A STUDY OF METHODS USED IN

MEASUREMENT AND ANALYSIS OF SEDIMENT LOADS IN STREAMS

REPORT U

AN INVESTIGATION OF A DEVICE FOR
MEASURING THE BULK DENSITY OF
WATER-SEDIMENT MIXTURES

AUGUST 1974

A Study of Methods Used in

MEASUREMENT AND ANALYSIS OF SEDIMENT LOADS IN STREAMS

A Cooperative Project
Sponsored by the
Subcommittee on Sedimentation
Water Resources Council

Participating Agencies

Corps of Engineers ** Geological Survey
Soil Conservation Service ** Bureau of Reclamation
Agricultural Research Service ** Tennessee Valley Authority
Environmental Protection Agency ** Federal Highway Administration
National Oceanic and Atmospheric Administration
Forest Service

REPORT U

AN INVESTIGATION OF A DEVICE FOR MEASURING THE BULK DENSITY
OF WATER-SEDIMENT MIXTURES

By

J. P. BEVERAGE AND J. V. SKINNER

Prepared for Publication by the Project Staff
at St. Anthony Falls Hydraulic Laboratory
Minneapolis, Minnesota

AUGUST 1974

DEFINITIONS

MASS (m)--The quantity of matter. A given quantity of matter does not change when moved away from the earth's surface. The unit of mass is the gram (cgs system) or the slug (American units).

WEIGHT (w)--The force of attraction between a mass and the earth. The weight of a given mass varies with distance from the earth and may also vary slightly over the earth's surface. The unit of weight is the dyne (cgs) or pound (American). Weight is the product of mass and the local acceleration rate due to gravity (i.e., $w = mg$).

DENSITY (ρ)--The mass of a substance per unit volume. Density is the property of a material that describes its concentration of mass. The common scientific unit is gram per cubic centimeter; while American engineering practice commonly uses both the pound per cubic foot and the slug per cubic foot.

BULK DENSITY (ρ_b)--The mass of a mixture per unit volume: the gross density. The density of a two-component mixture will be less than that of the most dense component and greater than that of the least dense component.

SPECIFIC GRAVITY (s)--The ratio of the density of one substance to that of another substance taken as a standard, usually pure water at 4°C. Note that specific gravity is a dimensionless ratio, while density has dimensions of mass per unit volume.

ABSTRACT

A special neutrally buoyant container is used to assess the feasibility of determining sediment concentration by measuring the bulk density of a sample placed inside a special container and lowered into a water bath. The displacement of an indicator rod is measured after equilibrium is reached. The displacement is a function of the suspended sediment and any dissolved matter. An analysis of errors is given as well as a discussion of empirically determined limitations.

TABLE OF CONTENTS

* * * * * * * * *

LIST OF ILLUSTRATIONS

AN INVESTIGATION OF A DEVICE FOR MEASURING THE BULK DENSITY OF WATER-SEDIMENT MIXTURES

I. INTRODUCTION

This report describes a feasibility study of the bulk-density method of determining suspended-sediment concentration. The density of the bulk sample is directly related to the dissolved and solid matter in the sample. Ordinarily, direct weighing of the bulk sample is not practical at low concentrations because the density is so nearly that of the pure liquid. The approach described herein involves submerging the sample in a neutrally buoyant container in water and obtaining the amount of sediment directly, in effect floating the water component of the sample mixture. A single weighing thus determines weight directly with reasonable accuracy.

The usual gravimetric determination of sediment concentration requires two to four weighings. The tare of the sampling container is subtracted from the gross weight of the sample and container to obtain the net weight of the water-sediment mixture. Next, the water and sediment are separated by decantation and evaporation or filtration; and the net sediment weight is obtained by subtracting the tare weight of the separation container from the gross weight. Finally, sediment concentration is the ratio of the sediment weight to the net weight of the mixture.

Many laboratories use average tares when satisfied that only minor changes occur between check weighings. The assumption of constant tare weights greatly reduces the workload, but an occasional chipped or dirty container will cause errors. Without the assumption of constant tare weights, two very accurate weighings of the sediment drying dish are necessary in order to obtain a net weight of reasonable accuracy.

This report contains an analysis of potential errors of the bulk density device constructed for this study. The tests described in this report were organized in order to verify the major sources of error and to provide data on sensitivity and accuracy of the device. This report conveys the results of these tests and recommendations for further instrumentation.

ACKNOWLEDGEMENTS

II. PREVIOUS APPROACHES TO THE PROBLEM

There have been many attempts to devise instrumentation to measure sediment concentration based on indirect methods. Some of the early approaches include the hydrometer, plummet, optical, and manometer methods, which were also used to determine particle-size distribution. Another example is the displacement method, which involves determining the weight difference between a volume of pure water and an equal volume of water-sediment mixture at the same temperature. A description of these methods may be found in Report No. 4: "Methods of Analyzing Sediment Samples," (Ref. 1).

Each of these methods has its advantages over other approaches. Each method also has one or more disadvantages. The displacement method requires adjustment of the volume to the same percentage accuracy as the weight determination, and requires an assumption of an average specific gravity of the sediment. The other methods fail when coarse sediment is present because the agitation required to maintain an even suspension disturbs the measurement. Additionally, the hydrometer and plummet tend to read too high because of sediment lodging on the sloping sides.

The disadvantages of the hydrometer and plummet can be minimized or eliminated by turning the hydrometer or plummet inside out. By putting the reference liquid on the outside and the mixture on the inside, heavy particles need not be maintained in suspension and settling on the inside walls would have no effect on the weight determination. The top should be left open to allow any air bubbles to leave the container. The problem now becomes one of preventing the escape of fine material from the container. The new problem cannot be completely solved, but careful design can minimize loss of material including dissolved salts. A correction factor based on conductivity will minimize the error due to dissolved salts.

One other density-measuring device deserving mention is the radio-isotope gage (Ref. 2). The instrument compares the attenuation of X-rays in reference water and river water. It may be calibrated in terms of sediment concentration. However, statistical error inherent to the radioisotope source becomes too great at low concentrations, especially for short counting periods. The instrument also is sensitive to dissolved materials which vary independently of the suspended sediment.

III. OPERATIONAL PRINCIPLES

Archimedes' principle of buoyancy equates the buoyant force on a body wholly or partly immersed in a fluid to the weight of the fluid displaced by the body. The force causing a floating object to float at a given position, preventing further sinking, is the buoyant force. An empty cup placed in water will seek an equilibrium position where the buoyant force balances the weight of the cup. Adding a small pebble to the cup will change the equilibrium position downward. The additional weight of the pebble causes the cup to displace more water. The equilibrium position of the cup is therefore a function of the weight of the cup and whatever it contains.

Proceeding further, if a water-sediment mixture is placed in a submerged cup, the weight of the sediment may be found without filtering or evaporating the mixture. By weighing underwater we have also reduced the requirement that the balance weigh heavy samples quite accurately. When sediment is weighed during the usual gravimetric concentration analysis, great accuracy is needed over a large weight range because the sediment is often the difference between two fairly large weights.

The bulk density ϱ_b of a water-sediment mixture of volume V can be expressed as

$$\varrho_b = \frac{m_s + m_w}{V} \qquad , \qquad (1)$$

where m_s and m_w refer to the sediment and water masses, respectively, in the mixture. The concentration of the mixture is customarily defined as (Ref. 3, p. 3):

$$C = \frac{\text{weight of sediment}}{\text{volume of water-sediment mixture}}$$

However to simplify development of relationships used in this report, the following defining equation will be used:

$$C = \frac{m_s}{m_s + m_w} \qquad , \qquad (2)$$

and by substitution,

$$\frac{m_s}{V} = \rho_b C \qquad . \qquad (3)$$

Because $V = V_w + V_s$, $m_w = V_w \rho_w$, and $V_s = m_s / \rho_s$, then

$$m_w = \rho_w (V - V_s) \qquad ,$$

$$= \rho_w V - \rho_w \frac{m_s}{\rho_s} \qquad . \qquad (4)$$

Substituting eq (4) then (3) into eq (1), we find

$$\rho_b = \rho_w + C\rho_b \left(1 - \frac{\rho_w}{\rho_s} \right) \qquad , \qquad (5)$$

which can be rearranged to

$$C = \frac{\rho_s}{\rho_b} \left(\frac{\rho_b - \rho_w}{\rho_s - \rho_w} \right) = \frac{\rho_s}{\rho_s - \rho_w} \left(1 - \frac{\rho_w}{\rho_b} \right) \qquad . \qquad (6)$$

Concentration is clearly a function of the bulk density. The density of sediment normally is taken as a constant, and the density of water is constant at a given temperature.

