A STUDY OF METHODS USED IN

MEASUREMENT AND ANALYSIS OF SEDIMENT LOADS IN STREAMS

FEDERAL INTER-AGENCY SEDIMENTATION PROJECT

REPORT GG

DETERMINING TRUE DEPTH OF SAMPLERS SUSPENDED IN DEEP SWIFT RIVERS

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A Cooperative Project

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DETERMINING TRUE DEPTH OF SAMPLERS SUSPENDED IN DEEP, SWIFT RIVERS

By

Joseph P. Beverage

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MULTIPLY	BY	TO OBTAIN
foot (ft)	0.3048	meter (m)
foot per second (ft/s)	0.3048	meter per second (m/s)
pound, avoirdupois (lb)	453.6	gram (g)

DETERMINING TRUE DEPTH OF SAMPLERS

SUSPENDED IN DEEP, SWIFT RIVERS

by Joseph P. Beverage

ABSTRACT

A sampler lowered into flowing water by cable will be pushed downstream by the water. The analysis of the cable profile and a computer program described in this study were originally devised to assist in testing sampler design modifications. The program shows merit for use in predicting the location of a sampler or sounding weight in deep, swift rivers, and for quantifying theoretical suspended-sediment sampling errors.

The BASIC program computes wetted suspension-cable length, cable tension, downstream drift, and the vertical angle of the cable at the surface. The program requires the total depth, mean velocity, Manning's roughness, and the wire drag coefficient, CDW. The forces of the sampler are computed and then balanced on each segment of cable from the sampler to the surface. Values of CDW of almost 5 were needed to match field data for sounding weights towed in a lake.

The error in suspended-sediment concentration caused by nonlinearity of the sampler's path through the vertical was found to be quite small for a limited set of conditions.

Measurements that require accurate depth placement need to use suitable instrumentation; depth sensors, heavy samplers, and a two-transducer fathometer scheme are suggested.

INTRODUCTION

In the fall of 1975 the Federal Interagency Sedimentation Project (hereafter called the Project) received a request to design a suspended-sediment sampler for use in the Amazon River. The sampler would have to obtain representative samples to depths of about 260 ft (feet), and velocities up to about 10 ft/s (feet per second). Standard United States samplers available at the time could not satisfy the requirements. The design of a new sampler raised a number of questions including those about the beneficial effect of added sampler mass, the desirability of adding wings for negative lift, the effect of added frontal drag, and the extent of downstream drift of the sampler. A computer program was devised to help answer these questions. The program provides information on the cable length, vertical angle of penetration at the water surface, the downstream drift, and cable tension when the sampler is placed at an arbitrary true depth in the stream. A wet-line correction table that uses the vertical angle and cable length has been used by stream gagers for many years to adjust the cable length to a true sounding depth. This study will result in data that will allow the table to be verified, modified, or improved.

Purpose and Scope

This report presents the problems in determining true sampler depth, analyzes the forces on the sampler and cable, and presents a computer program that incorporates the analysis. Also given in the report are a few applications of the computer program. The program does not take into account the effect of drag during the vertical movement of the sampler or sounding weight through the water column.

Standard suspended-sediment samplers have been used in deep, swift rivers for almost four decades. The limitations and errors that arise from engineering compromises in their design have been documented by the Inter-Agency Committee on Water Rescources (IACWR) (1940; 1941a, b; 1952). This study does not examine errors arising from the sampler design but rather errors caused by the downstream deflection of the sampler as it is lowered and raised through the water column, particularly in deep rivers. This study further assumes ideal conditions--that is, the sampler is assumed to collect a time-representative subsample of the water-sediment mixture as it traverses the flow. Therefore, this study is sampler independent.

Acknowledgments

The writer wishes to thank J. V. Skinner of the Project for many fruitful discussions over many years. Discussions with Drs. H. Stefan, C. Song, and J. Killen of the University of Minnesota St. Anthony Falls Hydraulic Laboratory staff are also appreciated. Dr. Neil Coleman and R. Darden of the Agricultural Research Service Sedimentation Laboratory at Oxford, Mississippi, helped by providing some of the computer time necessary. J.C. Futrell II and coworkers at the U.S. Geological Survey Hydrologic Instrumentation Facility at Bay St. Louis, Mississippi, kindly provided the submerged weight of the one-eighth-inch cable commonly used for sampling and discharge measurements in deep, swift streams.

PROBLEMS IN DETERMINING TRUE SAMPLER DEPTH

A sampler lowered into a stream does not trace a strictly vertical path. It drifts downstream, pushed by the water's force on the sampler and suspension cable. After the sampler enters the water its position is difficult to predict. The person lowering the sampler has only two indicators for estimating the sampler's depth: the cable-length counter and the vertical-angle indicator. The cable-length counter measures the length of cable unwound from the reel, not the true depth of the sampler in flowing water. The vertical-angle indicator is used to measure the angle θ shown in figure 1. This angle is also the cable's penetration angle into the water. A calculation of sampler depth with the cable straightened at the angle of entry will position the sampler too high in the flow (position A in fig. 1). Ignoring the angle and assuming the cable is hanging straight down will position the sampler too deep (position C in fig. 1). The true depth of the sampler (position B in fig. 1) is between these values and can be determined by instrumentation.

Sounding a river for a discharge measurement involves zeroing the cable-length meter when the sounding weight is just touching the water surface, lowering the weight until it just touches the stream bed, and then raising it until the full weight is taken by the cable. The cable length, CL, and the angle of the cable with the vertical, θ, are measured at that time. The cable length will be assumed to originate at the water surface for convenience in this report. The adjustments to the length of cable from the metering point to the surface must be taken into consideration in practical cases (see Buchanan and Somers, 1980, p. 47-

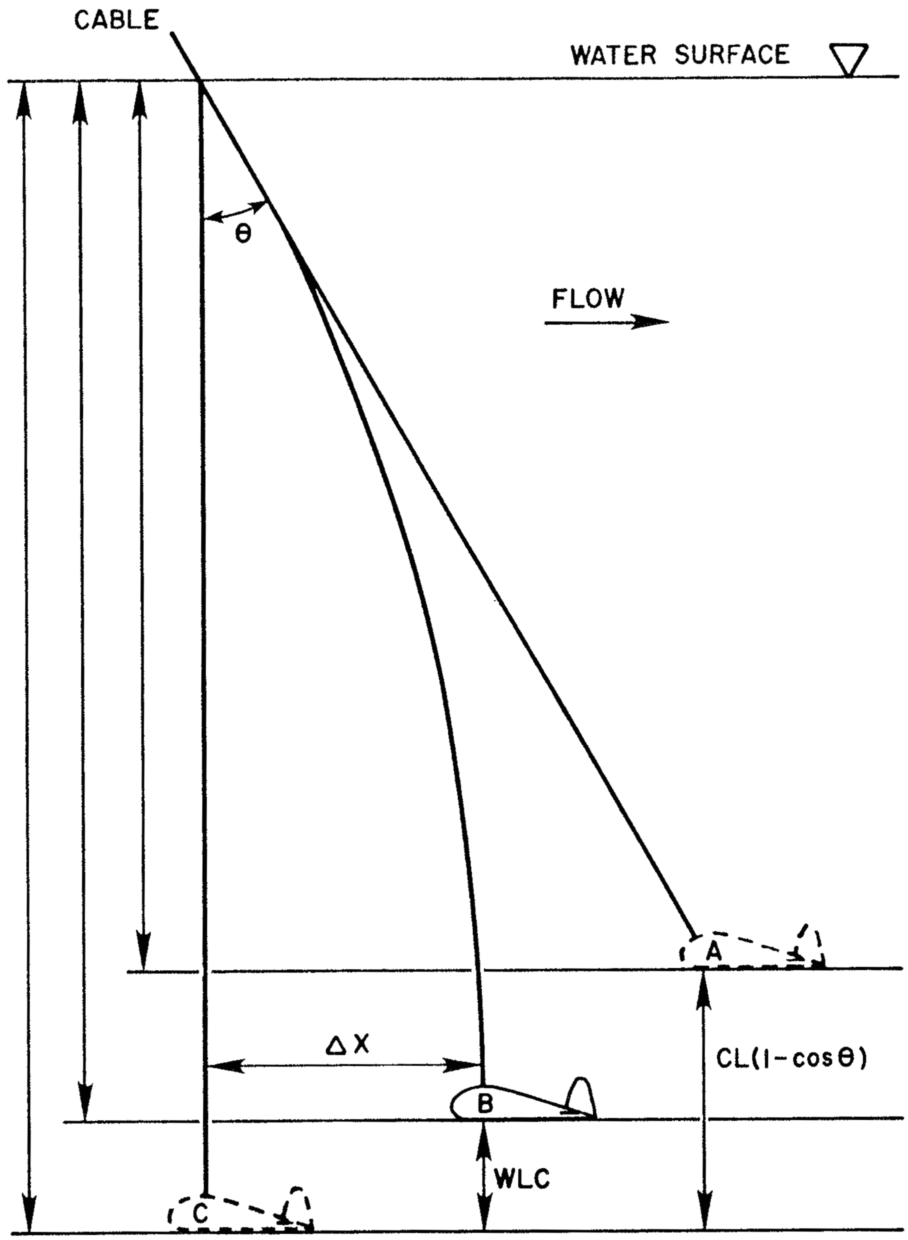

Figure 1.--A weight (B) suspended from a cable in flowing water at true depth D_t. CL is the wetted cable length and the distance to position C (in calm water). A is the position if the cable were a stiff rod at the vertical angle of penetration θ. The downstream drift of the sampler is ΔX. The wet-line correction, WLC, is the difference between CL and D_t.

53, for further explanation). The cable length still does not represent the true depth of the weight because the force of the water pushes the cable and weight downstream and upward into the flow. The forces also cause the cable to curve gradually from the angle at the surface to very nearly vertical at the sounding weight. Because the cable does not hang vertically, more cable is needed to allow the weight to rest on the bottom than would be the case if the water had been calm. This extra cable represents the error that must be corrected to find the true depth of the weight.

The usual method of determining true depth is to subtract the wet-line correction, WLC, obtained from a table (for example, Shenehon, 1900; Stevens, 1931; Corbett and others, 1957; Buchanan and Somers, 1980; Rantz and others, 1982) based on CL and the vertical angle, θ. In figure 1, WLC is the difference between CL and the true depth, D_t.

Shenehon (1900, p. 5329-5330) derived the static method of sounding the bottom by making two assumptions:

> It was found that with a properly designed sounding weight in the comparatively dead water near the bottom, P [the horizontal pressure on the weight] could safely be neglected; and that with a heavy weight and a very light wire the uplift of the current in ordinary work was small....

He also made the observation that:

> The variation of velocities from surface to bottom is so nearly the same at different stations and at

> different depths that a vertical curve may be platted that is typical. Its exact form is not essential to the validity of the results that enter into any of the soundings on which the discharge of the Niagara River depends, and no great error would result from the assumption that velocities were uniform from top to bottom.

Shenehon used a typical curve, however, and used the velocities for each tenth of depth to compute the depth-correction (WLC) table still in use at the present time. He did correct for uplift on the cable due to the current though. His table gives corrections for even angles from 2° to 36° and for even depths from 10 to 100 ft. He noted that 36° "marks the limit which is permissible" without explaining why. The 36° limiting angle was measured while sounding a depth of "nearly 70 feet" in a velocity of "at least 10 miles an hour," approximately 15 ft/s with a 600-lb (pound) weight suspended by "plow-steel wire of the highest grade, one-tenth of an inch in diameter." At this site in Niagara Gorge, his winch was on a bridge 240 ft above the water.

