and for irrigation, makes more pressing the
question as to the extent of reserves of ground water and the advisability and methods of
regulating its use. Proper regulation, of course, is conditioned upon the ability to
forecast with some degree of accuracy the future history of water levels in wells in a
given area. Mr. Theis here gives a clear picture of the factors that must be taken into
account in such forecasts, and concludes with a brief summary of recommendations for
"the ideal development of any aquifer from the standpoint of maximum utilization of the
supply."*

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For distribution in Ground Water Branch only

This paper discusses in a general way the essential factors that control the response of an aquifer to development by wells. A knowledge of these factors, including the role of time, is necessary for the interpretation of existing records of water levels, and can yield the only method of predicting the effect of ground-water development in an area where records of long duration are lacking. Some of these factors have been long recognized but others have come to light in the last few years, and the intensive work now being done in quantitative ground-water hydrology will doubtless still further refine our concepts.

The essential factors controlling the action of an aquifer appear to be (1) the distance to, and character of, the recharge; (2) the distance to the locality of natural discharge; and (3) the character of the cone of depression in the given aquifer. Figure 1 illustrates diagrammatically the controlling factors in one type of aquifer.

Conditions of Equilibrium in an Aquifer

All ground water of economic importance is in process of movement through a porous rock stratum from a place of intake or recharge to a place of disposal. Velocities of a few tens or a few hundreds of feet a year are probably those most commonly met with in aquifers not affected by wells. This movement has been going on through a part of geologic time. It is evident that on the average the rate of discharge from the aquifer during recent geologic time has been equal to the rate of input into it. Comparatively small changes in the quantity of water in the aquifer, with accompanying changes in water level, may occur as the result of temporary unbalance between discharge by natural processes and recharge, but such fluctuations balance each other over a complete season or climatic cycle. Under natural conditions, therefore, previous to development by wells, aquifers are in a state of approximate dynamic equilibrium. Discharge by wells is thus a new discharge superimposed upon a previously stable system, and it must be balanced by an increase in the recharge of the aquifer, or by a decrease in the old natural discharge, or by loss of storage in the aquifer, or by a combination of these.

Conditions in the Recharge Area

Recharge to the aquifer may result from the penetration of rainfall through the soil to the water table, or by seepage from streams or other bodies of surface water, or by movement vertically or laterally form another ground-water body. The latter process is more or less an incident in the movement of water underground, and will not be discussed here. Two possible conditions in the recharge area must be considered. The potential recharge rate may be so large in wet seasons or cycles, or even uniformly, as to exceed the rate at which water can flow laterally through the aquifer. In this case the aquifer becomes over-full and available recharge is rejected. The water table stands at or near the surface in the recharge area. There may be permanent or seasonal springs in low places discharging the excess water, or there may be marshes or other areas of vegetation drawing water from the zone of saturation and transpiring the excess. In such a case, it is evident that if use of ground water by means of wells can increase the rate of underground flow from the area, more water is available to replenish the flow. More

water will go underground and the springs will flow less, or through-flowing streams will lose more water, or the vegetation will become more sparse.

On the other hand the possible rate of recharge may be less than the rate at which the aquifer can carry the water away. The rate of recharge in this case is governed (1) by the rate at which the water is made available by precipitation or by the flow of streams, or (2) by the rate at which water can move vertically downward through the soil to the water table and thus escape evaporation. In recharge areas of this latter type, none of the recharge is rejected by the aquifer.

In attempting to determine where the water discharged by wells comes from, or, more accurately, what process serves to balance the hydraulic system after the new discharge of the wells is imposed on it, this difference between rejected recharge and unrejected recharge must be kept clearly in mind. If water is rejected by the aquifer in the recharge area under natural conditions, then pumping of wells may draw more water into the aquifer. On the other hand, no matter how great the normal recharge, if under natural conditions none of it was rejected by the aquifer, then there is no possibility of balancing the well discharge by increased recharge, except by the use of artificial processes such as water spreading.