The buoyant force exerted on a fully submerged water-sediment mixture is given by $F_b = gV\rho_w = g(V_w + V_s) \rho_w$, and the force due to gravity is $F_g = gV_w\rho_w + m_s g$. The net upward force necessary to achieve balance is $F_g - F_b$, or

$$F_n = g(V_w\rho_w + m_s - V_w\rho_w - V_s\rho_w)$$

$$= g(m_s - V_s\rho_w) \qquad . \qquad (7)$$

Substituting m_s / ρ_s for V_s,

$$F_n = g\left(m_s - \frac{m_s\rho_w}{\rho_s} \right) \qquad , $$

$$= gm_s \left(1 - \frac{\rho_w}{\rho_s} \right) \qquad . \qquad (8)$$

Measurement of F_n will enable one to compute m_s if values for ρ_s and ρ_w are known or can be assumed.

12

The first test model is shown in Fig. 1. The container was constructed of thin brass shim stock with a spring-loaded plastic plug in the bottom. A diagonal plate was added to the top of the container in order to minimize loss of the sample from the container because of circulation. Operation of the device was quite simple. The beam balance used for determining weight was adjusted to balance at the neutral point at which the empty container was just submerged. The material to be tested was added to the container and the deflection of the pointer was measured after the container's motion stopped.

Tests of the first model showed the method had promise for automation. The model was simple, yet had a mechanical accuracy on the order of one or two hundred parts per million. The mechanical accuracy was limited by the knife edges of the balance, the short moment arm, and the relatively large mass of the balance arm and indicator. The ambient liquid was not insulated from the environment so accurate control was difficult to achieve. The water-surface elevation had to be adjusted carefully to a fixed point each time because the measurement was affected by variations in the displacement of the suspension wire. One final limitation of this model was the large magnitude of the container's weight compared with the sediment in low concentration samples.

The next step was to simplify operation and construction by eliminating the balance and improve accuracy by providing thermal insulation and temperature control. The appeal of a neutrally buoyant container is obvious.

A diagram of this container is shown in Fig. 2. The container held a pint sample and was neutrally buoyant when filled with ambient liquid. The device was made from 0.015-inch, tin-plated steel. It was later coated with nickel to prevent corrosion at the joints and exposed edges. The air chamber was placed high on the container to provide flotation stability and was designed to give a small excess of buoyancy so that final trimming could be done with solder. On the first model F_n, eq (8), was supplied by the beam balance and suspension wire. On the second model the indicator rod generated the balancing force. The rod was made of stainless steel tubing which was crimped and soldered at each end. Three different diameters were used in order to provide a wide concentration range. The container was floated in a well-insulated three-gallon cylinder which had wire guides to keep the container from touching the side wall. A drain in the bottom of the cylinder allowed the sample to be flushed concurrently with the ambient fluid. Once the operation had been validated, further samples were retrieved by lifting the sample container from the water bath.

THE RELATIONSHIP BETWEEN SEDIMENT WEIGHT AND ROD DISPLACEMENT

Given a neutrally buoyant container, such as the second test model, what is the relationship between the sediment weight and the rod's displacement? Let us assume that the water density inside and outside the model are equal. If we submerge the container, it soon reaches an equilibrium position with the water surface at some point y_0 on the rod; y_0 being the initial rest point position. The weight of the water displaced by the container and rod equals the weight of the container and rod. Adding a small amount of sediment will cause the container to sink to a lower equilibrium position. The weight of the water displaced by the rod after sinking a distance Δy equals the weight of the added sediment less the weight of water displaced by the sediment. Expressed mathematically,

$$\Delta y \left(\frac{\pi d^2}{4}\right) \rho_w g = w_s \left(1 - \frac{\rho_w}{\rho_s}\right) , \qquad (9)$$

where Δy is the distance the water surface has moved up the rod and d is the outside diameter of the rod. Direct application of eq (9) requires that the water surface be within range of Segment I, Fig. 2. The densities of water and sediment are ρ_w and ρ_s, respectively, w_s is the weight of sediment, and g is the gravitational acceleration. The right side may also be expressed as $m_s g(s - 1)/s$, where s is the specific gravity of the sediment. Substituting and solving for m_s:

$$m_s = \rho_w \left(\frac{s}{s-1}\right) \left(\frac{\pi d^2}{4}\right) \Delta y , \qquad (10)$$

or, $\qquad m_s = K d^2 \Delta y$.

For constant sediment and water density, the sediment mass is determined only by the displacement Δy of the rod with diameter d. Knowing the weight or mass of the original sample mixture, the concentration of the sediment may be computed if w remains constant. The second equation above shows that the deflection for a given mass can be increased by decreasing the diameter of the rod. Reducing the rod diameter to increase sensitivity makes a flotation chamber essential. The device used in this study has an attached, enclosed air chamber and an indicator rod composed of three increasingly

larger diameter tube segments. The descriptive equation is now of the form,

$$m_s = K(\Delta y_1 d_1^2 + \Delta y_2 d_2^2 + \Delta y_3 d_3^2 + \ldots) \quad ,$$

where $\Delta y = \Delta y_1 + \Delta y_2 + \Delta y_3 \ldots$, and $K = \varrho_w \left[\dfrac{\pi s}{4(s-1)}\right]$. This form of eq (10) will be used in Section VI, Results.

THE EFFECT OF VARIATIONS IN s AND ϱ_w

How much error is introduced by assuming constant sediment and water density? In routine use the rod deflections would be calibrated for assigned values of both s and ϱ_w. A converse question might then be: what are the allowable limits in s and ϱ_w for a given tolerable error in the sediment mass? Assuming that the temperature of the device and its contents are identical to that of the ambient water, the error, Δm_s, caused by perturbations in both s and ϱ_w, is defined as the partial derivative of eq (10) with respect to s and ϱ_w:

$$\Delta m_s = \Delta s \frac{\partial m_s}{\partial s} + \Delta \varrho_w \frac{\partial m_s}{\partial \varrho_w} \quad , \tag{11}$$

$$= - \left(\frac{\Delta y \pi d^2}{4}\right) \varrho_w \frac{\Delta s}{(s-1)^2} + \left(\frac{s}{s-1}\right)\left(\Delta y \frac{\pi d^2}{4}\right) \Delta \varrho_w \quad ,$$

$$= \frac{\pi d^2}{4} \Delta y \left[\Delta \varrho_w \frac{s}{s-1} - \Delta s \frac{\varrho_w}{(s-1)^2}\right] \quad . \tag{12}$$

The mass error Δm_s represents the amount of sediment that must be added to or subtracted from the container to maintain Δy constant with respect to small changes in s or ϱ_w. The relative error is the ratio of eqs (12) and (10):

$$\frac{\Delta m_s}{m_s} = \frac{\Delta \varrho_w}{\varrho_w} - \frac{\Delta s}{s(s-1)} \tag{13}$$

Some values of relative error may now be estimated. For example, Bird, et al (Ref. 4), listed the common minerals comprising the bulk of suspended solids in normal streams and concluded that "the mean density of suspended solids is about 2.65 g/cm^3 and that deviations from this value greater than $\pm$ 0.05 g/cm^3 should be rare." The density of quartz is

2.65 g/cm^3 and suspended sediment is commonly assumed to have this average value (Ref. 1, p. 27). Substituting $s = 2.65$ and $\Delta s = 0.05$ into eq (13) indicates a relative error due to a variation of $\pm$ 0.05 units in s alone is about one percent. The assumption of a constant value for the average specific gravity of the sediment at a particular site is thus acceptable for most analyses.

The first term in eq (13) accounts for the effect of a variation of the density of the ambient liquid. For this term to have a one percent magnitude, the temperature would have to change nearly fifty degrees Celsius or the dissolved solids would have to increase by ten thousand milligrams per liter. Neither event is likely although some natural streams do have large concentrations of dissolved solids during low flows.

It should be recognized that eq (13) assumes identical temperatures inside and outside of the container and that the temperatures do not change during the measurement. The final assumption is that the initial rest point of the container does not change. In practice these conditions are not easily satisfied. The sample is an aqueous suspension usually at a temperature different than the ambient water. The sample temperature may be adjusted artificially or be allowed to approach the ambient temperature over a long period of time. Allowing the sample to remain for long periods in a water bath can be inconvenient, and heating or cooling the sample causes problems of mixing, air entrainment, and temperature sensing. Because the air chamber buoyancy is a function of water densities inside and outside the container, any uncertainty of inside or outside density is reflected directly as an uncertainty in Δy. The following section will deal with these sources of error.