Stevens (1931, p. 6) pointed out that Shenehon had not disclosed his method of computing uplift of the wire due to the current. Stevens noted that at 30° and 10 ft, the tabular correction was 0.03 ft greater than a value computed without uplift. Stevens stated (p. 6) that "vertical angles greater than 36 degrees have been measured in the field..." He mentioned that three offices were using graphical extensions of the table for angles greater than 36°.

Stevens (1931, p. 8) also discussed the problem of placing a current meter at a specific depth. He concluded that the table is not strictly applicable and gives too small a correction, but that the error is probably small. However, the table must be used " as there is no other known method for placing the meter."

Corbett and others (1957, p. 44-56)[1/], in the classic reference on stream-gaging procedure, expanded on Stevens' paper, and discussed the corrections for proper depth placement of a velocity meter. Two corrections were given. The air-line correction adjusts for the sheave-to-water error, in effect shifting the sheave to the water surface. The correction is the product of the sheave-to-water distance and the exsecant of the vertical angle. An exsecant is one less than the reciprocal of the cosine of the angle, $((1/\cos\theta) - 1)$. The analysis and table for the wet-line correction used by Corbett and others is Shenehon's. The wet-line correction is "the sum of the products of each tenth of depth and the exsecants of the corresponding angles derived for each tenth of depth by means of the tangent relation of the forces acting below any point" (Corbett and others, 1957, p. 48). The tangent of the angle above any point is taken as the sum of the horizontal forces below the point divided by the sum of the vertical forces below the point. When the sounding weight is just above the bottom of the stream where

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1/ This report has been revised and expanded (Rantz and others, 1982) but the treatment of this particular subject was minimal.

the velocity is presumed negligible, the horizontal force on the sampler can be neglected.

Most attempts to devise an analytical solution generally simplify the problem by assuming a uniform velocity field. The weight of the cable often is assumed to be negligible. Landweber and Protter (1944) used both assumptions to derive a set of differential equations describing the variation of forces along the cable. The assumption of a uniform velocity fit their subject (towed marine bodies) and allowed the use of constant drag per unit length of cable projected normal to the velocity. Pode (1948) extended their work to include the cable weight and a towed body with a large negative lift-to-drag ratio. Pode took as constant the ratio of unit lengths of drag parallel and normal to the stream.

R. H. Multer (written commun., 1983) gave a solution obtained independently. He assumed a uniform velocity field but included cable weight. Multer stated that uncorrected cable length had been used instead of true depth in computing river discharges on the lower Mississippi River.

> At high flows, the Mississippi River is deep and swift, and represents a situation where the submerged cable length and depth of submergence would tend to differ extremely and have the maximum potential influence on the accurate determination of discharge.

Multer calculated that a 100-lb streamlined body would drift 70 ft downstream when tethered by a 100-ft cable in 15 ft/s flow velocity. The sampler depth would be 70 percent of the cable length.

THEORETICAL BASIS FOR THE COMPUTER PROGRAM

Drag is the term given to fluid forces acting on an object to resist its motion through a fluid. Textbooks (for example, Olson, 1968, p. 280-285) give drag as the product of an empirical drag coefficient, the affected area, A, and the dynamic pressure:

$$\text{Force} = C\ A\ (\rho V^2/2), \qquad (1)$$

where C is the appropriate drag coefficient which is given a subscript denoting the particular force: C_f for skin friction (shear) drag and C_d for form (pressure) drag. Drag is measured in pounds (of force). The coefficient is dimensionless and area is in square feet. In the last term, ρ is the fluid density in slugs per cubic foot[2/], and V is the fluid velocity in feet per second.

The forces acting on a sampler or sounding weight can be resolved into a horizontal and vertical force. The horizontal force, F_{ds}, consists of skin and form drag which are computed from equation 1 with appropriate coefficients and values. Because skin and form drag act in the same direction, they can be added. A new coefficient can be defined which incorporates the constant or slowly varying terms of equation 1:

$$K_s = (C_f\ A_{skin} + C_d\ A_{frontal})\ \rho\ /\ 2. \qquad (2)$$

2/ ρ is the density of water in pounds (mass) per cubic feet divided by the gravitational constant, 32.174 ft/s^2. In the slug-mass system, a 1 slug mass is accelerated 1 ft/s^2 by a <u>force</u> of 1 lb.

Each area is constant for a given device, of course. C_f and C_d vary slightly with the Reynolds number at normal stream values (Olson, 1968, p. 160 and 240). The density, ρ, varies slowly with temperature (Olson, 1968, p. 15), but is a constant 1.94 slugs/ft^3 between 32°F and 60°F. Equation 1 may now be expressed as

$$F_{ds} = K_s V^2. \quad (3)$$

The U.S. P-61 point-integrating suspended-sediment sampler is a standard device commonly used in the United States for sampling deep rivers. Its frontal area is 0.307 ft^2 and its outer surface area is 3.55 ft^2. It is shaped for minimum drag. A nominal drag coefficient of 0.10 was chosen for this shape. The exact value of C_d is not too important because the cable's vertical profile in the water is not very sensitive to this value. The skin friction coefficient is about 0.0045. Substituting the above values into equation 3 gives K_s = 0.045275[3/] for the P-61 sampler. Skin friction is responsible for about one third of this value. The drag force on the sampler is thus,

$$F_{ds} = 0.045275 V^2. \quad (4)$$

A similar computation was made for the 100- and 200-lb sounding weights.

[3/] There are two philosophies regarding the number of significant figures to show in such a coefficient. One is to round the coefficient to the same number of significant figures as the value with the least number. The second philosophy endorses rounding of the final value. In this report, the second philosophy will be followed and the final computation will be rounded.

Their coefficients were 0.018872 and 0.028014, respectively.

The vertical force, F_{ys}, is primarily gravity. Under dynamic conditions of raising or lowering a sampler, the vertical drag force will add to or subtract from the vertical force. The effect has been neglected in the present analysis because most applications are amenable to a static treatment and because the relative effect should be small at the high velocities being considered.

F_{ys} is the weight in water. The air weight must be corrected for the water displaced by the metal and for the buoyancy of any voids. The P-61 weighs 100 lb in air. The sampler is cast from bronze with a specific gravity of about 8.9. The submerged weight is

$$100\ (8.9 - 1)\ /\ (8.9),$$

or about 88.8 lb. There is also a compression chamber in the body having a volume of 2.9 L (liter), a small cavity of about 0.1 L in the head, and the 0.5-L or 1.0-L volume of the sample container itself. The compression chamber will fill as the sampler is lowered. The sample container will fill when the sampling valve is opened. The buoyant force will be between 7.7 or 8.8 lb when the sampler enters the water and will then decrease to about 0.4 lb when the sample container is full. A buoyant force of 1.8 lb will be assumed as a first approximation. The vertical force due to gravity, F_{ys}, then becomes 87.0-lb of submerged weight.

The submerged weights of the 100- and 200-lb sounding weights are about 90.8 and 181.6 lb, respectively.

Next, the forces F_{ds} and F_{ys} acting on the sampler are translated into vector coordinates: a direction and a tension force magnitude. The direction, θ_1, of the sampler's force from the vertical is the arctangent of F_{ys}/F_{ds}. The tension force on the bottom of the first cable segment is:

$$T_s = \sqrt{F_{ys}^2 + F_{ds}^2}. \qquad (5)$$

Imagine a short segment of cable suspended in flowing water. The forces on such a cable segment are shown in figure 2. The forces acting on the ends of the segment are the tensions due to adjacent segments. These are vector forces directed away from the cable segment. The other forces which are shown are assumed to act at the center of the segment. These forces include the weight of the cable segment, form drag, and skin drag.

Some of the forces are calculable directly. The weight and drag forces are known or computable. The lower tension force is known if the forces on the sampler are equated to the tension on the lower end of the segment. Knowing these forces, the upper tension force and direction then may be calculated because the upper tension vector must balance all forces and moments below it.

This analytical approach requires that the sampler position be assumed first and then the calculations are carried out for each segment upwards towards the water surface.

At this point, a decision is required whether to take fixed increments of cable segment length or fixed increments of depth. Taking the latter approach ensures that all depth increments will be equal. Taking fixed cable segment lengths will result in the final segment being

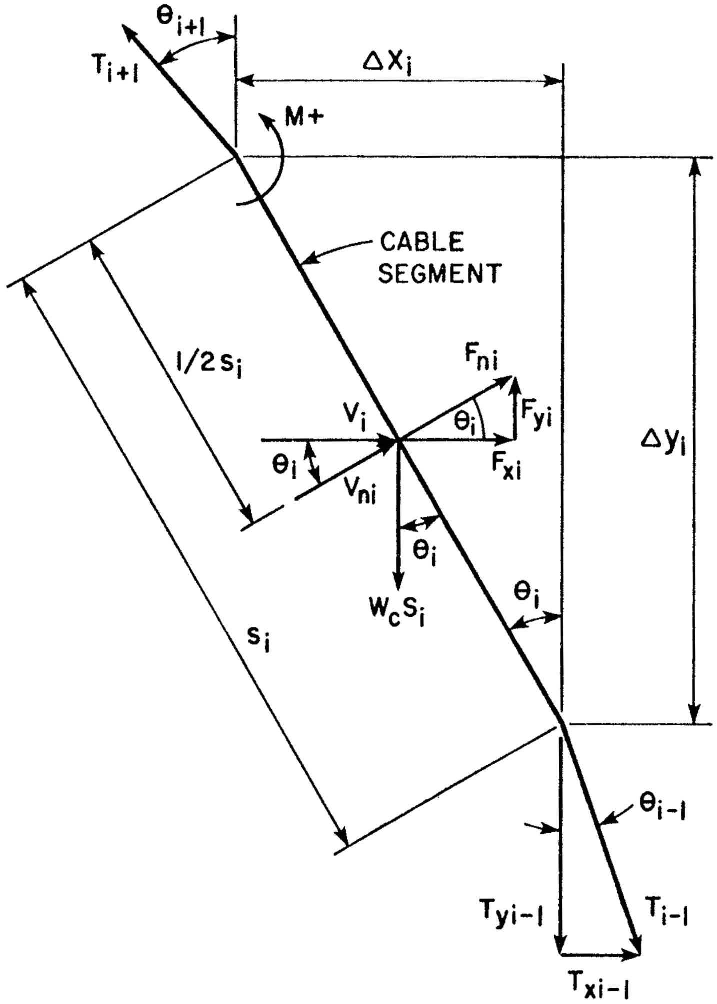

Figure 2.--Cable segment showing forces acting on the segment. Also shown are definitions of angles, partitioned forces, dimensions, and direction of summing moments. The subscript i refers to the present cable segment, i-1 indicates the previous lower segment, and i+1 refers to the next segment upward. The subscript n refers to the normal (perpendicular) velocity or force component. Dimension s is the segment length. The sum of all segment lengths is the wetted cable length, CL. Dimension Δx_i is the downstream drift for the segment, while Δy_i is the elevation increment for the segment. In this study, Δy_i is taken as a fixed difference: 1 ft in early computations and one-hundredth of the total depth in later computations.

shorter than the others. The approach used in the present study is based upon equal depth increments.