Figure 1 indicates diagrammatically the difference between these two conditions. Near the mountain border the water table is close to the surface, there is vegetation using ground water, and streams maintain their courses. This is the area of rejected recharge. A lowering of the water table in this zone will result in adding to the ground-water flow by decreasing the amount of transpiration and surface-water runoff. In the remainder of the area there is some recharge by rainfall, but the water table is so deep that no comparatively small change in its level can affect the amount of recharge. No recharge is rejected here and no lowering of the water table by pumping will cause more water to seep downward to the ground-water body.

The normal recharge of the aquifer is sometimes assumed to be the measure of the possible yield of the aquifer to wells. The theory is that if the wells take the recharge then the natural discharge will be stopped. Under certain conditions, and especially where the wells are located close to the area of natural discharge, this may be at least approximately true, but it is recognized that generally wells are not able to stop all the natural discharge. Whether or not the natural discharge can be affected, or whether the recharge can be affected without too great a lowering of water level in the pumping area, depend on the conditions of flow in the aquifer.

Conditions of Flow in the Aquifer

Ground water flows through an aquifer according to the simple law enunciated by Darcy in 1856. The rate of flow is proportional to the pressure gradient in the water. Thus the flow of ground water bears a close resemblance to the flow of heat by conduction in a solid, or the flow of electricity through solid conductors.
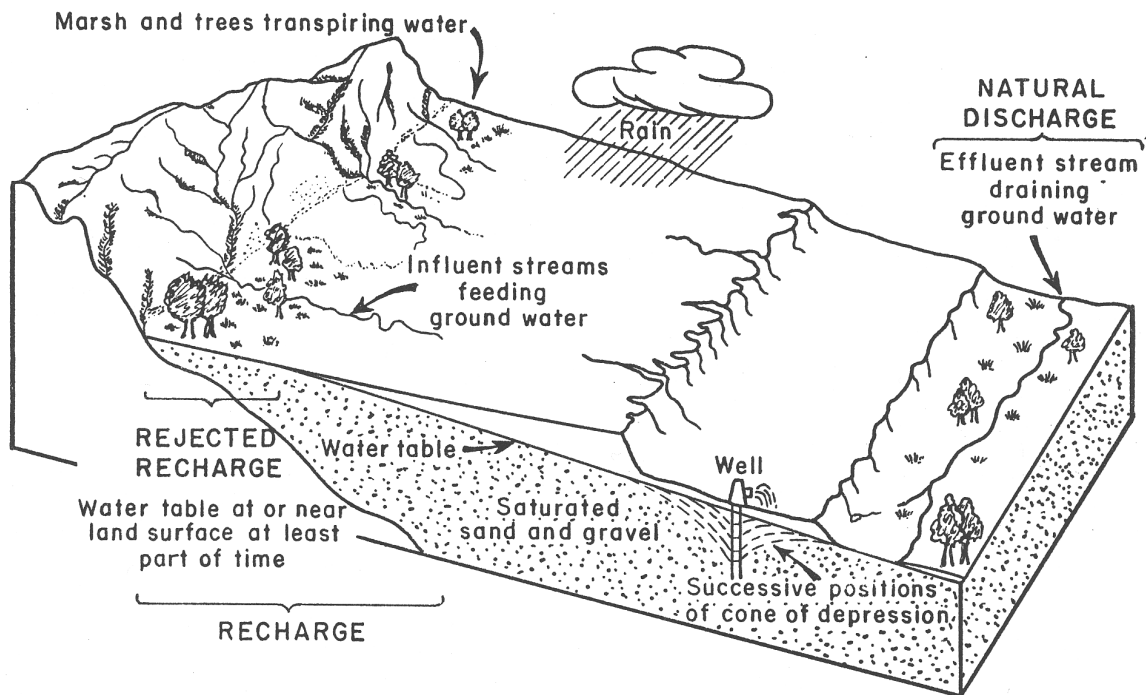
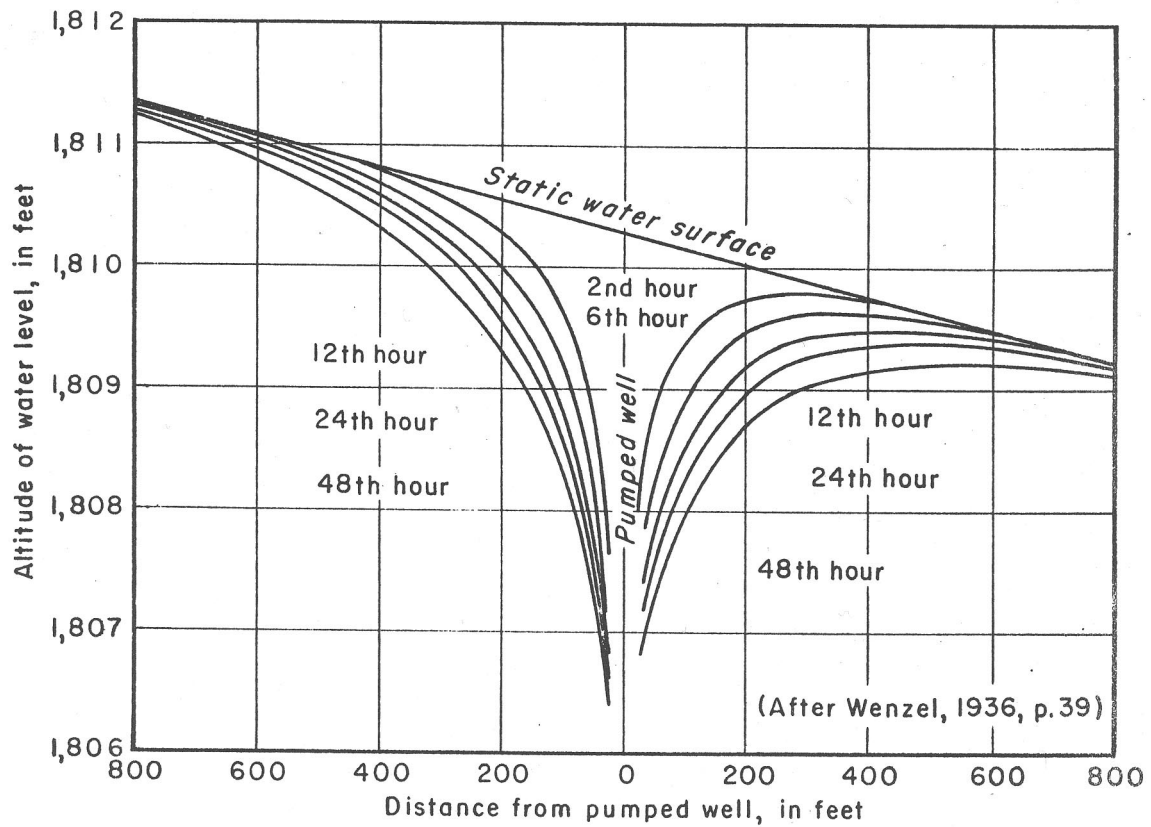Figure 1—Factors controlling response of an aquifer to discharge by wells

Figure 2—Growth of cone of depression during pumping test in Platte Valley, Nebraska

Under Darcy's law there is only one way of reducing the flow in the areas of natural discharge or of increasing the flow in the areas of recharge. This is by changing the pressure gradient or the thickness of saturation of the aquifer in those areas, which in turn means changing the height to which water levels rise in wells throughout the area between the producing wells and the areas of natural recharge or discharge. This means a lowering of water level everywhere between the wells and the areas of natural discharge or recharge. In turn this means a reduction of storage in the aquifer and an abstraction of water from it.