IV. ANALYSIS OF TEMPERATURE EFFECTS

One may now consider the effect of a temperature variation on the readings obtained from the second test model (Fig. 2). The answer is complex because we are dealing with a device intended to detect minute differences in density and a device which has a temperature-sensitive air chamber. As mentioned earlier, the device was fabricated from thin sheet metal which, in turn, has a thermal expansion effect. Therefore, the correction for a slight increase in temperature above calibrated point should include allowances for the following:
1) increased buoyancy of the air chamber because of:
   a) increased air pressure within the chamber.
   b) thermal expansion of metal walls of air chamber.
2) decreased buoyancy of the sample container because of expansion of the liquid and thus a reduction in liquid density.

The following analysis of the effects of temperature variation makes use of several assumptions. One is that the change in sediment density is negligible over a small temperature range. Another assumption is that corrections calculated for the container at its zero point (no sediment in the container) are applicable when the rod is partially submerged. Other assumptions will be stated as we continue the analysis.

The net upward force required to support the submerged container with volume V_c and air chamber volume V_a is:

$$F = g(-V_a \varrho_w + V_c \Delta \varrho + m_c) \quad , \quad (14)$$

where $\Delta\varrho$ is the difference in density, $\varrho_{wc} - \varrho_w$, between liquid densities inside and outside the container and m_c is the container mass. The change in force with respect to temperature is obtained by differentiation:

$$\frac{\Delta F}{\Delta T} = g \left[-\varrho_w \left(\frac{d\ V_a}{dT}\right) - V_a \left(\frac{d\ \varrho_w}{dT}\right) + V_c \left(\frac{d\Delta\varrho}{dT}\right) + \Delta\varrho\left(\frac{d\ V_c}{dT}\right) \right] .$$

Because the air chamber is elastic, the first term on the right side must be expanded:

$$\frac{\Delta F}{\Delta T} = g\left[-\frac{\rho_w}{\partial P}\frac{\partial V_a}{\partial T_o}\frac{d\ P}{dT_o} - \frac{\rho_w}{\partial T_o}\frac{\partial V_a}{\partial T_o} - \frac{V_a}{dT_o}\frac{d\ \rho_w}{dT_o} + \frac{V_c}{dT_o}\frac{d\Delta\rho}{dT_o} + \frac{\Delta\rho dV_c}{dT_o}\right], \quad (15)$$

where P is the air pressure in the chamber and T_o refers to the temperature outside the container. The terms within the parentheses describe the effects listed in the previous paragraph. In order to minimize the change of buoyancy with a change in temperature, we must minimize the parenthetical terms. This may be accomplished by reducing each term and by balancing positive terms against negative terms. In this regard let us now examine each term more closely.

The first term, $\rho_w\ \frac{\partial V_a}{\partial P}\frac{dP}{dT}$, describes the increase in buoyancy with a slight increase in pressure caused by increased temperature in the air chamber. The coefficient of expansion of air between 0° and 100^oC at standard atmospheric pressure is 3.67E-3, where (E-3) is a convenient notation for an exponent to base ten; ie, (E-3) $= 10^{-3} = 0.001$. Therefore, at standard atmospheric pressure of 1.013E6 dynes per square centimeter, dP/dT $=$ 3.72E3 dynes/cm^2/^{o}C. Let us now assume that the air chamber may be described by a spherical shell of equivalent volume. This is an approximation which yields a minimum ΔV_a limited by tension in the shell. The probable error in using this approximation should be negligible because the three conical surfaces describing the air chamber do approximate two spherical surfaces. Increasing the pressure within a sphere tends to cause an increase in the radius, a, given (Ref. 5) by:

$$\Delta a = \frac{\Delta P\ a^2\ (1-\nu)}{2\ M_e h} , \quad (16)$$

where M_e is the modulus of elasticity,
    h is the shell thickness,
    ν is Poisson's ratio (unit transverse contraction per unit axial elongation), and
    a is positive with outward displacement.

Differentiating the equation for the volume of a sphere, $V = 4/3\pi a^3$, we find that

$$\Delta V_a \cong 4\pi a^2 \ \Delta a$$

which becomes, after substituting eq (16) and rearranging:

$$\frac{\Delta V_a}{\Delta P} = \frac{2\pi a^4 \ (1-\nu)}{M_e h} \qquad . \qquad (17)$$

The air chamber volume was 187 cubic centimeters, so that an equivalent sphere's radius would be 3.55 cm. Substituting a = 3.55 cm, M_e = 2.07E12 dynes/cm^2, h = 0.0381 cm (0.015 in), and ν = 0.3 into eq (17) we obtain $\Delta V_a/\Delta P$ = 4.08E-8 cm^3/dyne/cm^2. Combining this with the previously determined value for dP/dT, we arrive at an estimate of the first term:

$$\varrho_w \ \frac{\partial V_a}{\partial P} \ \frac{dP}{dT} = 1.52E-4\varrho_w \ gm/^\circ C \qquad . \qquad (18)$$

The second term of eq (15), $\varrho_w dV_a/dT$, describes the increase in buoyancy caused by a slight increase in temperature which expands the metal walls. Two further assumptions are necessary for this calculation. The first is that the container shell may be treated as a solid volume, and any expansion expands the whole volume. The second assumption is that the internal stresses in the shell induced by a temperature change are negligible, so that the volume change is given by $\Delta V_a = \eta_v V_a \ \Delta T$. The volume thermal coefficient, η_v, is about three times the linear coefficient or 36 E-6 per $^\circ C$ for the metal shell (Ref. 6). The second term may be written:

$$\varrho_w \ \frac{dV_a}{dT} = \varrho_w \eta_v V_a = 6.73E-3 \ \varrho_w gm/^\circ C. \qquad (19)$$

The third term of eq (15), $V_a \ d\varrho_w/dT$, describes the change in buoyancy caused by expansion of the liquid. The change in density per degree is plotted in Fig. 3B. At 4°C, $d\varrho_w/dT$ is zero but increases to about - 2.0E-4 at 20°C. The third term, then, varies with temperature. Typical values for the product $V_a \ d\varrho_w/dT$ are -0.0168, -0.0374, and -0.0561 gm/$^\circ$C for temperatures of 10, 20, and 30°C, respectively.

The term $V_c d\Delta\varrho/dT$ is also temperature dependent and $\Delta\varrho dVc/dT = \Delta\varrho Vc\eta_v$. For now we will assume that the temperatures inside and outside the container are identical, so that $\Delta\varrho = 0$. The fourth and fifth terms then vanish.

Substituting all of the above equivalent terms into eq (15), we arrive at the following:

$$\frac{\Delta F}{\Delta T} = g \left[-3.72E3\varrho_w \frac{2\pi a^4}{M_e h} \frac{(1-\nu)}{} + \varrho_w \eta_v V_a + V_a \frac{d\varrho_w}{dT} \right] . \qquad (20)$$

For the device used in our studies, eq (20) becomes

$$\frac{\Delta F}{\Delta T} = g \left(-1.52E{-}4\varrho_w - 6.73E{-}3\varrho_w - 187 \frac{d\varrho_w}{dT} \right) . \qquad (21)$$

Collecting terms and noting that ϱ_w varies less than a quarter of a percent from 0.998 between 4° and $30^\circ C$, we find

$$-\frac{\Delta F}{\Delta T} = g \left(-6.88E{-}3 - 187 \frac{d\varrho_w}{dT} \right) , \qquad (22)$$

Obviously ΔF will vanish when the two terms within the parentheses are of equal magnitude. With the given air chamber, the balance point is reached at a temperature such that $V_a d\varrho_w/dT = 6.88E{-}3$, or when $d\varrho/dT = 36.7E{-}6$. This occurs at a temperature of about $6.4^\circ C$.

It is possible to design the apparatus for other temperature-insensitive balance points, where $\Delta F/\Delta T$ is zero. Each balance point is a compromise between the volume of the air chamber, the mechanical and thermal properties of the material used, and the temperature-density relation of the ambient liquid.