As an approximation, each cable segment will be taken as a straight line. If the sampler is at the bottom of a 100-ft stream, the cable shape can be described by 100 straight-line segments. As the sampler nears the surface, the number of segments per computation decreases, but the velocity profile also tends to become more uniform. One-foot changes in sampler elevation were used in this study (although later versions of the program use 100 segments regardless of the depth). Choosing a value smaller than 1-ft will increase computer time. A 1-ft elevation change usually is so small that the change in velocity from one end of the cable segment to the other is negligible except near the streambed. The velocity at the mid-segment elevation is taken from the von Karman log-defect law (IACWR, 1941a, P. 28):

$$V_i = V\,(1 + (9.521\ n/D^{1/6})\ (1 + \log_e\ (Y_i/D))), \tag{6}$$

where V is the mean velocity in the vertical, in feet per second;

n is Manning's roughness coefficient,

D is total stream depth, in feet; and

Y_i is the mid-segment elevation, in feet.

The cable normally used with the P-61 is a 1/8-inch diameter, stranded steel, aircraft cable with a submerged weight of about 0.0223 lb/ft. While the drag coefficient of a smooth cylinder normal to the flow is nominally 1.2, a stranded cable's C_d is lower, about 1.04 (Hoerner, 1958, p. 4-5). However, Pearce (1986, p. 26) stated:

> In practice the drag coefficient of a cable is higher

than 1.2 because it vibrates or strums as it is towed through the water with an amplitude of about 2 cable diameters. This is driven by a mechanism known as vortex shedding and drag coefficients have been measured as high as 3. The drag forces the cable and the payload backwards and upwards reducing the transducer depth.

No source was given for the high C_d value.

The drag force acting perpendicular to the cable is used:

$$F_d = C_d A_w \rho (V_i \cos\theta_i)^2/2, \qquad (7)$$

where A_w is the area of the segment perpendicular to the velocity component, that is, $A_w = s_i d$, where s_i is the segment length and d is cable diameter; and where $V_{ni} = V_i \cos\theta_i$ is the velocity component perpendicular to the segment. The cable skin friction, which acts along the cable, will be ignored because it is so small. For example, the skin drag per foot of cable in 15 ft/s water at an angle of 35° is less than 0.003 lb while the form drag under the same conditions is about 1.3 lb.

Two conditions must be satisfied for the cable segment to remain in static equilibrium. The sum of the vector forces on the segment must be zero, and the sum of moments must be zero. If F_{xi} (see fig. 2) is the horizontal component of the upper tension force,

$$F_{xi} = C_d(s_i d)(\rho/2)(V_i \cos\theta_i)^2(\cos\theta_i) + T_{i-1}(\sin\theta_{i-1}), \qquad (8)$$

where T_{i-1} is the lower cable tension force, and θ_{i-1} is the angle of that tension as shown on figure 2. The second term in equation 8 is the horizontal component of the lower tension force. For the first cable segment, T_{i-1} and θ_{i-1} describe the tension vector of the sampler.

In like manner, for F_{yi}, the vertical component of the upper tension force,

$$F_{yi} = -C_d(s_i d)(\rho/2)(V_i \cos\theta_i)^2(\sin\theta_i) + T_{i-1}(\cos\theta_{i-1}) + W_c s_i, \qquad (9)$$

where $W_c s_i$ is the submerged cable weight.

Next, the moments about the upper end of the cable segment are summed counterclockwise. A moment is the product of a force times the distance (the moment arm) through which it acts. The moments produced by the upper cable tension components (which are not known yet) become zero with respect to the upper end because they act at zero distance from the reference point. Both the cable drag and cable weight are at a distance of 1/2 s_i, and the lower segment tension is at distance s_i from the upper end. Summing,

$$(s_i/2)C_D(s_i d)(\rho/2)(V_i \cos\theta_i)^2 - (s_i/2)W_c s_i(\sin\theta_i) - s_i(T_{i-1})\sin(\theta_i - \theta_{i-1}) = 0. \qquad (10)$$

This equation can be simplified by noting that

$$s_i = \Delta y_i / \cos\theta_i,$$

where Δy_i is the increment of cable segment elevation, taken in this study to be 1 ft. Thus,

$$s_i = (\cos\theta_i)^{-1}, \qquad (11)$$

where s_i is in feet.

Substituting (11) into (8), (9), and (10) gives:

$$F_{xi} = C_D(d)(\rho/2)(V_i\cos\theta_i)^2 + T_{i-1}(\sin\theta_{i-1}), \quad (12)$$

$$F_{yi} = -C_D(d)(\rho/2)(V_i^2)\cos\theta_i\sin\theta_i + T_{i-1}(\cos\theta_{i-1}) + W_c/\cos\theta_i, \text{ and} \quad (13)$$

$$(C_D/2)(d)(\rho/2)V_i^2 - (W_c/2)\tan\theta_i/\cos\theta_i - T_{i-1}(\cos\theta_i)^{-1}\sin(\theta_i - \theta_{i-1}) = 0. \quad (14)$$

The above three equations contain three unknown parameters: F_{xi}, F_{yi}, and θ_i. Taking θ_{i-1} as a first estimate of θ_i and substituting this into equations 12 and 13, a second estimate of θ_i can be obtained from:

$$\theta_i = \arctan (F_{yi}/F_{xi}). \quad (15)$$

The new estimate of θ_i is used as an initial estimate to solve equation 14 by iteration. The iteration proceeds until the ratio of the difference between successive estimates of θ_i becomes less than 0.000001. The final value of θ_i is used to compute F_{xi}, F_{yi}, and the upper tension force:

$$T_i = \sqrt{F_{xi}^2 + F_{yi}^2}. \quad (16)$$

The next step is to compute the segment length from equation 11 and add it to previous values to obtain computed cable length ($CL = \Sigma\, s_i$). The downstream drift (in feet) or displacement for this cable segment is:

$$\Delta x_i = (\sin\theta_i)^{-1}, \quad (17)$$

and this value likewise is added to previous values ($\Delta X = \Sigma\, \Delta x_i$). The computational process is then repeated for the next cable segment. The process continues until the surface is reached. This completes the computation of the cable shape, or profile, for the sampler at one position.

Raising the sampler 1 ft (later, one-hundredth of the depth) and repeating the computational process will lead to a large table of computed values. The final values of ΔX for each initial sampler elevation, y_o, can be used to plot the equilibrium path of the sampler or sounding weight through the vertical. This path is the same whether the weight is lowered or raised. In real life, the path of the weight will be further downstream during lowering and will be upstream of the predicted path during raising. This occurs because the program neglects the lifting or depressing effect of the weight's vertical progress.

The analysis given in the theory section has been translated into a BASIC-language computer program which is described in Appendix I. Figure 3 is an example of the computation for an 87.0-lb (submerged) sampler in a 100-ft stream with a mean velocity of 15 ft/s and a Manning's n of 0.040. The 'path' of the sampler is the plot of downstream drift, ΔX vs. elevation. This curve has the most pronounced hook at the bottom with a maximum drift which is 4.6 ft greater than when the sampler is on the bottom. The path has a distinct 's' shape compared to a straight line from the surface to the bottom point. As the CL curve nears the surface it asymptotically approaches a line drawn from the surface to 100 ft at the bottom. The hook in the CL curve is only 0.7 ft, but the CL at the maximum point is 34 percent greater than the depth. The θ curve hooks only 1° but has a value of 65° when the sampler is near the streambed.

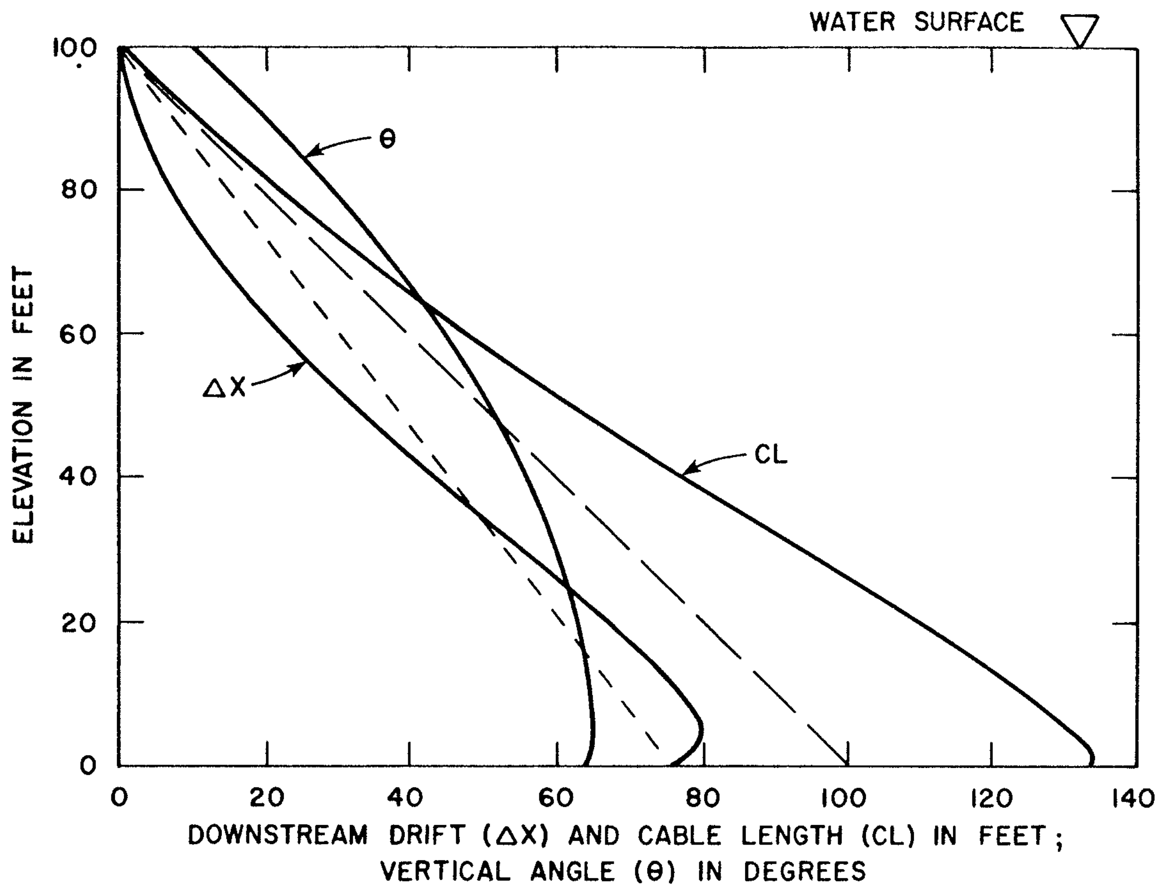

Figure 3.--Relations between sampler elevation above the streambed and downstream drift, ΔX, vertical angle at the surface, θ, and cable length at the surface, CL. The sampler weighs 87.0 lb (submerged) as it is raised in a 100-ft deep river with a mean velocity of 15 ft/s. Manning's roughness coefficient is assumed to be 0.040. The ΔX curve can be read as the 'path' of the sampler as it moves vertically. In reality, however, it is a series of static equilibrium positions neglecting the dynamic effects of vertical lift and drag as the sampler travels through the water column. The dashed lines are to assist the eye in estimating divergence from a straight line: the short-dash line is for the ΔX curve, and the long-dash line is the asymptote for the CL curve.

CORROBORATION OF COMPUTER ANALYSES BY FIELD DATA

There are few field data available for checking the program. One source is Shenehon (1900), who gave two velocity profiles (stations 6 and 8). Using these with an assumed value of 3.0 for the wire's drag coefficient, the computed wet-line correction agrees within 0.02 feet of the published values (see table 1). However, the test conditions did not cover a wide range. Depths at the two stations were 33.6 and 34.0 ft, and velocities were about 6.13 and 5.74 ft/s, respectively. The maximum angle was only 9.8° and the maximum WLC was only 0.16 ft.