There are two fundamental physical properties of any aquifer which largely control the movement of water through it. The first is the ease with which it transmits the water, analogous to the thermal conductivity of a solid in the theory of heat, or the electrical conductivity of an electrical circuit. This characteristic of the aquifer as a whole is called the coefficient of transmissibility and is defined as the number of gallons of water that will pass in one day through a vertical strip of the aquifer 1 foot wide under a unit pressure gradient.

The other important characteristic of the aquifer is the amount of water that will be released form storage when the head in the aquifer falls. This has been called the coefficient of storage, and is defined as the amount of water in cubic feet that will be released from storage in each vertical column of the aquifer having a base 1 foot square, when the water level falls 1 foot. For non-artesian aquifers the coefficient of storage is nearly identical with the specific yield of the material of the aquifer. For artesian aquifers the coefficient depends on the compressibility of the aquifer or of included or stratigraphically adjacent shaly beds and is much smaller.

The Cone of Depression

Consider a broad flat slab of metal that has been brought to a uniform temperature and one of more edges of which are continuously maintained at that temperature. Somewhere near the middle of this slab let us place a colder rod and draw off heat through this rod at a uniform rate. The temperature of the plate in the vicinity of the rod will be reduced, and the depression of the temperature at any particular place will depend on the thermal conductivity of the metal, its specific heat, and its thickness. When a well is drawn upon a closely analogous process occurs. Water levels are drawn down in the vicinity of the well. Some water is removed form the vicinity concurrently with this reduction in water levels, and a so-called cone of depression is formed. The shape of this cone is determined principally by the ease with which water flows through the aquifer—the coefficient of transmissibility—and by the coefficient of storage.

Figure 2 shows the position of the water table in the vicinity of a pumped well at several times during the course of pumping; that is, it shows the successive shapes and positions assumed by the cone of depression. With continued pumping the cone deepens and broadens. It is evident that the well is taking water out of storage in the vicinity and that as more and more water is removed by the well, the cone of depression affects more and more distant parts of the aquifer.

On the simplifying assumption that the removal of water is exactly analogous to the removal of heat from a metal plate, an equation for the drawdowns caused by pumping a well may be derived. That this equation is essentially true is shown in Fig. 3 by the comparison of computed and observed drawdowns after 48 hours of pumping in the test made by Mr. Wenzel. The observed values shown are the averages of all drawdowns measured in all the observation wells at the given distances form the pumping well. Throughout most of the cone the difference between observed and computed values is less than 0.01 ft, and the maximum error is less than 0.05 ft.

This formula for the cone of depression in the ideal homogeneous and isotropic aquifer assumed is:

$$ s = \frac{114.6\ Q}{T} \int_{z}^{\infty} (e^{-u}/u)\,du $$

in which

s = drawdown at any point, in ft
Q = rate of discharge of the well, in gal per min
T = coefficient of transmissibility, in gpd per ft
z = $1.87\ r^2 S/Tt$
r = distance between pumped well and point of observation, in ft
S = coefficient of storage, dimensionless
t = time the well has been discharging, in days
u = a dimensionless quantity varying between the limits given

Some of the simplifying assumptions used in developing this formula are not rigidly realized in nature. However, the tolerance of the assumptions made appears to be sufficient for the purposes of this paper.