Eq (22) may be used to estimate corrections for the instrument when the ambient temperature has drifted slightly from the calibrated value. For ease in application the correction in F can be converted to equivalent sediment mass by noting that ΔF in eq (22), is equivalent to ΔF_n obtained from eq (8),

$$\Delta F_n = g \Delta m_s \left(1 - \frac{\varrho_w}{\varrho_s} \right) = \Delta F \qquad .$$

Making appropriate substitutions, eq (22) becomes

$$\frac{\Delta m_s}{\Delta T} \approx -11.1E\text{-}3 - 301 \frac{d\varrho_w}{dT} \qquad . \qquad (23)$$

Fig. 3 shows $d\varrho_w/dT$ is $-210E\text{-}6$ and $\Delta m_s/\Delta T$ is 0.052 for an ambient temperature of 20°C. In other words, if the initial rest point, y_o, had been obtained when the ambient liquid was 20°C and if the ambient temperature had drifted, say, to 20.5°C, then about 26 milligrams (52 mg/$^\circ$C times 0.5°C) would have to be subtracted from the calculated mass. Another way of interpreting the correction value is that at this temperature there is an error of 3.05 mg per tenth of a degree. At low sediment concentrations, an uncorrected 3 mg error would have a large effect on the calculated concentration. Empirical results plotted in Fig. 5 show a shift from -0.29 to -0.4 when the temperature changed from 20°C to 22°C. This corresponds to 0.055 gm/$^\circ$C which within the limits of experimental error verifies results from eq (23).

Continuing the analysis, let us now suppose that the device is filled with water at a temperature T_i which is slightly colder (more dense) than the ambient water at temperature T_o. The denser water must be inside to prevent circulation currents. We will further assume that the container is also at temperature T_o. If only T_i is allowed to vary, eq (14) becomes

$$\Delta F = gV_c\Delta\varrho_{wc} \qquad , \qquad (24)$$

with ϱ_{wc} the only temperature dependent variable. Eq (24) becomes, after differentiation,

$$\frac{\Delta F}{\Delta T_i} \approx g\, V_c\, \frac{d\varrho_{wc}}{dT_i} \qquad . \qquad (25)$$

Again employing eq (8) to convert ΔF to an equivalent Δm_s we obtain

$$\frac{\Delta m_s}{\Delta T_i} \approx \frac{V_c \dfrac{\partial \varrho_{wc}}{\partial T_i}}{1 - \dfrac{\varrho_w}{\varrho_s}} \qquad . \qquad (26)$$

Substituting nominal values for V_c, ϱ_w, and ϱ_s, we obtain

$$\frac{\Delta m_s}{\Delta T} \doteq 748 \frac{\partial \varrho_{wc}}{\partial T_i}$$

Eq (26) yields corrections of -0.067, -0.15, and -0.224 gm/°C at ambient temperatures of 10°, 20°, and 30°C, respectively. This translates into a correction of 15 mg per tenth of a degree difference at 20°C, for instance. Again, at low sediment concentrations this large an uncorrected error would lead to large errors in calculated concentrations. However, the samples could be left in the ambient liquid a suitable length of time for water inside the container to warm up to ambient. No operator time is required for this procedure and it should not add too much complexity to an automated apparatus.

From the foregoing analysis, it is evident that temperature constancy is of utmost importance. A shift in the overall operating temperature will produce errors of about 52 mg/°C. A mismatch between temperatures inside and outside the container is more serious and will produce errors in the order of 150 mg/°C. The ratio of these errors is of the same order of magnitude as the ratio between volume of the air chamber and the volume of the sample chamber. This relationship exists because a change in the overall operating temperature produces a change in buoyant force produced by the air chamber. A change in differential temperature from inside to outside affects the gravitational force exerted on the contents of the entire sample chamber.

V. TEST PROGRAM

The purpose of the test program was to verify the mathematical model derived in the previous sections.

Mechanical Operation

The first part of the test program on the final test model was conducted at constant ambient temperature. The goal was to determine the limitations of the method without the influence of changing temperature.

The insulated support container was filled with tap water mixed slightly warmer than room temperature. A heater and controller assembly maintained the slightly elevated temperature. A mechanical mixer distributed the heat evenly, but was turned off during a measurement. Samples were poured into the cup after preparation and the cup placed carefully into the ambient liquid.

The distance from the water surface to the top of the rod was measured to the nearest hundredth of an inch. The accuracy was on the order of two or three hundredths. Later measurements were read to the nearest half tenth (0.05 in). The half-tenth reading gave consistency between observers and represented an economically obtainable readout increment for automatic measuring equipment.

Temperature Variation

The effect of temperature was considered in two parts. First was the effect due to a temperature difference between the sample and ambient fluid. The second was the particular temperature range for the ambient: would a high or low range give optimal results?

The first part of the problem was attacked from two directions. The rate of temperature change with time was obtained for the case where the inside water was initially a few degrees cooler than the ambient water. The time required to reach a near equilibrium temperature was often half an hour or more. Please note also that the sample will almost always be lesser volume than the container volume, and will be brought up to full container volume with ambient water. The admixture of ambient water with the sample will reduce the time required to reach ambient temperature.

The second part of the problem was largely a matter of defining optimum conditions. Operation with the ambient liquid near freezing intuitively was indicated, but would it be difficult to maintain a temperature near freezing? Would the analytical procedure be more difficult or less accurate?

Tests were made with the ambient liquid at two temperatures: near room temperature and near freezing. At both temperatures, the water in the container was at the surrounding liquid's temperature. The cold ambient temperature was easily maintained with snow and ice. There was no particular difficulty in maintaining the chilled temperature and the analytical procedure was unchanged except for the time allowed for the sample water to cool to ambient.

VI. RESULTS

This section of the report deals with the results of tests on
the second model, shown in Fig. 2. The first test model was
considered useful only for orientation and problem definition.

The results of the initial tests on the second model are summa-
rized in Fig. 4. The plotted points include data obtained at two
different times and temperatures. A standard error of estimate
of about 75 mg/L was obtained from empirical data plotted in Fig. 4.
The errors appeared to be spread uniformly over the entire range
of concentrations; that is the errors did not seem to change with
concentration.

Sensitivity

The slope of each line segment in Fig. 4 is proportional to
the sensitivity of the instrument when the water surface is at
that segment. The largest diameter rod segment is the least
sensitive, and the smallest rod segment is the most sensitive.
Sensitivity will be defined as the displacement per unit mass.
Sensitivity may be calculated from eq (9):

$$\frac{\Delta y}{m_s} = \left(\frac{4}{\pi d^2}\right)\left(\frac{\varrho_s - \varrho_w}{\varrho_s \, \varrho_w}\right) \qquad . \qquad (27)$$

In the cgs system of units, $\varrho_s = 2.65$ gm/cc and $\varrho_w = 0.998$ gm/cc
near room temperature. Substituting these values into eq (27):

$$\frac{\Delta y}{m_s} = 0.795 \, d^{-2} \ \text{cm/gm} \qquad , \qquad (28)$$

where the diameter, d, is expressed in centimeters. For the three
rod segments used in this model, the following sensitivities were
computed:

TABLE I

Rod Segment	Diameter	Sensitivity
I	0.059 in (0.150 cm)	13.9 in/gm (35.2 cm/gm)
II	.083 (.211)	7.01 (17.8)
III	.148 (.376)	2.21 (5.61)

Obviously, any reasonable sensitivity may be obtained by selecting
the proper rod diameter. The rod diameters used on this model were
chosen within physical restraints set by available rod diameters to
give about the same resolution within each weight range when measuring
displacement to the nearest 0.05 in (0.127 cm). The theoretical values
in Table I were used to compute quantities of sediment and equivalent
concentrations shown in Fig. 2. Empirical data shown in Fig. 4 showed
close agreement.

The computation of sediment mass becomes more difficult when the
second or third rod is immersed. The mass is no longer the simple
product of a constant coefficient and the net displacement, Δy. This
still suffices for mass in the range of segment I but will not work for
masses in the range of segments II and III. Therefore, let us now
exchange our net displacement parameter, Δy, for net protrusion distance,
Δh, which we shall define as $h_o - h$, which is the change in protrusion
length before and after sediment has been added. Fig. 2 shows these
terms graphically. The change in displacement, Δy, is identical to the
change in protrusion distance, Δh, but we are able to measure more easily
the protrusion distance than displacement, the distance below the water
surface. The initial rest point with no sediment present, h_o, must be
obtained because it will vary with the density of the ambient fluid
which in turn varies with temperature and to a slight degree with dis-
solved solids as well as fine suspended matter possibly lost to the
water from prior samples.