Table 1. Comparison of Shenehon's wet-line correction (WLC) with computed values.

True Depth (feet)	Cable Length (feet)	Angle (degrees)	Shenehon's WLC (feet)	Computed WLC (feet)	Difference WLC (feet)
100-lb COLUMBUS SOUNDING WEIGHT					
STATION 6 VELOCITY DATA					
33.4	33.54	9.8	0.16	0.14	0.02
30.0	30.13	9.6	.14	.13	.01
20.0	20.06	7.2	.05	.06	-.01
10.0	10.01	4.0	.01	.01	.00
STATION 8 VELOCITY DATA					
33.8	33.92	8.7	.14	.12	.02
30.0	30.10	8.4	.11	.10	.01
20.0	20.04	6.3	.03	.04	-.01
10.0	10.01	3.5	.01	.01	.00
200-lb COLUMBUS SOUNDING WEIGHT					
STATION 6 VELOCITY DATA					
33.4	33.44	5.0	.04	.04	.00

Another set of test data is found in a recent study (Coon and Futrell, 1986) which was obtained by towing weights in a deep lake. In this case, a uniform constant velocity is assumed. The range of conditions, though, is wide enough to give a good test of the program's capabilities. The maximum cable angle was 52.5° and the maximum velocity was 14.76 ft/s. Two weights (100 and 200 lb) were lowered to depths up to 97 ft. Table 2 summarizes the test data and computed values for depths of 20 ft and greater. The first five columns are abstracted from Coon and Futrell (tables 1 and 2, p. 6-9). Apparent depth, velocity, and vertical angle (cols. 1, 2, and 5) were assumed to be accurate as given for use in the computations. Because the wire's drag coefficient, CDW, was unknown, a graphic interpolation scheme was used to determine the true depth and CDW given the cable length and vertical angle.

First, though, a series of calculations were performed attempting to close on the approximate true depth and CDW. Then a further array of computations was made to bracket the solution. Finally, accurate values were obtained by graphic interpolation.

Table 2.--Summary of published and computed data for corroboration of BASIC program. The first seven columns are from Coon and Futrell (1986, p. 6-9).

[A dash indicates a missing or uncomputable value; ft is feet; and ft/s is feet per second.]

Apparent Depth (ft)	Average Velocity (ft/s)	True Depth (ft)	Change in Depth (ft)	Vertical Angle (degrees)	Computed True Depth (ft)	CDW (Best Fit)	Computed Wet-line Correction (ft)	Shenehon Wet-line Correction (ft)	CL(1-cos θ) (ft)	Computed Downstream Drift (ft)
TEST WITH 100-LB COLUMBUS WEIGHT										
20.0	4.88	20.2	0.0	6.0	19.96	3.82	0.04	0.03	0.11	1.11
20.0	9.76	20.0	-0.2	20.0	19.54	3.29	.45	.41	1.21	3.76
20.0	14.76	18.2	-2.0	26.5	19.16	1.90	.84	.73	2.10	5.19
40.0	5.44	40.2	0.0	14.0	39.58	3.74	.42	.39	1.18	5.09
40.0	7.89	38.2	-2.0	25.0	38.61	3.33	1.39	1.29	3.75	9.23
40.0	9.59	37.2	-3.0	33.0	37.49	3.14	2.50	2.33	6.45	12.35
40.0	10.12	36.2	-4.0	32.0(41.0)[1]	35.95	3.81	4.01	--	9.81	15.51
40.0	13.24	--	--	48.5	34.02	2.89	5.98	--	13.5	18.86
70.0	5.00	68.1	-2.2	23.0	67.98	4.39	2.01	1.90	5.56	14.62
70.0	6.56	67.3	-3.0	29.0	66.70	3.32	3.29	3.08	8.78	18.66
70.0	8.26	64.3	-5.5	38.0	64.05	2.96	5.90	5.46	14.84	24.87
70.0	9.46	60.9	-9.4	46.5	60.66	3.06	9.33	--	21.82	31.04
97.0	5.78	93.3	-4.2	35.0	90.19	3.95	6.79	6.39	17.54	31.37
97.0	8.66	85.2	-12.3	52.5	79.78	3.31	17.21	--	37.95	49.18
97.0	7.55	90.4	-7.4	43.0	86.20	3.09	10.77	--	26.06	39.32
97.0	8.20	88.2	-9.6	44.5	85.25	2.76	11.66	--	27.81	40.86
TEST WITH 200-LB COLUMBUS WEIGHT										
40.0	5.01	40.0	-0.2	8.0	39.86	4.92	.14	.13	.39	2.91
40.0	6.58	--	--	12.0	39.79	4.30	.31	.29	.87	4.41
40.0	9.87	39.6	-0.6	20.0	39.12	3.24	.89	.82	2.41	7.43
40.0	13.40	36.6	-3.6	27.5	38.26	2.49	1.73	1.57	4.52	10.41
40.0	8.07	39.8	-0.2	17.0	39.37	4.11	.63	.58	1.75	6.24
70.0	5.09	70.7	-0.7	13.0	69.36	4.54	.63	.60	1.79	8.21
70.0	7.61	69.3	-2.1	25.0	67.60	4.10	2.41	2.26	6.56	16.02
70.0	10.32	67.0	-4.4	37.0	64.43	3.61	5.58	5.18	14.10	24.25
70.0	12.53	61.5	-9.9	41.5	62.80	2.86	7.24	--	17.57	27.58
95.0	3.86	94.9	-0.1	10.0	94.5	4.46	.50	.48	1.44	8.53
95.0	4.94	94.3	-0.7	16.0	93.7	4.43	1.30	1.22	3.68	13.70
95.0	6.90	92.1	-2.9	29.0	90.6	4.40	4.45	4.18	11.91	25.24
95.0	9.39	85.1	-9.9	41.0	85.5	3.74	9.44	--	23.31	36.52

[1]Published value appeared anomalous; value in parentheses was obtained by interpolation and used initially.

As an example, consider the measurement by Coon and Futrell using the 100-lb sounding weight with CL = 97.0 ft, velocity equal to 5.78 ft/s, and $\theta = 35.0°$. Rough approximations of the desired values were computed as shown in table 3. The parameters D_t and CDW were adjusted until the computed values of CL and θ were reasonably close to the values in table 2. A bracketing array of computations also could have been used in this case.

Table 3.--Successive computations of CL and θ for a 100-lb weight

[CL = 97.0 ft, V = 5.78 ft/s, and $\theta = 35.0°$.]

Run No.	Estimated True Depth (feet)	Estimated CDW	Computed Cable Length (feet)	Computed θ (degrees)
1	93.0	3.0	97.3	28.2
2	92.0	3.5	97.7	32.1
3	91.0	3.6	96.8	32.5
4	91.0	3.8	97.5	34.1
5	90.0	3.6	95.6	32.2
6	90.0	3.8	96.2	33.8
7	90.0	4.0	96.9	35.3

On the basis of these computations, 16 more computations were made attempting to bracket and refine the best value more closely. These results are plotted in figure 4. Each of the computations used a different combination of CDW and D_t, and was plotted on figure 4 with the computed cable length and angle for the combination. Lines were drawn connecting computations with the same D_t or CDW. Then the cable length and angle given in table 2 were plotted on the graph. The final values were interpolated graphically for the intersection of CL = 97.0 ft and $\theta = 35.0°$: $D_t = 90.19$ ft and CDW = 3.95. More accurate determinations are not warranted considering inaccuracies in measuring the vertical angle. This same procedure was used for computing each of the values shown in columns 6 and 7 of table 2.

One test of the program is to compare the computed true depths (col. 6 in table 2) with those given by Coon and Futrell (col. 3). Figure 5 shows this comparison. Both the 100- and 200-lb sounding weight data agree very well, although the 100-lb data were low at depths greater than 80 ft. The coefficient of determination (r^2) is 0.9991.

Another test of the program's competence is obtained by comparing the computed wet-line correction with those given by Shenehon's table. The difference between CL and D_t is the computed wet-line correction, WLC_c. Table 2 (col. 9) also lists Shenehon's WLC_s for comparison. Figure 6 shows WLC_c plotted against Shenehon's correction, WLC_s. The agreement is excellent. The least-squares regression equation is:

$$WLC_c = 0.01784 + 1.0604 \, WLC_s \tag{18}$$

with $r^2 = 0.9995$. This means that the computed WLC is six percent higher

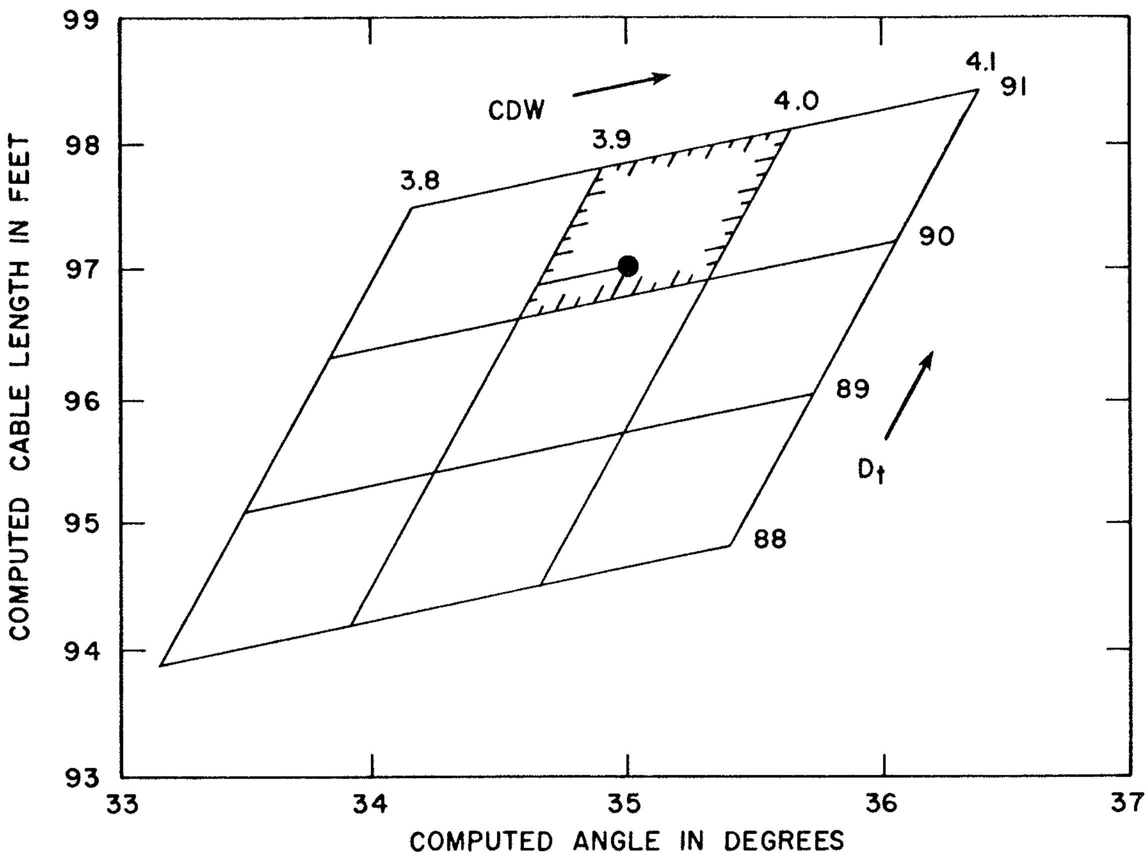

Figure 4.--Computed array for 91.8-lb (submerged) sounding weight with a cable length of 97.0 ft, vertical angle of 35°, and towed velocity of 5.78 ft/s. The lines of constant true depth, D_t, and wire drag coefficient, CDW, are drawn through computed data. The dot at 97.0 ft of cable length and at 35° is used to interpolate an accurate solution: D_t = 90.19 ft and CDW = 3.95.