The characteristics of this formula should be noted. The quantity represented by the definite integral has a value depending only on the value of the lower limit, z, which involves distance, time, transmissibility, and storage ability. This quantity in effect determines the virtual radius of the cone of depression. The two factors outside the integral cause a variation in drawdown proportional to themselves. Specifically, the rate of pumping causes a proportional variation in the depth of the cone but does not affect its radius. The coefficient of storage, S, because of its relation to time, affects the rate of lateral spread of the cone, the rate of lateral growth being inversely proportional to its value. The coefficient of transmissibility affects both the radius of the cone and its depth, the radius for any given time increasing with increasing transmissibility, and the depth being inversely proportional to the transmissibility. The important general principle is that, according to the formula, which appears to hold except for very short periods of pumping, the rate of growth and the lateral extent of the cone of depression are
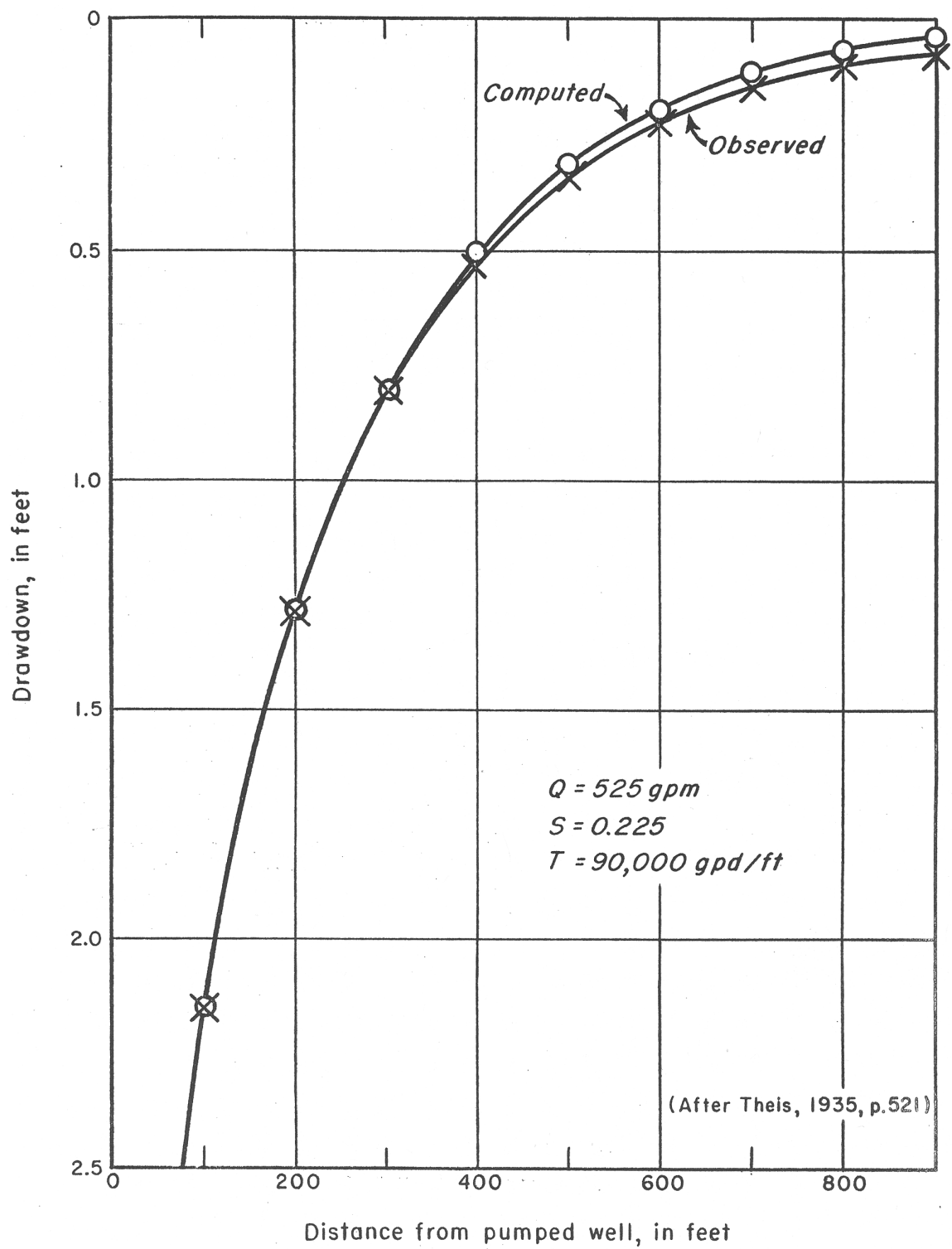
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Figure 3—Observed and computed drawdowns in the vicinity of a well
after pumping 48 hours

independent of the rate of pumping. If we pump twice as hard the cone will be twice as deep at any point, but it will not extend to any more distant areas. The disturbance in the aquifer created by the discharge of the well may be likened to a wave: the amplitude depends on the strength of the disturbance but the rate of propagation depends only on the medium in which the wave is formed. The reservoir from which the well takes water is almost as closely circumscribed by time as it would be by any material boundary, and until sufficient time has elapsed for the cone to reach the areas of natural discharge and rejected recharge a new equilibrium in the aquifer cannot be established.