The calculation of sediment mass is therefore a function of h and
h_o:

Rod Segment	h, inches	m_s, in grams
I	$8.30 \geq h > 6.00$	$0.0719 (h_o - h)$
II	$6.00 \geq h > 3.10$	$0.142 (6.00 - h)$ $+ 0.0719 (h_o - 6.00)$
III	$3.10 \geq h \geq 0.00$	$0.0719 (h_o - 6.00)$ $+ 0.412 + 0.452 (3.10 - h)$

It is possible to use the above equations to compute the error in
the mass owing to the increment of h chosen: 0.05 in, as previously
mentioned. Assuming h is in error by half this amount or $\pm$ .025 in
and that the water surface is in the mid-range of each segment, the

absolute error and the relative error, assuming a 0.400 liter sample suspension, are shown below:

Rod Segment	Error owing to $\pm$ .025 in. error in h	
	Absolute	Relative
I	0.0018 gm	2.2 per cent
II	.0035	.9
III	.0113	.9

The relative error in this case was defined as the ratio of the absolute error to the calculated mass at the mid-range of that particular segment.

Of course, along with the high accuracy (low error) of the smallest diameter rod goes the limitation on range. That is, in order to measure a convenient range of concentration, the smallest rod diameter would have to be quite long and therefore unwieldy. For example, if one wished to measure up to 5,000 mg/L (2.0 gm in 0.4 liter) using a single rod of 0.059-in diameter, the rod would have to be over two feet long. The maximum amounts of sediment measurable by the three rods of the second test model are 140, 553, 1950 mg, respectively.

Effect of Temperature

The control of sample and ambient fluid temperatures is obviously essential to this method. Unless the air chamber is at the calibration temperature, the chamber will expand or contract, and the rest point of the container will be modified slightly. Fig. 5 shows the temperature effect on the rest point, converted to an equivalent weight of sediment, with equal temperatures in and out of the container. The curve is similar to the variation of water density with temperature as shown in Fig. 3. An important difference, however, is the broadened hump which has been shifted to the right about 3.3°C. The theoretical point of zero slope, Fig. 5, agrees with that obtained empirically. In physical terms the shift in the inflection point is caused by a combination of two factors. As the temperature drops the liquid density increases, reaches a maximum at 4°C, then begins to decrease as temperature

approaches the freezing point. If the air chamber volume had remained
constant, Fig. 5 would have duplicated Fig. 3a. Lowering the temper-
ature from 20° to $0^\circ C$ would have caused the container to first rise,
reach a peak at $4^\circ C$, then settle slightly as the temperature approached
freezing. Operating in conjunction with density changes is the steady
contraction of the air chamber as the temperature drops. If fluid
density had remained constant the displacement would have steadily
decreased with a lowering in temperature, and as a result the container
would have settled deeper into the liquid.

When the sample temperature differs from ambient a correction must
be applied to the computed mass. The correction chart would be
determined individually for each instrument because of variations
in air chamber volume, weight, and length of indicator rod. There is
also some difficulty caused by convection currents and thermal
gradients which makes the use of a chart difficult if temperature
differences exceed more than one-half degree.

The results of the study of temperature effects therefore point
to operation of the device in the temperature range from six to eight
degrees Celsius, primarily because of the simple control requirements
needed to maintain the ambient fluid in this range. Adequate temper-
ature control eliminates the need for a temperature correction.

Other Sources of Error

Air bubbles clinging to the container below water and water droplets
clinging to the rod are the next two most important sources of error.
Both are reduced by using clean, degassed water for the ambient fluid.
A small amount of surfactant will reduce surface tension; and tapping
the container lightly will prevent clinging by droplets or bubbles.

Loss of sample through the top of the container is minimized by
designing a narrow opening in the top and by maintaining the sample
and ambient fluids at nearly the same temperature. The inside temper-
ature should be slightly cooler than ambient.

Minor errors due to non-uniform indicator rod diameter or changes
in volume of the container are eliminated by calibration of the indiv-
idual instrument.

Finally, disturbances of the container preceding and during the

displacement measurement will contribute errors. A long rest period would eliminate this type of error but conflicts with the need for a short rest period to minimize convection losses and expedite sample measurements.

The operational use of the instrument without a correction chart was tested with prepared samples of sands, clays, and dissolved salt. Measurements were made only after sample temperatures reached ambient. The results of the test are given in Fig. 6. Of the 35 samples, about a quarter of the computed weights were more than five percent in error. Five of the computed weights were more than ten percent in error. The results are acceptable considering that most of the samples were variations of worst-case samples; that is, they were samples of material fine enough to be carried out of the container by small convection currents.

VII. CONCLUSIONS

The test model was a neutrally buoyant container with an indicator rod on top. The tests were arranged to determine the feasibility of this approach for manual operation and for possible automation. Measurement of the height of the indicator rod was easier technically and less expensive than direct weighing of the container under water.

Within the limitations of the investigation, the following conclusions were obtained:

1. Of the various sources of error, temperature variations are the largest. Variations must not exceed approximately $\pm 0.5^{\circ}C$. If temperature constraints can be met, the bulk density measuring system is sufficiently accurate for routine rapid sediment concentration measurements, provided that the concentrations exceed 1000 mg/liter and that the volume of suspension exceeds 0.4 liter.
2. Temperature control can be relaxed slightly if the system is operated between $4^{\circ}C$ and $10^{\circ}C$. Operation at low temperature improves accuracy and is possible with commercial refrigeration equipment.
3. Careful design and operation can minimize the effects of air bubbles, surface tension, and water droplets (on the indicator rod).

4. Empirical results verified the mathematical model within the limits of experimental error.

The measurement was termed rapid in reference to the short time needed to make the measurement. The time necessary to equalize sample and ambient temperatures is not counted, because even with a manual measurement, temperature equalization does not require an operator's attention.

An expected probable error on the order of 50 mg/liter was found with the relatively crude apparatus used in this study. Further mechanical refinements and closer temperature control should reduce this figure.

Any future investigations of this approach should consider: a low density device to eliminate or reduce the volume of the air chamber, and a uniformly tapered rod to eliminate surface tension effects at the segment junctions. The mathematical model and error analysis should prove helpful in evaluating modifications. Where the accuracy of the present model is adequate, future study should be toward automating or refining the readout system, the application of automatic direct weighing of the container, and an automatic sample transport system. Further means of eliminating air bubbles should also be considered at that time.

LIST OF REFERENCES

1. Inter-Agency Committee on Water Resources; A study of methods
    used in measurement and analysis of sediment loads in streams.
    Report No. 4, Methods of analyzing sediment samples, 1941.

2. Ziegler, C. A., Papadopoulas, J., and Sellers, B.; Radioisotope
    gauge for monitoring suspended sediment in rivers and streams:
    Internatl. Jour. Applied Radiation and Isotopes, 1967, v. 18,
    p. 585-593.

3. Guy, H. P.; Laboratory theory and methods for sediment analysis,
    Techniques of water-resources investigations of the U. S.
    Geological Survey, bk. 5, ch. C1, 1969, p. 3.

4. Bird, L. L., Papadopoulas, J., Rex, R. A., and Ziegler, C. A.;
    Radiological mechanisms for geophysical research. Annual report
    of Parametrics, Inc. (now Panametrics, Inc.) to the Div. Isotopes
    Development, U. S. Atomic Energy Comm., 1963, p. 3.

5. Zaid, M., and Kolb, R. P.; Mechanics of materials in Mechanical
    design and systems handbook, H. A. Rothbart, ed.: New York,
    McGraw-Hill Book Co., 1964, sect. 15, p. 50.

6. Bolz, R. E., and Tuve, G. L.; Handbook of tables for applied
    engineering science: Cleveland, Ohio, Chemical Rubber Co., 1970.