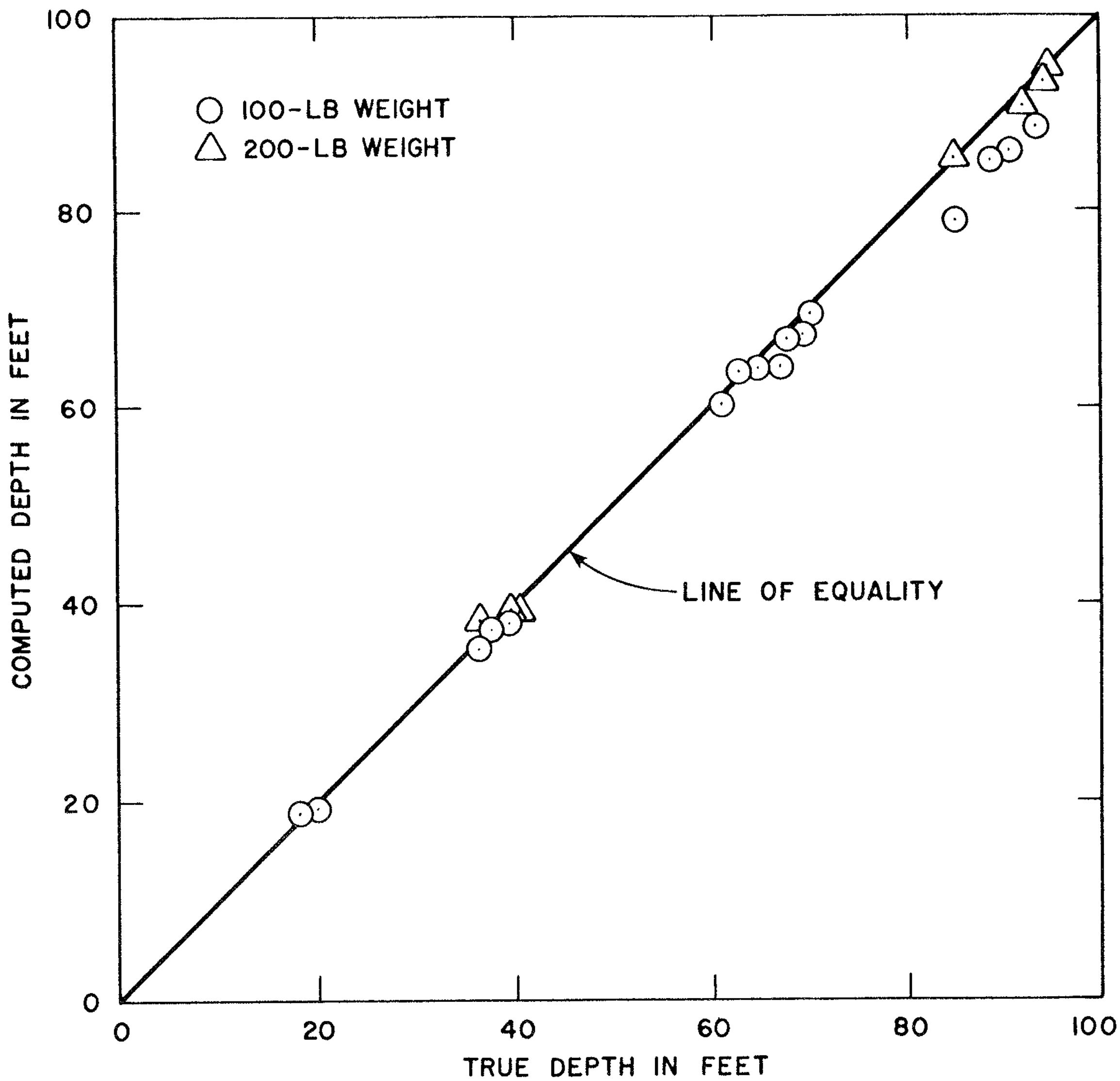

Figure 5.--Computed depth compared to true depth given by Coon and Futrell (1986).

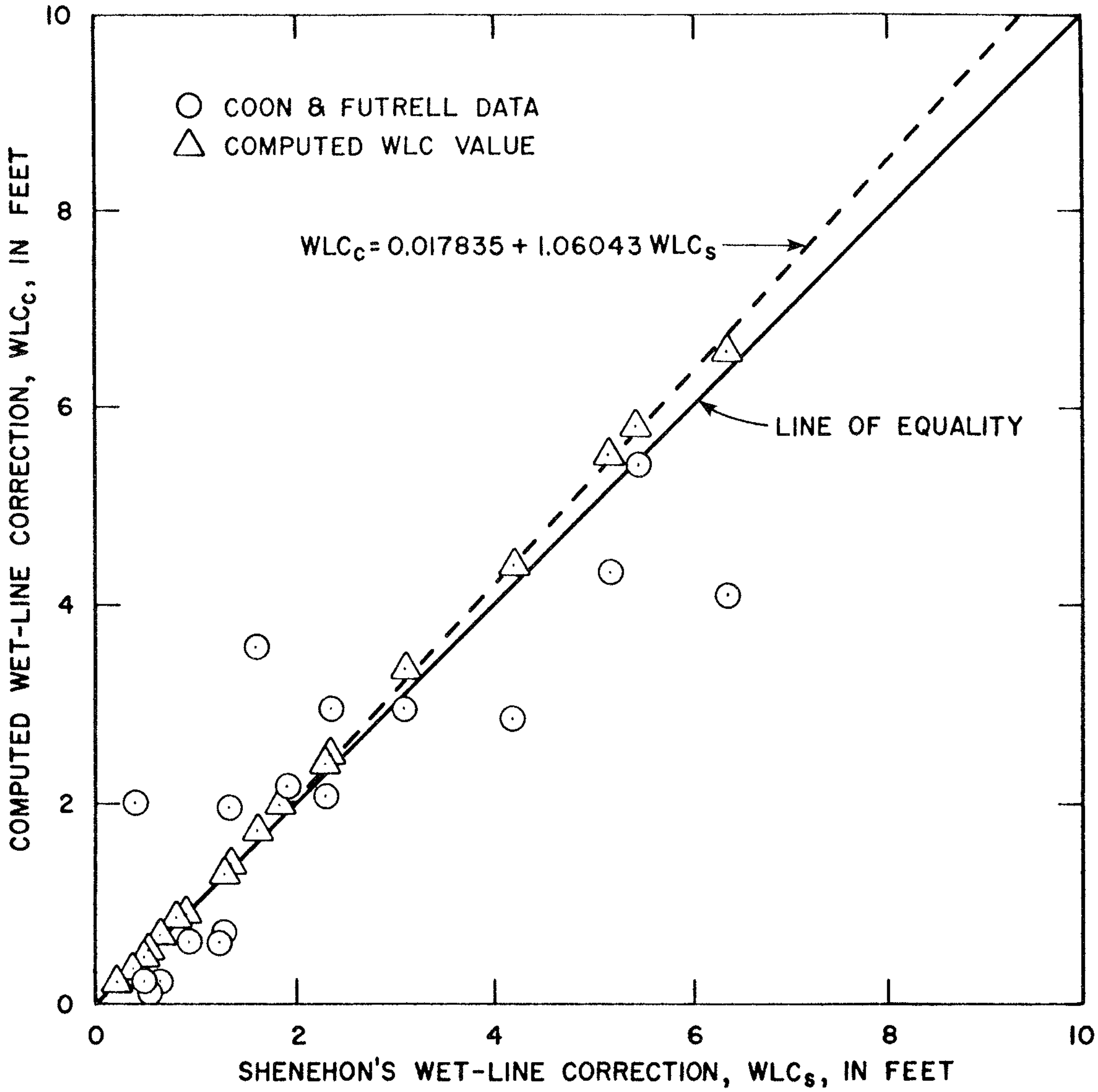

Figure 6.--Relation between Shenehon's tabular wet-line correction, WLC_s, and graphically computed wet-line correction, WLC_c, for data from Coon and Futrell (1986). The least-squares regression (dashed line) through this data has an $r^2 = 0.9995$. The circled points are those as given by Coon and Futrell and were not included in the regression computation.

than the table values (except for a 0.02-ft offset) over the range the table values could be compared. The six-percent difference is possibly the result of using 100 segments to compute the forces on the cable instead of Shenehon's 10 segments.

The circled points plotted on the figure are values given by Coon and Futrell. The scatter in the circled points is believed to be the result of inaccuracies in determining the depth of the sounding weight. The authors reported the accuracy of the true depth readings, by fathometer, to be ± 0.1 ft. However, there will be an error due to the geometry of the measurement unless the fathometer is directly above the weight. Figure 7 diagrams the problem. Their data were collected from a 16-ft boat, but computed downstream drift in table 2 is as great as 49 ft. Therefore, the transducer A could not possibly be directly above the weight when the drift was greater than 16-ft. A second, passive transducer B trailing transducer A by a fixed, known distance would have allowed the accurate determination of true depth, D_t, and downstream drift, Δx. Appendix II gives a description and the derivation of this computation.

Inspection of table 2 reveals a relation between CDW and velocity. Figure 8 shows the general relation between the two parameters. The values of CDW generally decrease with increasing velocity. The plot shows the least squares regression lines for the 100- and 200-lb weight data separately and as an the aggregate. The equations are:

100-lb weight: $\quad CDW = 4.6700 - 0.16245V,$ (19)

200-lb weight: $\quad CDW = 5.7484 - 0.22707V,$ and (20)