The importance of this time effect varies with the characteristics of the aquifer and distance from the well to the areas of recharge and natural discharge. An idea of the order of magnitude of the effect may be gained from Figs. 4 and 5. These are drawn for an aquifer whose coefficient of transmissibility is 100,000 and whose coefficient of storage or specific yield is 20 per cent. These values are in the range of magnitude of the respective coefficients for most important non-artesian aquifers. The rate of pumping is 100 gallons per minute (gpm) or about 160 acre-feet a year. As the drawdown is directly proportional to the rate of pumping, the drawdown for any other rate of pumping can be readily computed.

Figure 4 compares drawdown with time at several distances from the pumped well. Time is shown on a logarithmic scale. There is a definite time lapse after pumping begins before the effects are felt at any given distance from the well. After a period of adjustment the fall of the water table proceeds approximately at a logarithmic rate. If the aquifer is extensive areally, and all the water withdrawn from the well is represented by a loss of storage in the aquifer, the drawdown at a distance of 1 mile from the pumped well in the first 10 years of pumping is over half of what it will be in 100 years.

Figure 5 plots the same data for several times against the distance from the pumped well. These are profiles of the cone of depression, with distance expressed on a logarithmic scale. Through most of their extent, these lines on the semi-logarithmic graph are practically straight. Within the radii represented by the straight portions of these lines, the aquifer is acting essentially as a conduit, merely carrying the water from more distant areas with only insignificant additions along the way. The significant additions are made in the regions where the lines are curved. This is the part of the aquifer that acts largely as a reservoir. Although theoretically the profiles of the cone of depression are asymptotic to the zero line, that is, the original position of the water table, and never quite reach it, except at the boundaries of the aquifer, practically speaking the cone has a definite edge beyond which neither the movement of the water nor its quantity is affected by the well. This edge, however, is constantly retreating and is not fixed, as is implied in some of the texts on ground-water hydrology.