7. Hodgman, C. S., ed.; Handbook of chemistry and physics,
    33rd ed.: Cleveland, Ohio, Chemical Rubber Publishing Co.,
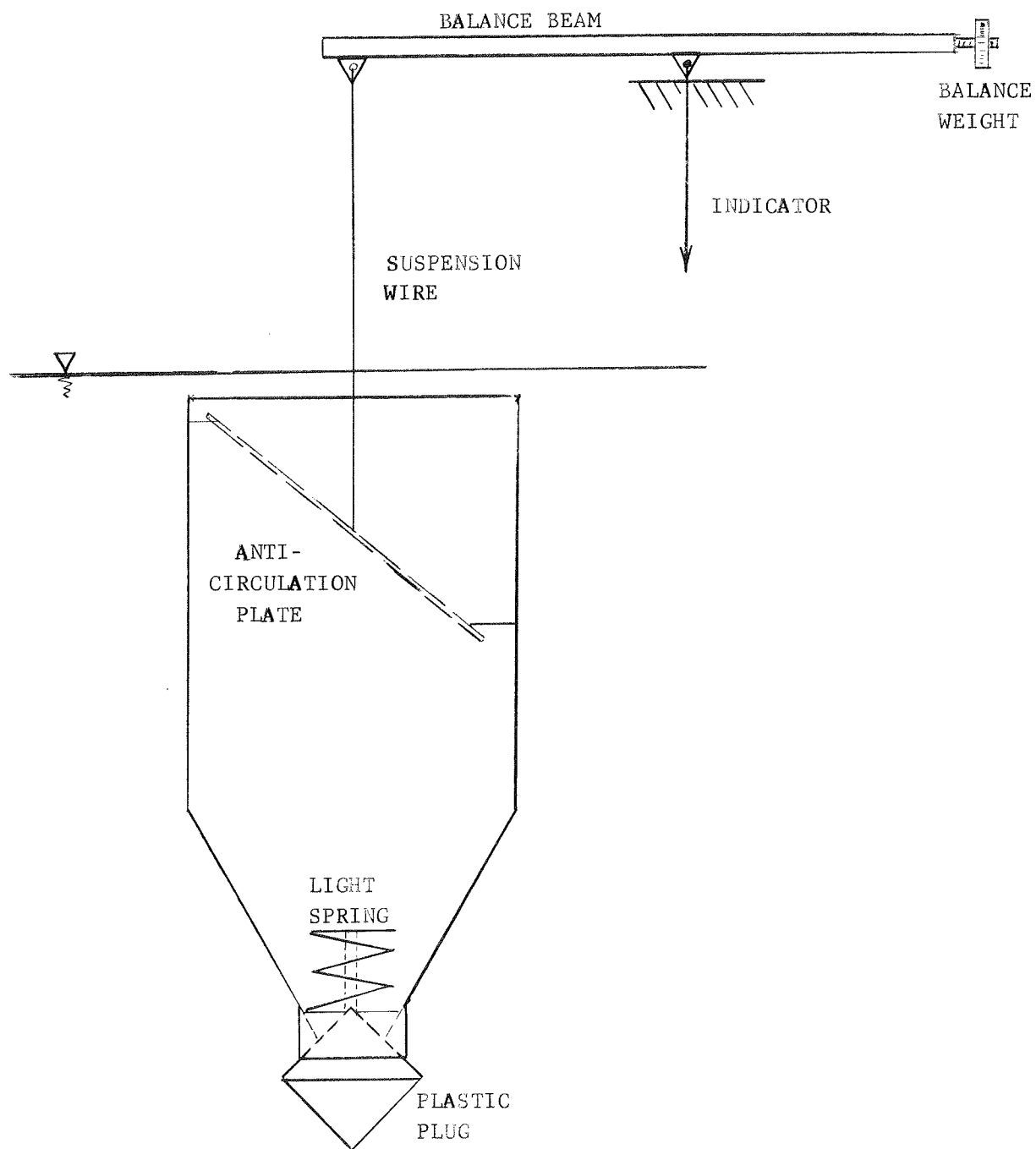   1951, p. 1790.

Fig. 1--Diagram of the preliminary test model. The anticirculation plate was added to reduce loss of sample due to recirculation. Air bubbles may escape through the open top.

1950	4880		INDICATOR
			ROD SEGMENT III
			O.D. 0.148 in (0.378 cm)
1220	3060		Length 3.1 in (7.87 cm)
			▽ WATER
			⋛ SURFACE
553	1380		
347	867		ROD SEGMENT II
			O.D. 0.083 in (0.211 cm)
			Length 2.9 in (7.37 cm)
140	350		
70	175		ROD SEGMENT I
sedi-	concen-		O.D. 0.059 in (0.15 cm)
ment	tration		Length 2.3 in (5.84 cm)
mass, (2)			
mg (1)			

(1) Quantity of
   sediment required
   to produce deflections
   at midpoint and upper
   end of each segment.
   Computed from Table I

(2) Equivalent
   concentrations,
   mg/liter based
   on 0.4 liter
   sample suspension.

(3) 0.9 cm typical

OPEN TOP

(V_a) (V_a) AIR CHAMBER (V_a) 187 ml

SAMPLE CHAMBER (V_c) 464 ml
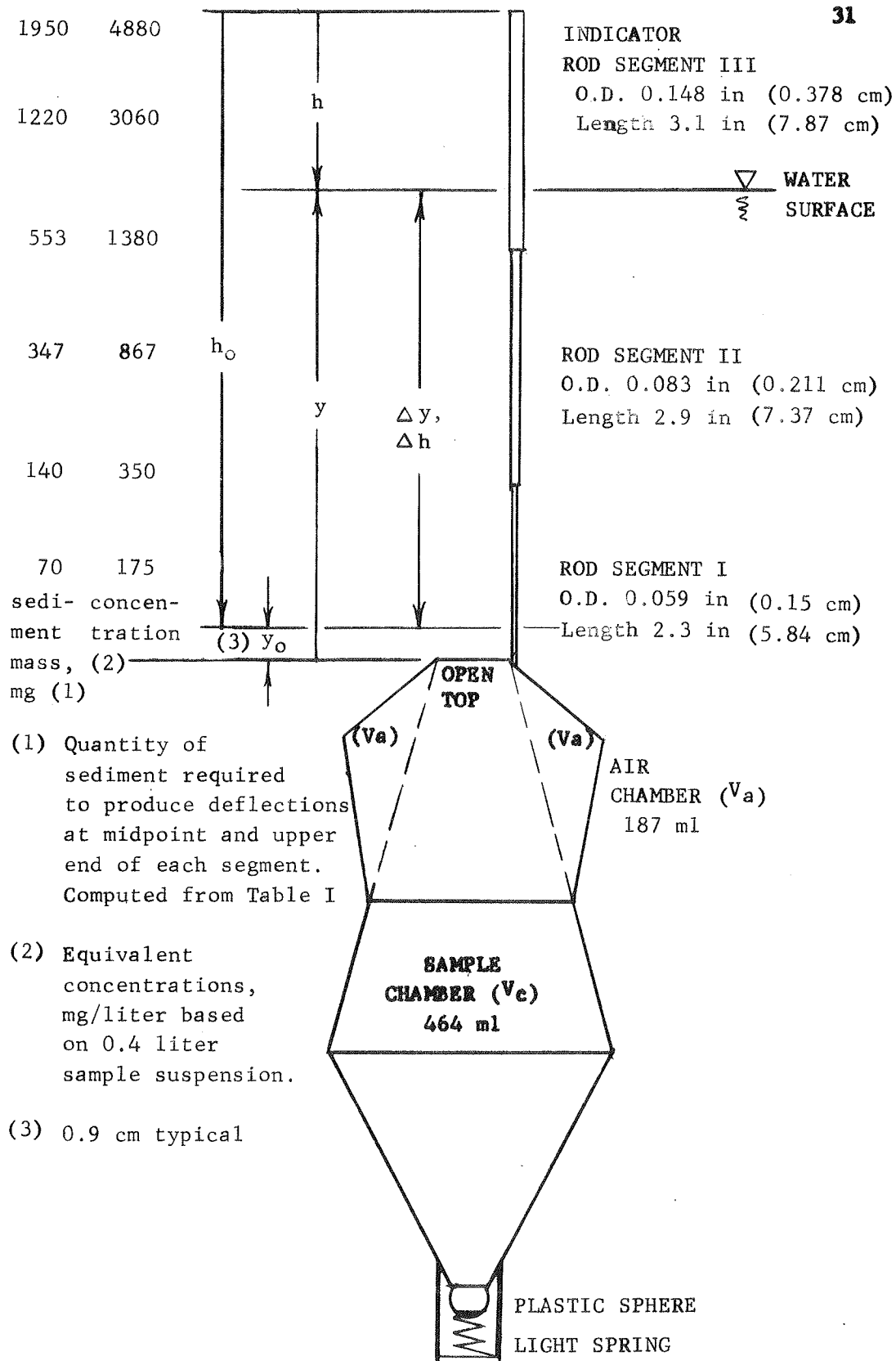
PLASTIC SPHERE
LIGHT SPRING