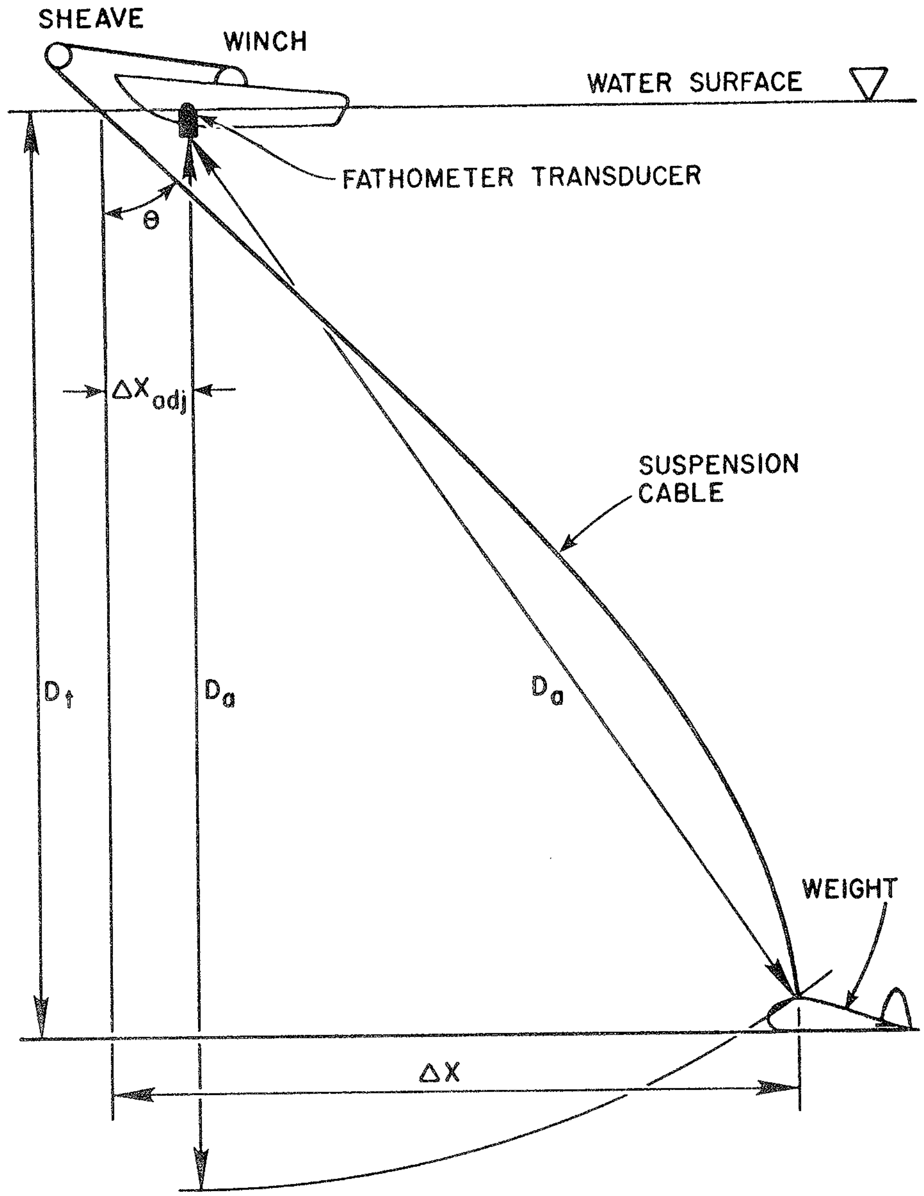

Figure 7.--Measurement arrangement in lake study. Apparent depth, D_a, is shown as the reflection distance and as the apparent vertical depth. The greater the downstream drift, ΔX, the greater will be the difference between D_a and true depth, D_t. The distance between the transducer and the point at which the cable penetrates the water is ΔX_{adj}. This drawing is not to scale.

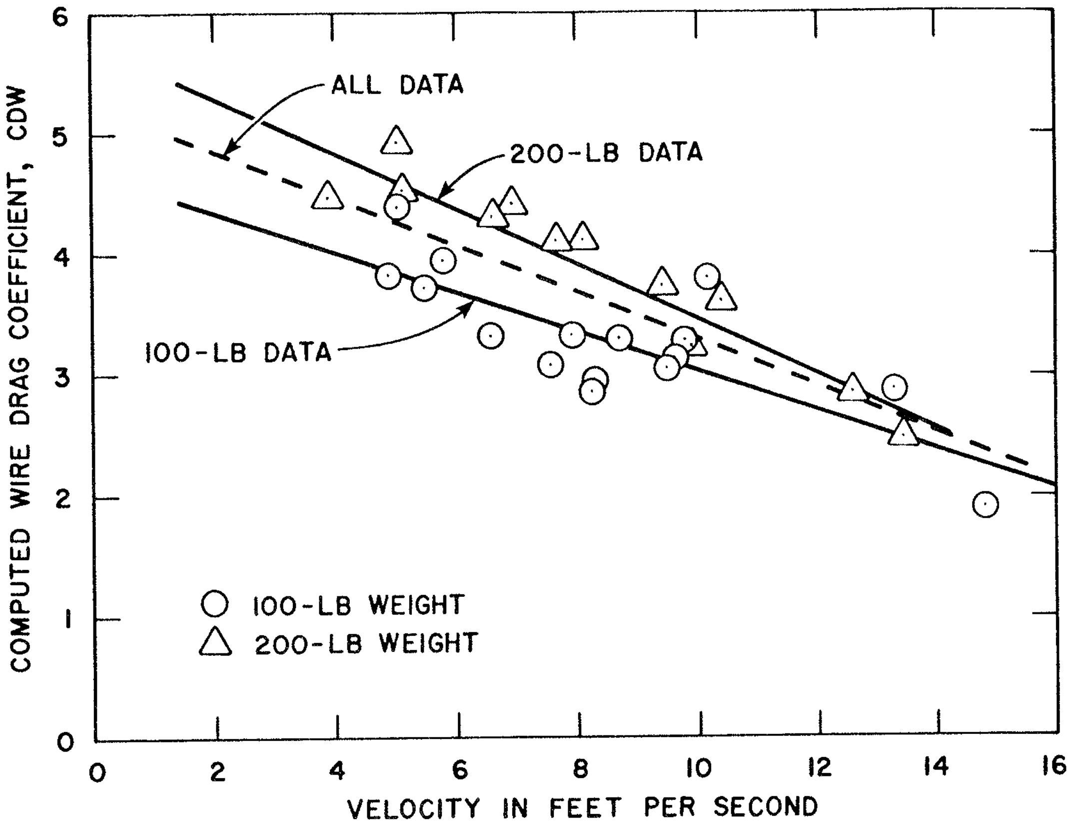

Figure 8.--Relation between computed wire drag coefficient, CDW, and velocity for computations based on data given by Coon and Futrell (1986).

all data: $CDW = 5.2489 - 0.20208V.$ (21)

The r^2 values are 0.609, 0.906, and 0.652, respectively.

A final point regarding table 2. The largest CDW value was 4.9 and all but 6 of the 29 values in the table exceed even the value of 3.0 mentioned by Pearce (1986, p. 26). Coon and Futrell (p. 6-9) listed measured strum amplitude for each run. A few of these values exceed three times the diameter of the cable, so the large CDW values obtained empirically in the present study do not seem out of line. The reason for the abnormally large CDW values at low velocities and the fact they appear to decrease with increasing velocity (fig. 8) is unknown.

APPLICATIONS OF PROCEDURE

Locating sounding weights in the vertical

The historic need for locating sounding weights and, later, samplers in the vertical has attracted many people to a study of the problem. Interestingly, no one has looked at the case where no wet-line correction is necessary. The true depth of the sounding weight is always between CL and CL cos θ. This statement follows from an examination of figure 1. The true depth is always equal to or less than the wetted length of cable, CL. This would be the case where the weight is hanging straight down, as in slack water. Also, the true depth is always equal to or greater than the product of the cable length and the cosine of θ, CL cos θ. This is the case where the cable acts as a rigid rod penetrating the water at the angle θ. If the difference between CL and CL cos θ is sufficiently small, then no correction to CL is necessary. If one needs to know the depth to the nearest 0.1 foot in a stream which is about 100 feet deep, then $(CL - CL \cos \theta) = CL(1 - \cos \theta)$ must be less than 0.1. Dividing by CL, this reduces to $(1 - \cos \theta) < 0.001$, which is satisfied when θ is less than 2.6 degrees. An angle of less than 5.7 degrees satisfies the 0.1-ft condition when the depth is about 20 ft. Figure 9 is a plot of the $CL(1 - \cos \theta)$ function mapped onto the CL-θ plane.

One might ask if WLC is related to the difference between CL and CL $\cos \theta$. For values computed by the cable program from Coon and Futrell's data and from Shenehon's two velocity profiles, WLC relates very well with $CL(1 - \cos \theta)$ as shown in figure 10. The data represent a wide range of values for the drag coefficient of the wire (1.9 to 4.9), uniform velocities to 14.76 feet per second, depths to 97 feet, and vertical angles to 52.5 degrees. The data closely follow a fourth-order polynomial of the form: $y = a_0 + a_1x + a_2x^2 + a_3x^3 + a_4x^4$, where $y = WLC_c$ and $x = CL(1 - \cos \theta)$. The coefficients of the polynomial are:

$a_0 = 1.223\,776 \times 10^{-2}$,

$a_1 = 3.883\,869 \times 10^{-1}$,

$a_2 = -8.331\,002 \times 10^{-4}$,

$a_3 = 6.321\,678 \times 10^{-5}$, and

$a_4 = 1.491\,841 \times 10^{-7}$.

If the five uppermost data points are ignored, the remaining data are related linearly. Figure 11 shows a least-squares regression line (r^2 = 0.9940) drawn through the remaining data. The equation for the line is:

$$WLC_c = 0.40268\ CL(1 - \cos \theta) - 0.079716. \qquad (22)$$

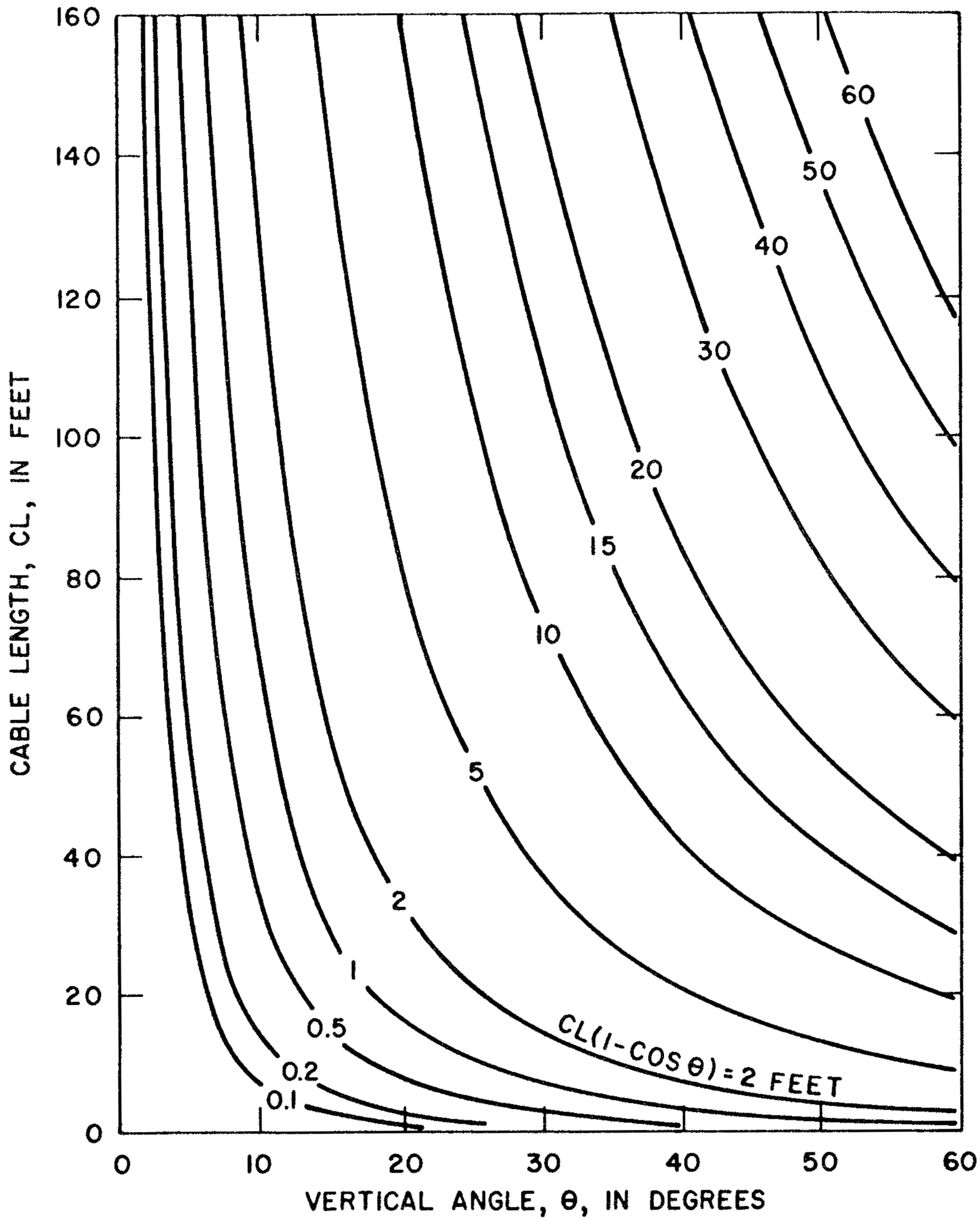

Figure 9.--The function CL(1 - cos θ). The function represents the difference in elevation between the cable hanging vertically and angled stiffly at angle θ.

