Fractal topography and subsurface water flows from fluvial bedforms to the continental shield

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[1] Surface-subsurface flow interactions are critical to a wide range of geochemical and ecological processes and to the fate of contaminants in freshwater environments. Fractal scaling relationships have been found in distributions of both land surface topography and solute efflux from watersheds, but the linkage between those observations has not been realized. We show that the fractal nature of the land surface in fluvial and glacial systems produces fractal distributions of recharge, discharge, and associated subsurface flow patterns. Interfacial flux tends to be dominated by small-scale features while the flux through deeper subsurface flow paths tends to be controlled by larger-scale features. This scaling behavior holds at all scales, from small fluvial bedforms (tens of centimeters) to the continental landscape (hundreds of kilometers). The fractal nature of surface-subsurface water fluxes yields a single scale-independent distribution of subsurface water residence times for both near-surface fluvial systems and deeper hydrogeological flows. Citation: Wörmann, A., A. I. Packman, L. Marklund, J. W. Harvey, and S. H. Stone (2007), Fractal topography and subsurface water flows from fluvial bedforms to the continental shield, Geophys. Res. Lett., 34, L07402, doi:10.1029/2007GL029426.

1. Introduction

[2] The interaction of surface waters with groundwater influences the global cycling of solutes of both natural and anthropogenic origin and affects aquatic ecosystems [Allan, 1995; Alley et al., 2002]. For example, diffuse nutrient pollution of river networks—currently a pressing problem worldwide—involves transport and reaction of nitrogen as it moves from agricultural fields through shallow groundwater and then through sediments beneath stream channels [Winter et al., 1998; Peterson et al., 2001]. Management practices aimed at decreasing nitrate loads in rivers require detailed understanding of the relationships between river flow, channel geomorphology, and interactions with ground water. Much deeper groundwater circulation must be considered in many other important problems such as assessing the safety of high-level radioactive waste storage facilities [Gascoyne, 2003].

[3] A common challenge in problems of contaminant migration is characterization of subsurface water flow paths that differ markedly in length, depth, and duration of the subsurface flow. Many studies have demonstrated that mountain and river topographies have fractal geometry [Rodriguez-Iturbe et al., 1992]. It has also previously been observed that the residence times of water and solutes in the subsurface follow fractal distributions both in river networks [Kirchner et al., 2000] and in the hyporheic zone underlying stream channels [Haggerty et al., 2002]. However, previous investigations have not explored the basic processes that control this behavior sufficiently to develop a general theory of topographic control of surface-subsurface water interactions. Here we evaluate the effects of fractal landscape topography on groundwater flows and the resulting subsurface residence time distributions. The well-known fractal nature of topography is characterized in two dimensions using a new spectral technique [Wörmann et al., 2006] and related with different methods to the three-dimensional groundwater flow in various parts of the Fennoscandian Shield, the North American Continent and several small fluvial systems.

2. Methods

[4] Topography provides a basis for predicting surface-subsurface interactions because the groundwater surface tends to follow the ground surface in humid climates [Tóth, 1963; Zijl, 1999] especially in glacial boreal landscapes, and streambed topographic features such as sediment cobbles and bedforms control hyporheic pore water flow [Thibodeaux and Boyle, 1987; Harvey and Bencala, 1993; Elliott and Brooks, 1997; Packman et al., 2000]. In order to reveal the commonality between subsurface flow patterns in these disparate systems, the forcing of the surface topography, Z(x, y), on groundwater flow is assessed using an analytical solution based on the spectral method with assumption of a homogenous subsurface, and deep groundwater circulation is further evaluated using a numerical model that includes known variability in temperature and permeability over the depth of the crust. We consider flow domains with flat top surfaces and homogeneous permeability over a finite depth ε. Subsurface head boundary conditions are then obtained as imposed by the surface topographies. For terrestrial systems, it is assumed that the phreatic surface follows the ground surface. This assumption is generally acceptable in humid climates with sufficient hydrologic input and for analysis at a resolution much larger than the thickness of the unsaturated zone. For fluvial systems, the boundary head distribution is phase-shifted and damped relative to the topography, as described by Elliott...
and Brooks [1997]. Hence, the surface topographical spectrum is related to a stationary and linear groundwater flow field in terms of the head distribution [Wörman et al., 2006]:

$$h(x, y, z) = (h) + \sum_{i=1}^{N_x} \sum_{j=1}^{N_y} (h_m)_{i,j} \theta(z) \sin(2\pi/\lambda_x, x) \cdot \cos(2\pi/\lambda_y, y)$$

(1)

where $h$ = hydraulic head, $(\ldots)$ denotes the areal mean value, $\lambda_x$ and $\lambda_y$ are the wavelengths in the x- and y-directions, $h_m =$ the amplitude coefficients and the auxiliary decay function with depth $\theta(z) = \exp\left[\sqrt{(2\pi/\lambda_x)^2 + (2\pi/\lambda_y)^2} z\right] + \exp\left[\sqrt{(2\pi/\lambda_x)^2 + (2\pi/\lambda_y)^2} (-2\epsilon - z)\right]/\left[1 + \exp(-2\epsilon)\right] \sqrt{2\pi/\lambda_x} + \sqrt{2\pi/\lambda_y} \epsilon$).