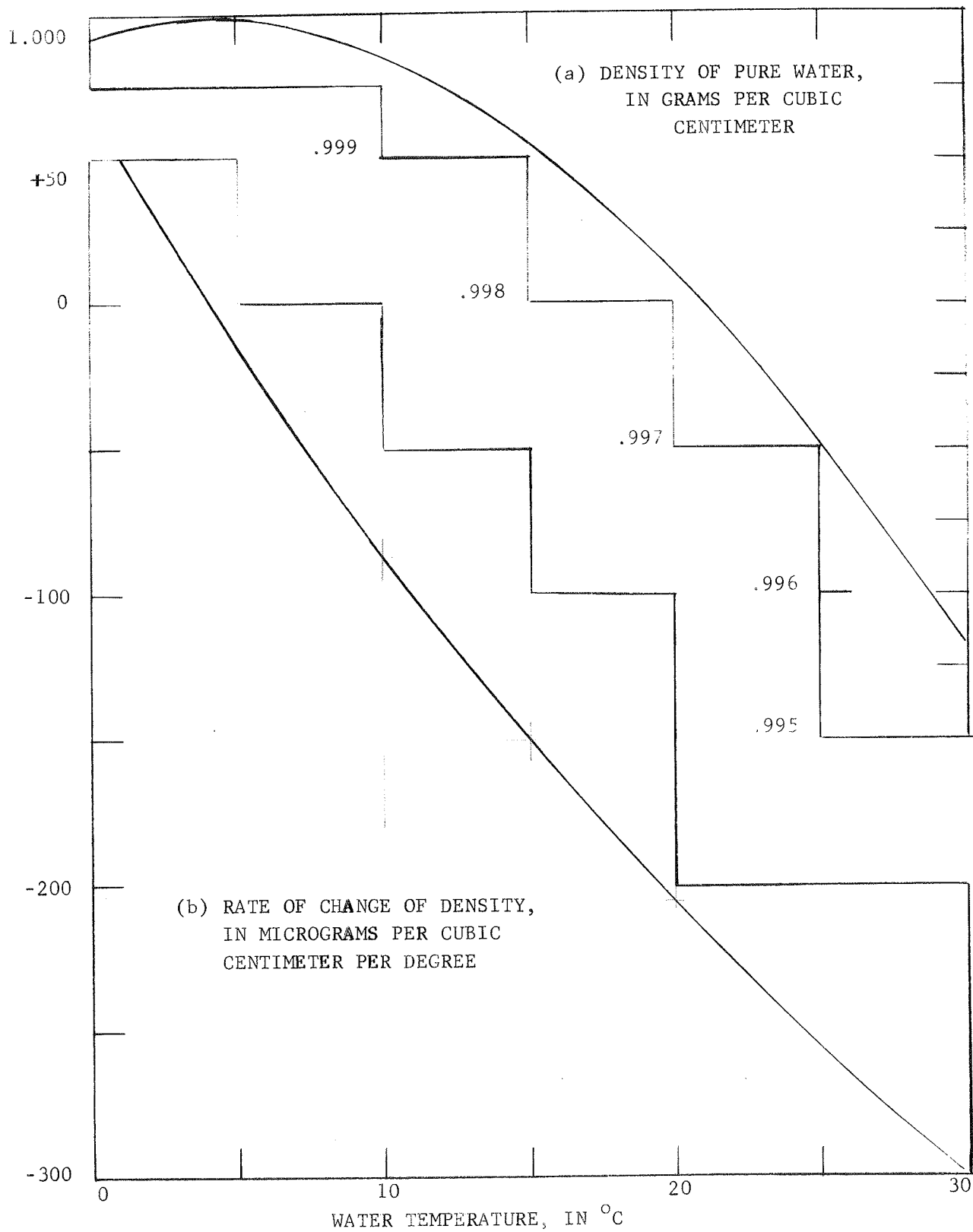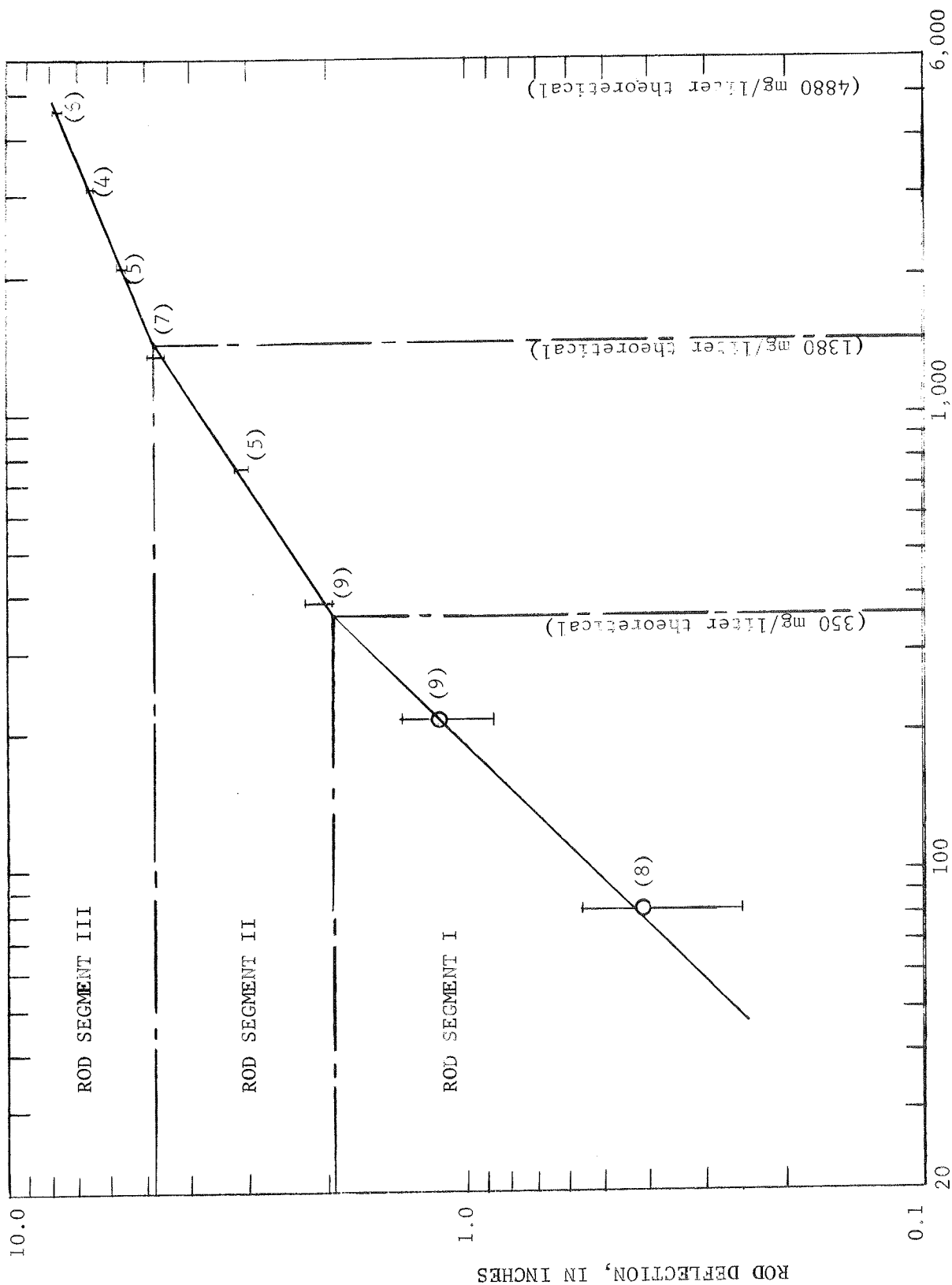Fig. 2--Diagram of final test model of the bulk density device.

Fig. 3--Graphs showing the relationship (a) between water density
and temperature; and (b) between rate of change of water
density and temperature. Density data obtained from Ref. 7.