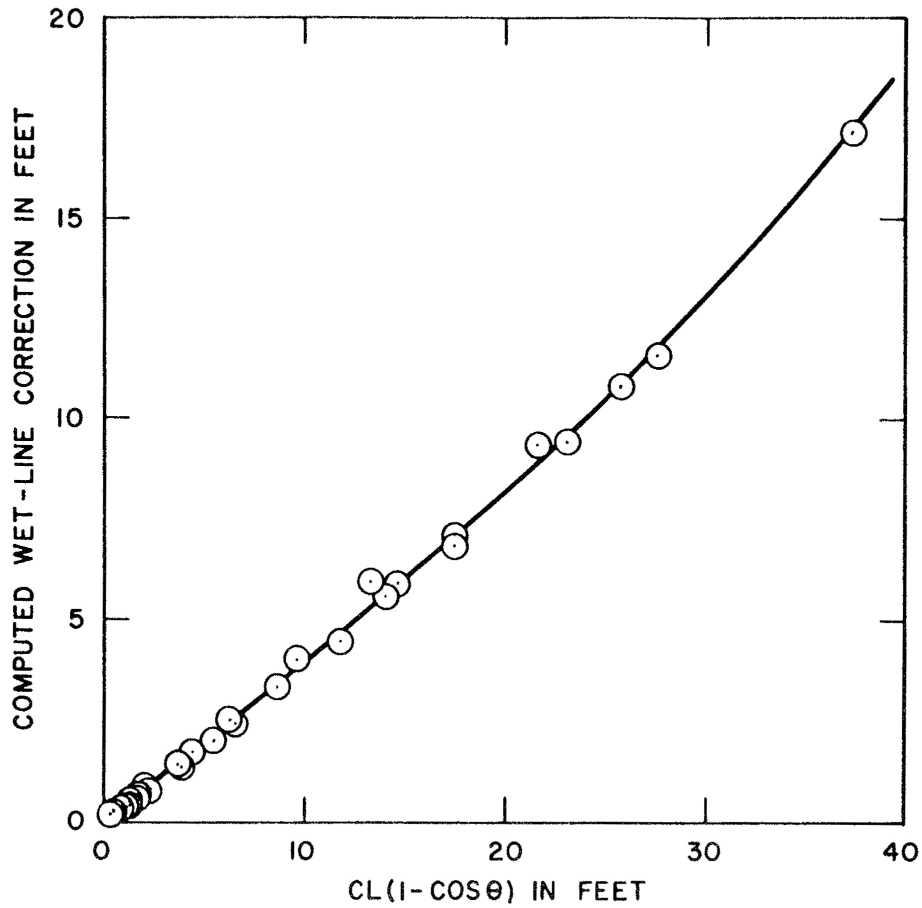

Figure 10.--Relation between CL(1 - cos Θ) and computed wet-line correction for all data in table 2. The line represents a fitted fourth-order polynomial.

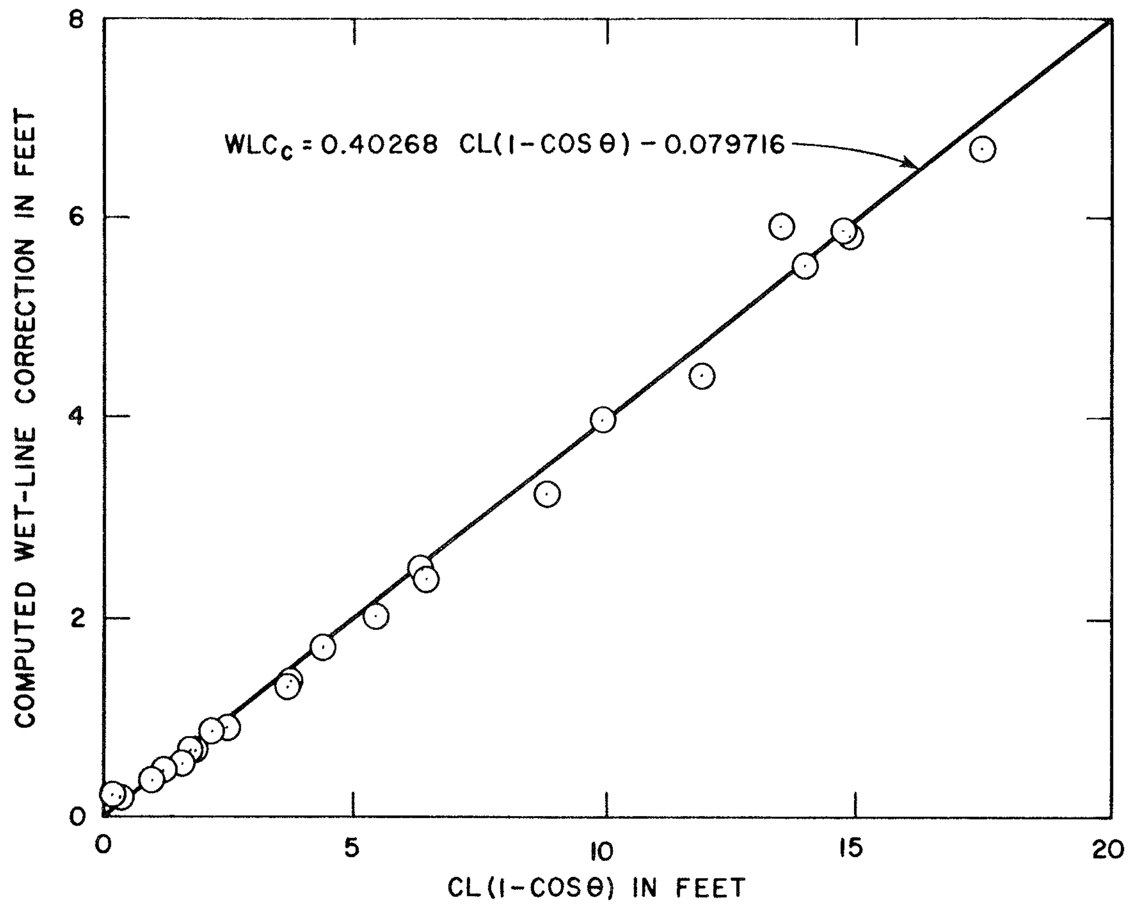

Figure 11.--Relation between CL(1 - cos θ) and computed wet-line correction, WLC_c, for CL(1 - cosθ) less than 20. The least-squares regression line shown has an $r^2 = 0.9940$.

Determining errors in suspended-sediment concentration

The determination of theoretical sampling errors requires the use of the table of CL and ΔX for initial sampler positions, y_o. This table or the relations $y_o = f_1$ (CL) and $\Delta X = f_2(y_o)$ are required for determining the midpoint coordinates of each 1-ft segment of cable beginning at the surface.

These coordinates are then used to compute velocity, concentration, and then their product for summation into "samples." A broader discussion of sampling practice should clear up any misunderstandings.

Let us assume that a P-61 is being used to collect a depth-integrated sample from a stream 100-ft deep. In normal practice the sampler will be operated such that perhaps five subsamples will be collected in the vertical, each subsample representing 20-ft increments of depth. For our purposes the composite of the group of subsamples will be considered the sample. In our ideal stream, the concentration does not vary with time, only with depth.

Let us make some additional assumptions. First, that the sampler is being lowered vertically at a rate of 1 ft/s. This is primarily for convenience, but it is also a typical vertical transit rate. Secondly, let us assume that each container is filled with a series of "snapshot" subsamples collected at the mid-point of each cable segment rather than a continuously collected sample. We define a "snapshot" as a subsample which is the product of local velocity and concentration collected for one unit of time, say one second. The sampler is motionless while collecting the sample. We will further assume that a snapshot sample is identical to a continuously collected sample through a 1-ft range in depth. This last assumption is reasonable because of the large number of snapshots, about 100, which will be integrated into our sample, and because the velocity and concentration are uniform over a small range in depth.[4/]

The sum of mass of the "snapshot" subsamples is taken as the total mass of an equivalent depth-integrated sample. The sum of all the velocities is proportional to the volume of such a sample because each snapshot is taken for one unit of time.

Because the velocity varies with depth in a stream, the effect of drag on the cable and sampler also varies with depth. This combination leads to a non-linear sampling path as shown in figure 3, for example. A non-linear sampling path will cause sampling errors. Any deviation of the sampler's path from a straight line will cause an error in the sample. The line does not have to be vertical to minimize error, just straight.

There is, however, one additional source of error. As the sampler is pushed downstream by the pressure of the stream, there is a small part

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[4/] Please note that the concept of a snapshot sampling scheme is for illustrative purpose only. Such a scheme would not collect satisfactory samples. When a nozzle and valve system is opened for only one second per snapshot, the flow in the nozzle is changing dynamically during a relatively large portion of that time, and representative sampling will be in error.

of the flow not sampled (for downward sampling). Imagine the snapshot subsample as being a horizontal cylinder of water-sediment mixture. If the sampling time is one second and if the local velocity is ten feet per second, then the subsample would seem to consist of a cylinder ten feet long and having a diameter equal to that of the nozzle. This is not the case. The sampler drifted downstream a distance of Δx_i during that one second. Therefore the cylinder length is Δx_i shorter than 10 ft in length ($10 - \Delta x_i$). If we had been sampling in an upward direction, the subsample cylinder would have been Δx_i longer than 10 ft ($10 + \Delta x_i$). We can determine the magnitude of this error with our calculation procedure.

An equation used to estimate suspended-sediment concentration was developed by Rouse (see Vanoni, 1975, p. 76):

$$C_y = C_a[\{(D-Y)/Y\}\{a/(D-a)\}]^z, \tag{23}$$

where C_y is the concentration at elevation Y,

C_a is the concentration at reference elevation a, and

D is total depth.

The exponent z is the ratio of the fall velocity of the sediment, w, to the shear velocity, U_*, and the von Karman constant, k:

$$z = w/(kU_*), \tag{24}$$

where k = 0.40 and $U_* = \sqrt{gDS}$. A typical size distribution was chosen and divided into five subclasses. A reference concentration was estimated and an exponent determined for each subclass. The concentration at each elevation was the sum of the concentrations of the subclasses at that elevation.

The weight of sediment multiplied by the local velocity and a coefficient and the volume of each snapshot subsample were summed separately, as were the sediment weight and subsample volume owing to downstream drift.

Finally, a similar computation was made assuming the sampler was lowered or raised vertically in elevator fashion, i.e., without any downstream drift. This concentration is taken as the true value for comparison purposes.

Table 4 summarizes the computations described in this section. The most striking observation regarding the table is the very low percentage error computed. We may conclude that this possible error mechanism may be disregarded.