A crucial limitation on deeper hydrogeologic flows is caused by the interplay between the thermodynamic properties of water under conditions found at depth in the Earth’s crust and the eight- to twelve-order decrease of rock permeability with depth in the crust. Despite the fact that temperature increases and viscosity decreases substantially over the depth of the crust, it is known that water exists in a liquid phase throughout the crust and deep groundwater flows do in fact follow the normal hydro-mechanical laws for liquid flow in a porous medium [Ingebritsen and Manning, 1999]. A finite difference model is used to evaluate the effects of the variation of temperature and hydrogeologic properties as well as water flow field over the depth of the Earth’s 40 km thick crust on groundwater circulation (see the auxiliary material). Subsurface water residence time distributions are evaluated numerically by particle tracking based on the calculated subsurface velocity field.

The interfacial flux (volume of water moving into the subsurface per unit area per unit time) can readily be expressed as $W(x, y, z = 0) = -K \frac{\partial h}{\partial z}|_{z=0}$, where $K$ is hydraulic conductivity. Thus, for isotropic harmonic functions ($\lambda_x = \lambda_y$), each harmonic of the spectrum contributes to the maximum value of the interfacial flux as

$$\max(W_{i,j}(x, y, z = 0)) = -2\sqrt{2\pi KB \frac{(h_m)_{i,j}}{\lambda}}$$

(2)

in which $B = [1 - \exp(-2\sqrt{2\pi \epsilon/\lambda})]/[1 + \exp(-2\sqrt{2\pi \epsilon/\lambda})]$. The B-factor reflects the blocking effect of the lower impermeable surface on the impact of landscape wavelengths on the surficial water flux. The B-factor rapidly approaches unity as the depth to the impermeable surface exceeds 1/3. All spectral simulations were conducted with modeled domain depths much greater than this.

Both the finite difference model and the spectral model are implemented with vertical closed boundaries located sufficiently far from the region where the surface-subsurface interaction is evaluated to ensure that the boundaries did not significantly affect the flow in the region of interest [Wörman et al., 2006]. In order to ensure that a similar range of scales is included in each analysis, the spectral method is applied with a constant resolution $N_x = N_y = 140$ in all simulations of terrestrial landscapes (1,600 elevation values in a regular grid), and with 28 frequencies in each horizontal direction. Laboratory-generated fluvial topographies are evaluated with a similar number of points in the longitudinal direction (140), but with variable resolution of the transverse behavior as appropriate for each system configuration. A small stream, Sugar Creek, is evaluated at a resolution of $20 \times 20$ points. See also the auxiliary material.

3. Results

3.1. Surface-Subsurface Interactions Over a Wide Range of Scales

Three-dimensional simulations indicate that subsurface flows are induced by the relative location of topographic highs and lows [Wörman et al., 2006] (see the auxiliary material). Further, it is apparent that superposition of flows induced by various topographic features yields a wide distribution of flow path lengths in both terrestrial and fluvial systems. It is important to note that the same resolution was applied in all analyses except Sugar Creek, so that the topographic distributions and flow fields are resolved to the same extent in each system and therefore encompass an equivalent range of scales.

3.2. Fractal Nature of Surface-Subsurface Interactions

Figure 1 shows surface topographies and subsurface head gradients for different Swedish landscapes and a variety of fluvial forms observed both in a small stream and in laboratory model systems. The results reveal fractal scaling in surface-subsurface interactions over a wide variety of landscapes and spatial scales. These power spectra were obtained from a 2D Fourier series that not only represents the topography but also provides an exact solution for the groundwater flow field (equation 1). As shown in Figure 1 (top), the power spectra of the surface topographies follow power-law scaling in the form $h_m \sim \lambda_x^\beta$ in which the power-law exponent (scaling factor) $\beta \approx 2/3$. Note that the amplitude coefficients plotted in Figure 1 is the marginal distribution $h(\lambda_x)$ obtained as an average over $\lambda_y$ from two-dimensional analysis, i.e., $h_m(\lambda_x, \lambda_y)$. The conventional power spectrum evaluated along one-dimensional landscape transects yields a slope of $1.6$ (see the auxiliary material) equivalent to $0.8$ in terms of the Fourier coefficient spectrum, which is within the range of previous results found for various terrestrial and fluvial landscapes [Rodriguez-Iturbe et al., 1992; Turcotte, 1997; Nikora, 1997; Jerolmack and Mohrig, 2005]. While this type of self-similarity behavior has been observed previously in terms of the fractal distributions of stream path lengths, river drainage networks, relief in glacial landscapes, and large dunes in rivers the results presented in Figure 1 provide the new observation that this topographic scaling also extends to the distribution of the smallest scale sedimentary features within stream channels.