Fig. 4--Plot of experimental values obtained with second test model. Numbers in parentheses indicate number of measurements at that concentration, and open circles indicate mean values. Equivalent sand concentrations based on 400gm sample, but were measured with copper weights (s=8.89) then adjusted to quartz sand (s=2.65) equivalence. Initial rest point, h_o=7.95 in.
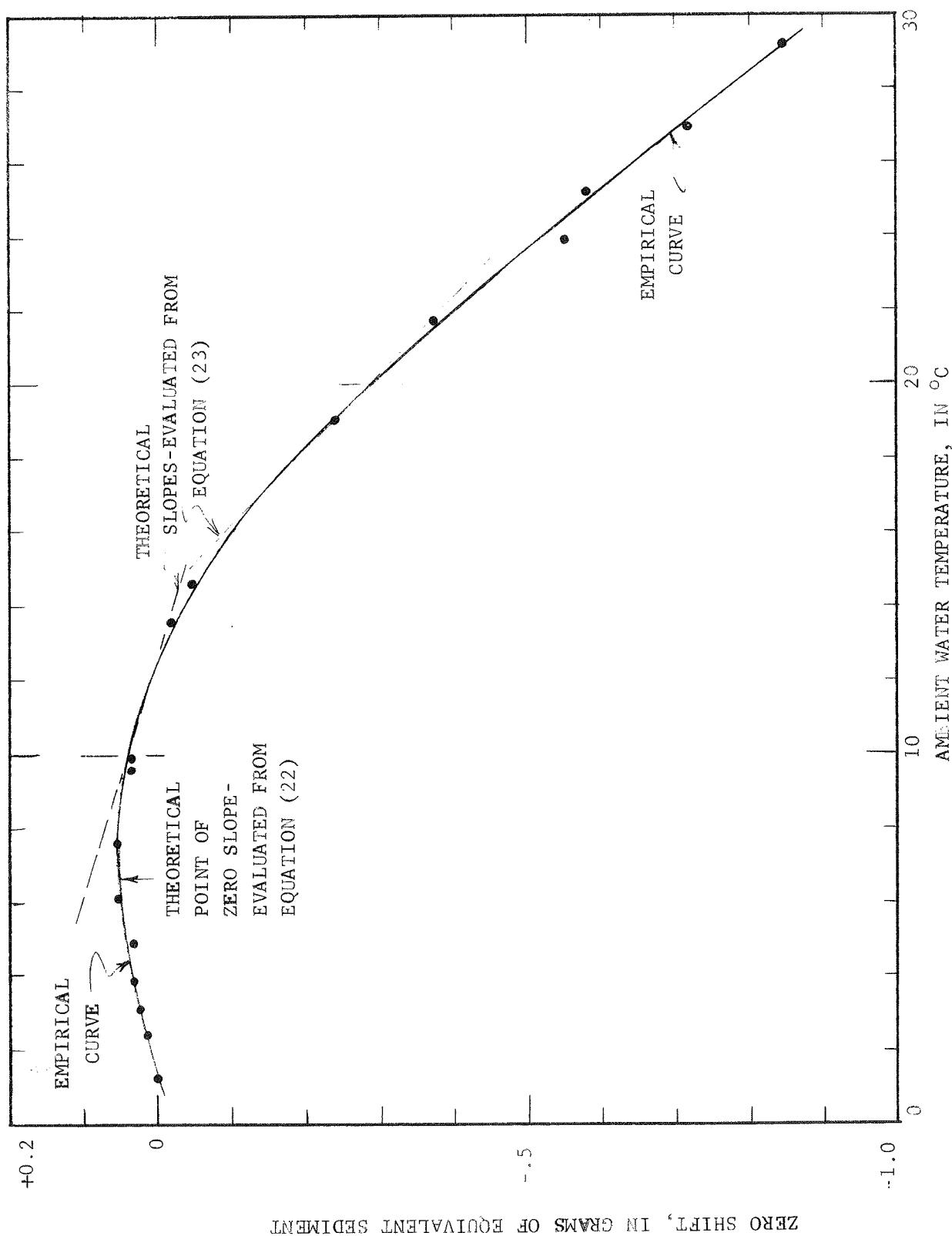
Fig. 5--Graph showing the change of the initial rest point, or zero shift, with change of ambient water temperature. There was no sediment in the container, and the temperatures inside and outside the container were identical. The curve is drawn through the experimental data points. The dashed lines are theoretical slopes computed for 10° and 20° C, respectively.

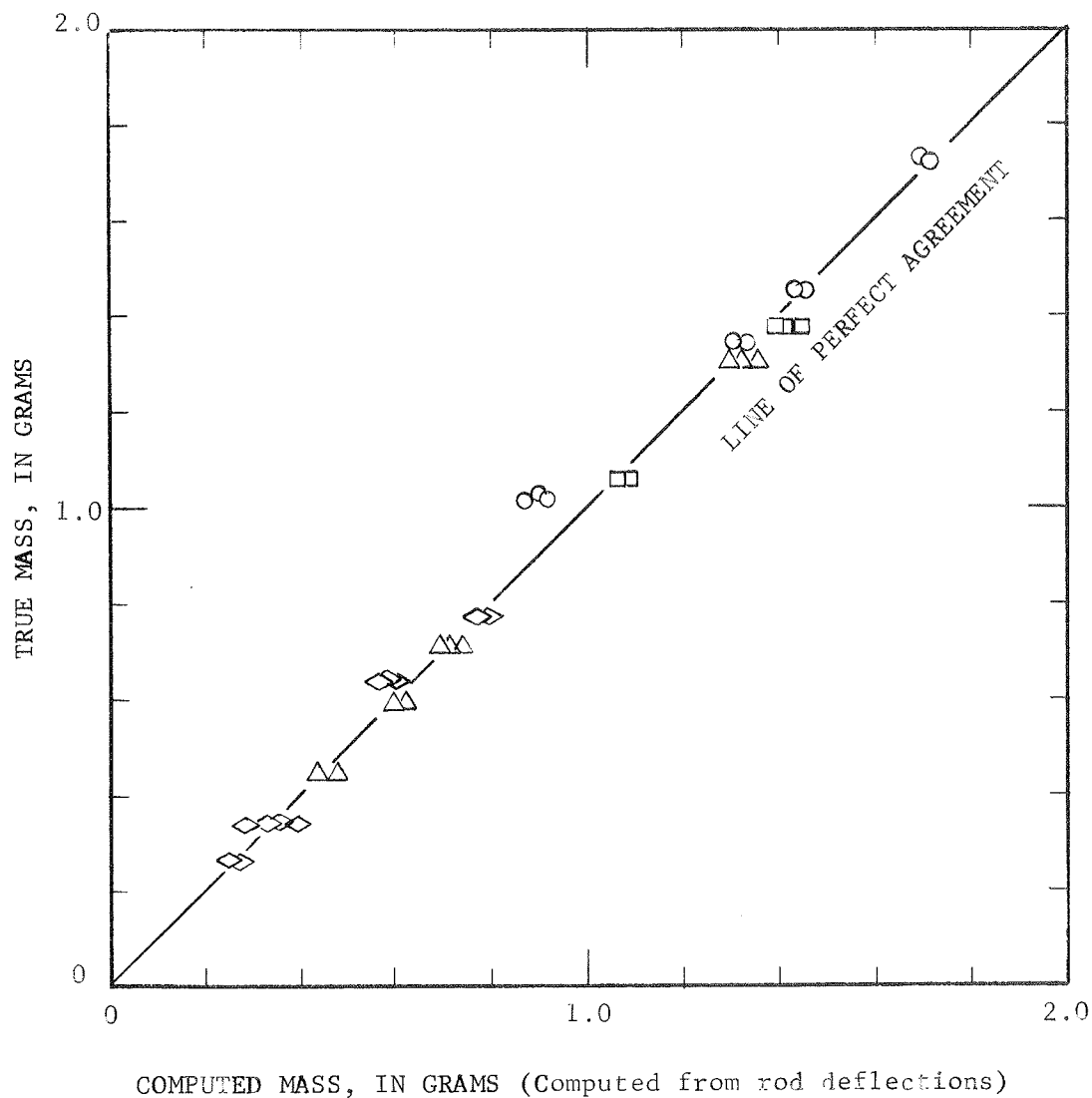COMPUTED MASS, IN GRAMS (Computed from rod deflections)

Fig. 6--Graph comparing computed sediment mass with mass of sediment
added to container. Squares indicate dissolved sodium chloride,
triangles are blasting sand (500 to 539 micron), circles are a
natural suspended-sediment from West Bitter Creek near Chickasha,
Oklahoma (60 percent silt and clay), and diamonds are from the
Rio Puerco near Bernardo, New Mexico (silt and clay). Computed
values were corrected for temperature differences: sample and
ambient temperatures varied from 28 to 30°C.