It has been said that the rate of growth of the cone is inversely proportional to the coefficient of storage. This point is of importance to the present discussion chiefly in its bearing on the difference between artesian and non-artesian aquifers. In artesian aquifers the coefficient of storage is dependent on the compressibility of the aquifer and probably
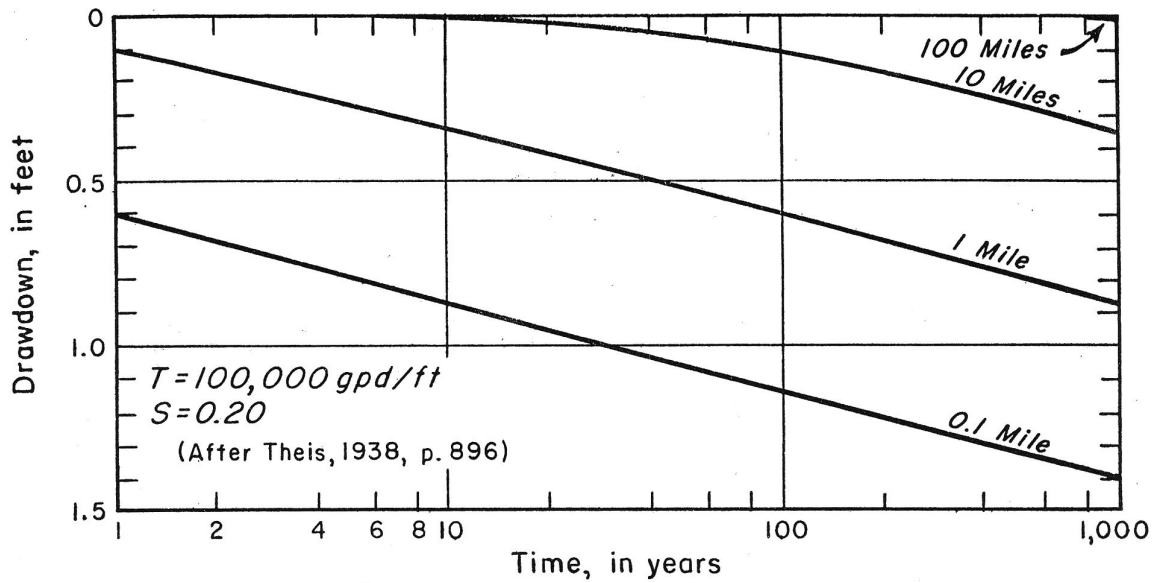
Figure 4—Drawdown in an ideal aquifer caused by continuous discharge of a well at the rate of 100 gallons per minute
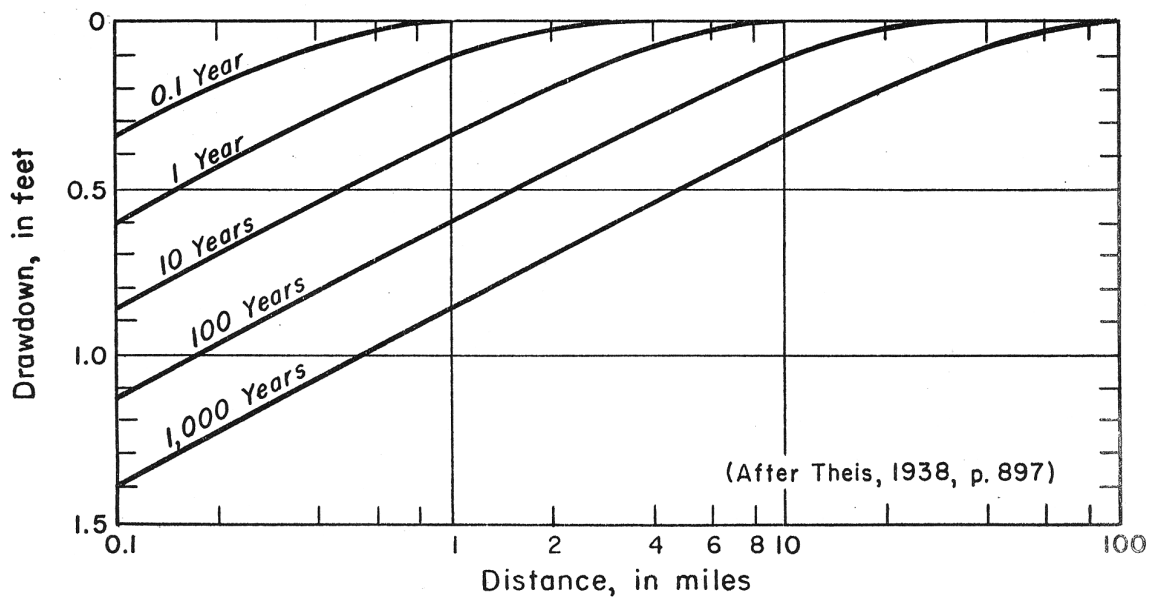

Figure5—Drawdown for same conditions as those of Figure 4, plotted against distance from discharging well

included or adjacent shaly beds, and is of a magnitude only a few per cent or a fraction of 1 per cent of that of non-artesian aquifers. Hence the cone of depression in artesian aquifers grows very roughly 100 times as fast as it does in non-artesian aquifers. Hence artesian aquifers, excluding the very extensive ones, are brought into a new equilibrium almost immediately and most of the effects of a new ground-water development are soon felt.