Equation (1) suggests that the water flux across the surface and general magnitude of the circulation is propor-

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1Auxiliary materials are available in the HTML. doi:10.1029/2007GL029426.
The amplitude (Fourier coefficient) spectrum or power spectrum taken in one direction of the 2D Fourier representation of the surface topography of several glacial and boreal landscapes, and fluvial systems. This is the marginal distribution $h(\lambda_x)$ obtained as an average over $\lambda_y$ from two-dimensional analysis, i.e., $h_m(\lambda_x, \lambda_y)$. All topographic spectra follow power-law scaling, indicating that the surface topographies as well as velocity components of surface-subsurface water fluxes all have fractal distributions. (b) Distributions of boundary head gradients induced under each system. All of the distributions show power-law scaling over a wide range of spatial scales and hydrological systems with a slope between $-1.1$ and $-1.2$. The power-law tail appears to widen only because of numerical limitations.

The fractal nature of both the landscape topography and resulting head gradients suggests that a common scaling should exist for surface-groundwater interactions over a wide range of scales. The fractal scaling of topography, $h_m(\lambda_x)$, implies that the head gradients scale as $h_m(\lambda_x) \sim \lambda_x^{-1/3}$. Thus, the surface-subsurface water flux is dominated by a decreasing impact of topography with landscape wavelength as indicated by equation (2) and the depth limitation (blocking factor in equation (2)) provides a secondary scaling mechanism that slightly enhances this tendency. Figure 2 (bottom) shows that depth limitations cause the residence time distributions to deviate only slightly from the homogeneous case.

This analysis indicates that while the interfacial flux depends to some extent on all topographical scales in any point in the landscape, it is likely to be dominated by smaller-scale topographic features. However, the effects of small features are only propagated to shallow depths. From equation (1) we can see that when the wave numbers $k_x = k_y$ the subsurface potential decreases with depth $z$ as $e^{-\sqrt{2k_z}z}$. Further, the vertical head gradient $(dh/dz)$ decays with depth as $\ln \left(-\sqrt{8\pi^2}/\lambda_z\right)$. Therefore, the effects of the surface topography decay relatively rapidly, and the flows found at increasing depths must be induced by larger-scale topographic features.

Figure 2 shows the distribution of residence (transit) times, $T$, along the flow paths from inflow to outflow sections in all systems. For the glacial and fluvial landscapes considered here, a single dimensionless subsurface residence time PDF is found regardless of scale, indicating consistent self-similar behavior in the temporal scaling of surface-subsurface interactions. Similar behavior is also found for the lower resolution applied to Sugar Creek (see the auxiliary material), which may suggest that not all topographical scales in the spectrum have equal importance for the residence time distribution. Nonetheless, the surface-subsurface flow interaction is clearly fractal because the power spectrum of the surface-subsurface flux follows a power law. Similar fractal behavior has been observed previously in the distribution of solute efflux from watersheds and in hyporheic exchange in both pristine mountain streams and lowland agricultural streams [Kirchner et al., 2000; Haggerty et al., 2002; Wörman et al., 2002]. The results presented here indicate that such scaling in surface-groundwater interactions can be explained by forcing imposed by the fractal topography that occurs from the bedform scale to the continental scale.

The fractal nature of surface-subsurface fluxes of solutes are important in applications ranging from microbial ecology to nutrient dynamics to contaminant transport. Previous studies have analyzed subsurface solute residence time distributions as power laws, log-normal distributions, or exponential distributions [Choi et al., 2000; Kirchner et al., 2000; Haggerty et al., 2002; Wörman et al., 2002]. While specific topographical forms can produce other residence time behavior, such as log-normal distributions, it is now clear that the fractal distribution of topographical features that control surface-subsurface water interactions should be expected to generally produce power-law scaling in solute residence times and solute efflux from watersheds.
3.3. Effect of Geological Structure on Deep Groundwater Circulation

[15] The shift towards shorter timescales as evident from Figure 2 (bottom) is caused by the substantial decrease in permeability and increase of temperature with depth in the Earth’s crust [Ingebritsen and Manning, 1999]. This effect is particularly prominent in the largest system, a large part of the North American continent. Nonetheless, the overall self-similarity of the subsurface residence time distribution essentially holds from the smallest fluvial bedforms up to the scale of the continental shield. Local heterogeneities in subsurface strata, such as fracture zones, can have additional specific, local effects on the subsurface flow. However, the statistical expectation of the generic problem includes stochastic perturbations in permeability as a second-order effect [Gelhar, 1986]. Consequently, the universal scaling of surface head gradients shown in Figure 1 still produces the dominate scaling in surface-groundwater interactions, as described by equation (1). While local heterogeneities in permeability can affect local flow patterns, self-similarity is still found in the mean behavior of the residence time PDF because of the critical forcing provided by the surface topography.