After the cone of depression reaches areas of rejected recharge or natural discharge, it is modified by the effects of adding water in the former or preventing it from escaping in the latter. If the rate of pumping does not exceed the amount of water added in the recharge area and that prevented form escaping in the discharge area, the cone will eventually reach equilibrium, at least practically speaking. The approximate effects that occur after the cone has reached the boundaries of the aquifer can be estimated by means of various mathematical analyses. The effects of discontinuous pumping can also be evaluated.

In summing up this technical discussion from the standpoint of ground-water conservation and statutory or other regulation to that end, the following points should be emphasized:

1. All water discharged by wells is balanced by a loss of water somewhere.

2. This loss is always to some extent and in may cases largely from storage in the aquifer. Some ground water is always mined. The reservoir from which the water is taken is in effect bounded by time and by the structure of the aquifer as well as by material boundaries. The amount of water removed from any area is proportional to the drawdown, which in turn is proportional to the rate of pumping. Therefore, too great concentration of pumping in any area is to be discouraged and a uniform areal distribution of development over the area where the water is shallow should be encouraged, so far as is consistent with soil and marketing or other economic conditions.

3. After sufficient time has elapsed for the cone to reach the area of recharge, further discharge by wells will be made up at least in part by an increase in the recharge if previously there has been rejected recharge. If the recharge was previously rejected through transpiration from non-beneficial vegetation, no economic loss is suffered. If the recharge was rejected through springs or refusal of the aquifer to absorb surface waters, rights to these surface waters may be injured.

4. Again, after sufficient time has elapsed for the cone to reach the areas of natural discharge, further discharge by wells will be made up in part by a diminution in the natural discharge. If this natural discharge fed surface streams, prior rights to the surface water may be injured.

5. In most artesian aquifers- - excluding very extensive ones, such as the Dakota sandstone - - little of the water is taken from storage. In these aquifers, because the cones of depression spread with great rapidity, each well in a short time has its

maximum effect on the whole aquifer and obtains most of its water by increase of recharge or decrease of natural discharge. Such an artesian basin can be treated as a unit, as is done in the New Mexico ground-water law, and the laws of some other western states that follow this law. In large non-artesian aquifers, where pumping is done at great distances from the localities of intake or outlet, however, the effects of each well are for a considerable time confined to a rather small radius and the water is taken from storage in the vicinity of the well. Hence these large ground-water bodies cannot be considered a unit in utilizing the ground water. Proper conservation measures will consider such large aquifers to be made up of smaller units, and will attempt to limit the development in each unit. Such procedure would also be advisable, although not as necessary, in an artesian aquifer.

6. The ideal development of any aquifer from the standpoint of the maximum utilization of the supply would follow these points:

   (a) The pumps should be placed as close as economically possible to areas of rejected recharge or natural discharge where ground water is being lost by evaporation or transpiration by non-productive vegetation, or where the surface water fed by, or rejected by, the ground water cannot be used. By so doing this lost water would be utilized by the pumps with a minimum lowering of the water level in the aquifer.

   (b) In areas remote from zones of natural discharge or rejected recharge, the pumps should be spaced as uniformly as possible throughout the available area. By so doing the lowering of the water level in any one place would be held to a minimum and hence the life of the development would be extended.

   (c) The amount of pumping in any one locality should be limited. For non-artesian aquifers with a comparatively small areal extent and for most artesian aquifers, there is a perennial safe yield equivalent to the amount of rejected recharge and natural discharge it is feasible to utilize. If this amount is not exceeded, the water levels will finally reach an equilibrium stage. It is exceeded, water levels will continue to decline.

   In localities developing water from non-artesian aquifers and remote from areas of rejected recharge or natural discharge, the condition of equilibrium connoted by the concept of perennial safe yield may never be reached in the predictable future and the water used may all be taken form storage. If pumping in such a locality is at a rate that will result in the course of ten years in a lowering of water level to a depth from which it is not feasible to pump, pumping at half this rate would not cause the same lowering in 100 years. Provided there is no interference by pumping from other wells, in the long run much more water could be taken from the aquifer at less expense.