[16] For the Swedish landscapes, the scale-independent residence time distribution found here reveals an extraordinarily broad range of groundwater transport timescales, with a very small fraction of groundwater being retained at least 1.5 to 2 million years. Such old water has been found in the Fennoscandian shield based on interpretation of chloride isotope composition of brine water [Laaksoharju and Wallin, 1997]. The exponential decay of hydraulic head with depth embodied in equation (1) indicates that largest features of the continental landscape control the circulation of such old and deep groundwater.

4. Discussion

[17] The 2D spectral technique employed here demonstrates that fractal landscape topography induces fractal patterns in groundwater circulation over a wide range of spatial scales. Both the spectral technique and a separate numerical analysis suggest that the subsurface flow also exhibits a self-similar distribution of residence times over a wide range of temporal scales ($\lambda/K$) associated with the spectrum of landscape topography. The fractal nature of surface-subsurface interactions means that the flow at any point in the subsurface is influenced to some extent by the entire spectrum of surface topography. The topographic scaling factor $\beta \approx 2/3$ found consistently for both terrestrial and fluvial systems indicates that interfacial flux predominantly scales as $\lambda^{1/3} \approx \chi^{1/3}$. This means that smaller-scale topographic features tend to produce larger boundary head gradients and, as a result, the surface-subsurface water flux tends to be controlled by smaller-scale topography in the fractal landscape.

[18] A competing impact on the groundwater flow pattern arises because the effect of topography attenuates with depth faster for the shorter wavelengths, i.e. $dh/dz \sim \ln (-\sqrt{8\pi}z/\lambda)$. The flow at each depth is most strongly influenced by a specific topographical wavelength, and the dominant wavelength increases with increasing depth. The three-dimensional solution employed here reveals that the effects of small-scale topographical features are more rapidly attenuated with depth than suggested by previous analyses [Elliott and Brooks, 1997]. Thus, deeper flows are increasingly likely to be controlled by larger-scale topographic features, and will not necessarily follow local
topographic divides. The entire range of topographical scales can significantly influence the magnitude and spatial variability of water flow in the deep subsurface, which will be of primary interest for hydrological applications such as siting of nuclear waste repositories in bedrock. The interactions between shallow and deeper subsurface flows also explain previous observations that spatial patterns and fluxes of water in streambeds (hyporheic exchange) are embedded within and modulated by larger patterns of groundwater discharge and recharge in rivers (e.g., gaining vs. losing reaches).

[19] The spectral analysis not only reveals that both fluvial and terrestrial systems have similar fractal surface topography and surface-subsurface water fluxes, but also indicates that topographic forcing tends to produce a corresponding fractal nature of the subsurface flows at all scales up to and including the continental scale. From numerical simulations we find that geological stratification, including the lower bound imposed by the decrease of permeability with depth and the thickness of the earth’s crust, exerts an important limitation on large-scale groundwater circulation by restricting the longest flow paths and residence times. However, topography remains a primary control even for deep groundwater circulation, and groundwater flows induced by surface features still show self-similarity in residence time distributions over a wide range of scales despite the variation of temperature, pressure, and permeability over the depth of the Earth’s crust. Therefore, while local geologic constraints can modify the scaling behavior, and features such as aquitards even lead to effective cutoffs at particular spatial scales, broad connectivity in surface-subsurface interactions should still be expected. Essentially, the theoretical analysis and results presented here show that surface forcing will tend to produce fractal patterns in subsurface flows unless particular local subsurface structural features prevent this behavior.

[20] The breadth and generality of fractal scaling in surface-subsurface interactions brings an entirely new perspective to the analysis of a wide range of geophysical and biogeochemical processes that are influenced by surface-subsurface exchange flows. Our work provides a theoretical framework for linking two-dimensional topographical distributions with three-dimensional subsurface flows. Such a framework can facilitate advancement of analytical approaches for hydrological and environmental applications. This work specifically provides a generalized framework for constraining the effects of larger-scale flows on the groundwater behavior of local sites, which is important for site-based evaluations of nutrient and contaminant transport. While bounded model domains can be applied in a meaningful way when the system is known to be highly constrained, the underlying fractal nature of the problem makes it critical to carefully evaluate which spatial scales are important for any site or application of interest. We provide a conceptual basis for assessing these effects and emphasize the result that both surface topography and subsurface flows show fractal scaling over a very wide range of systems and scales: from small fluvial bedforms to the breadth of the continental landscape.

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