Table 4.--Summary of suspended-sediment concentration computations.

Sampler Type	Mean Velocity (ft/s)	Sampler Concentration (mg/L)	Elevator Concentration (mg/L)	Sampler Concentration Error (%)
P-61	10	14.94	14.96	0.1
	12	17.91	17.95	.2
	14	20.86	20.94	.4
P-63	10	14.97	14.96	- .1
P-50	10	14.95	14.96	.1
P-61	10	1098.	1117.	1.7

NOTES: 1) Buoyancy and dynamic lift of the sampler and cable skin friction forces were ignored.
2) Sampler concentration adjusted for effect of downstream drift. Effect changed concentration 0.01 to 0.07 percent.
3) Stream depth was 100 ft and Manning's roughness was 0.035, except for the last line where the roughness was 0.085.
4) Submerged weights used were, respectively: P-61 (88.5-lb), P-63 (177-lb), and P-50 (266-lb).
5) Sampler concentration error is the elevator concentration less the sampler concentration divided by the elevator concentration times 100.

This is not to say that a closely allied error mechanism is also to be disregarded. The extra downstream drift of the sampler due to lift while it is being lowered and the decreased drift due to drag while it is being raised will also cause errors in concentration in a similar fashion to those computed for the table. However, as stated in the introduction, this analysis did not incorporate the dynamic aspects of sampling.

There is reason to believe that errors due to dynamic motion could be appreciable. As noted in the IACWR report on sampler tests in the Grand Canyon (1951, p. 25-26), when the sampler is suspended from a bridge or cable car, the total downstream drift should be measured from the point at which the sampler enters the water. That point is directly below the sheave. The distance from this point to the point where the cable penetrates the water must be added to the downstream drift computed in the present study. Only point-integrated samples are immune to this error. Use of very heavy samplers can minimize this error for depth-integrated samples.

The computer program used in the present study should be modified to provide estimates of the magnitude of concentration errors due to dynamic motion of the sampler. Errors due to the suspension point being well above the water surface can only be minimized by educating field personnel and by providing suitable equipment.

Sampler design

The original purpose of the analysis given in this report was to aid in sampler design. The computer program based on this analysis allowed preliminary inexpensive testing of the effects of adding extra mass, negative-lift wings, and other design options. Pearce (1986) used a similar approach in developing an oceanographic towed body for instrumtation. He investigated the benefits of a negative-lift wing and faired cable to achieve maximum depression of the towed body.

The Amazon River sampler mentioned in the introduction had to perform at such a great depth that the compression chambers of standard samplers were too small. After reviewing several alternative schemes, an external compression chamber was chosen to supply the extra air volume required. Frontal area, surface area, and submerged weight of this chamber were computed once the physical dimensions were fixed. The computer program then was used to construct a 'flight' path through the sampling vertical and to test the effect of added mass and of negative-lift wings in straightening the path. The program was adjusted to print out cable tension and the force on the sampler at each sampler position in the vertical. The maximum force on the sampler could be compared to the estimated strength of fittings and connectors.

CONCLUDING REMARKS

Computations based on the analysis presented in this report compare favorably with the historic values of Shenehon (1900), while providing an extension to the range of application. The present analysis is, in fact, just a more sophisticated extension of Shenehon's analysis: the progressive summation and balancing of forces from the sampler or sounding weight upward to the surface. Lacking a full scale test, the program does seem to be corroborated by the data of Coon and Futrell (1986). Their conclusion that Shenehon's table of wet-line corrections gives values which "may be in error" is in agreement with computations given in table 2.

The graph of the sampler's path through the stream vertical (fig. 3) has a hook shape near the bed as a result of lower velocity on the cable and sampler in that region. Such a nonlinear relation must cause errors in sampling. The nonlinearity is in the region with the highest concentration and the highest rate-of-change of concentration. The results of the suspended-sediment-concentration error computations that were summarized in table 4, are small. The error increases with increasing velocity and with increasing roughness. Downstream drift and nonlinear sampling path do not produce the expected large error, at least for the conditions studied herein. However, a stream with coarser sediment in high concentrations would surely yield larger sampling errors. The effect of lifting force on the sampler during the vertical traverse was not studied. This, and a traverse with varying buoyancy force, are good subjects for further study. Another possibile modification to the program is adjustment of sampler and wire drag coefficients based on the local Reynolds number.

The ability to compute a cable profile raises an interesting question. Is it possible to determine the exact depth of a sampler or sounding weight from the two measurable parameters, cable length and vertical angle? No, not precisely. The sampler depth is also dependent on the velocity, its vertical distribution, and the drag on the sampler and cable. Refer again to figure 1. The straight-angle depth is shown as position A and the cable-length depth is shown as position C. The true depth, position B, is between positions A and C. The difference in depth between A and C is a function only of cable length and vertical angle. The plot of lines of constant difference $CL - CL \cos \theta$ (fig. 9) provides a means to determine whether a correction is needed under specific conditions. For example, if a sampler or sounding weight were suspended in a stream and the cable length from the surface was 65 ft at an angle of 10° with the vertical, the true depth would be between 64 and 65 ft. The 1.0 ft difference line passes through the 65 ft and 10° coordinates. A correction might be needed in this case if the stream depth were being determined; but possibly no correction would be needed in other circumstances. Figure 9 also might be used to determine rules-of-thumb for field personnel. "No correction is necessary if the vertical angle is less than eight degrees--regardless of the cable length" is one possibility for a site-specific rule.

The correction needed to adjust the cable length value to obtain the depth of a sampler or sounding weight is usually obtained from Shenehon's wet-line correction table (Corbett and others, 1957, p. 50-52). Stream depth values computed for this study compared reasonably well with table values. Interpolating for a vertical angle of 35° gives a wet-line correction of 6.59 ft at an indicated 100-ft cable length. This is very close to the 35.33 degrees and 6.5-ft correction computed for a P-61 in a mean velocity of 10 ft/s. The closeness of these values may be only a coincidence, however. A similar comparison for a P-63 and a P-50 showed differences of 0.6 and 0.8 ft, respectively. The table, therefore, is not completely accurate.

The wet-line correction table was devised for determining the stream depth during a discharge measurement. The table also is used to locate the current meter in the flow, and occasionally to locate a point sampler in the flow. In this case, the forces on the meter and sounding weight, or on the sampler cannot be ignored. The wet-line correction table was compared with values computed for various samplers suspended ten or more feet above the stream bottom: table values averaged 25 percent lower than the difference between computed values of depth and cable length.

A requirement for accurate depth such as point sampling, therefore, makes the use of a depth-measuring device necessary. A pressure meter or an ultrasonic meter aimed upwards from the sampler would satisfy the need. Another possibly is the use of a two-transducer scheme such as described in Appendix II. The only other alternative is to use a very heavy sampler such as the P-63 (200-lb air weight) or P-50 (300 lb) to ensure small downstream drift and minimal depth error. Of course, heavier samplers require increasingly heavier, stronger, and more powerful lifting and support equipment as well.

CONCLUSIONS

The position of a sampler or sounding weight in the water cannot be determined solely and exactly from cable length and vertical angle. The drag on the body and on the cable are important also. These, in turn, are dependent on drag coefficients and on the velocity along the cable.

The BASIC program predicts quite well the true depth of weights towed in a lake. In this case, the program computes wet-line corrections that are about six-percent higher than the historic values given by

Shenehon (1900) which are in daily use around the world.

Other conclusions reached during the course of this study are based on the computations made possible by the program:

1. For samplers or weights suspended in the flow (above the bottom) of a deep, swift river, standard wet-line table values can be 25 percent too small.
2. The wet-line correction is related to the function $CL(1 - \cos \theta)$.
3. Lumped wire drag coefficients of almost 5 were computed based on data provided by the lake study of Coon and Futrell.
4. Any measurement which must be made at a particular depth in flowing water, such as a point velocity or a point sample, should be positioned with suitable instrumentation or else a very heavy weight or sampler should be used.
5. Use of the analysis in the present report provides a means to quantify possible sampling errors that were intuitive previously.
6. Depth-integrated sediment sampling errors were relatively small for the limited case studied. However, other aspects of sampling which can lead to large errors were discussed.
7. Field data are not presently available to adequately test the assumptions and coefficients used in this report.

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APPENDIX I. BASIC program with outline schematic and modifications

The program begins by computing the forces acting on the sampler or sounding weight, then resolves these forces into a tension vector which acts on the bottom of the first cable segment. Next, the program computes the first estimate of the angle of the first cable segment. This first estimate is used as the starting point in a successive approximation iteration which computes the angle of the cable segment such that the forces on the segment balance. Once the angle is determined, the forces on that segment are resolved into a tension vector for the next segment upward. The length of cable segment and downstream drift are computed for summation with succeeding segmental values. The program then loops back to computing the angle of the next upward segment using the angle of the present segment as it's first estimate and the upper tension vector as it's lower tension force. The program continues until the surface is reached.

Note that this program computes the static or equilibrium position of the cable. That is, the program does not account for the lifting effect on the sampler or sounding weight when it is being lowered, nor does it account for the dragging effect when it is being raised through the water column.

In the version shown in table I, line 1120 asks the operator for data describing the hydraulic conditions in the stream. After asking for the data file name in line 1140, the next several lines set constants and initial values of parameters and variables. The depth and Manning's n are incorporated into a lumped velocity coefficient at line 1300. The depth is tested next because in this version of the program the velocity is linear below an elevation of one foot. Using compound curves is sometimes necessary because the standard log-defect equation will yield negative velocities near the streambed. After computing the velocity in line 1320 (linear) or 1360 (log-defect), the force of the water on the sampler (line 1370) and then the magnitude of the tension vector (line 1380) is computed. Next the angle of this vector is determined (line 1390). These values then are printed.

The initial values for elevation are reset: the sampler's to zero and the first cable segment's to one-half foot. The latter is so the velocity and forces on the cable segment are computed for it's midpoint. These computations occur on lines 1490-1540.

The moments on the cable have not been balanced, however. They are balanced by repeating the secant method successively until the ratio of the most recent estimate of the angle to the difference between angles computed on two successive tries is less than 0.000001. Three passes normally suffice to converge on this value. Lines 1560-1670 contain the application of the secant method, also called Newton's method.

Once the angle of the cable segment is known the final values for that segment may be calculated (lines 1680-1730). The tension for the segment is tested to see if the previous maximum has been exceeded (line 1740) and, if so, the new values for maximum tension and the segment and sampler elevations are stored (lines 1750-1770).

Table I.--BASIC program for computing P-61 suspension-line profile.

```
1000 '***************************************************************
1010 '                 PROGRAM:    CABLE
1020 '
1030 '        MBASIC VERSION                              8 NOV 84
1040 '
1050 '     P-61 SUSPENSION LINE PROFILE CALCULATION W/ REMARKS
1060 '
1070 'NOTE: THIS VERSION USES LINEAR VELOCITY PROFILE BELOW ONE FOOT
1080 '
1090 '***************************************************************
1100 '
1110  PRINT CHR$(27)"*"                    ' CLEARS SCREEN
1120  PRINT "TYPE DEPTH, MANNING'S N, AND MEAN VELOCITY"
1130   INPUT D,MN, VBAR          :  PRINT D,MN,VBAR
1140  PRINT "WHAT IS THE DATA FILE NAME?"
1150   INPUT D$                  :  PRINT D$
1160  DIM Y0(100),V0(100),CL(100),DX(100),AN(100)
1170 '           SET INITIAL VALUES
1180  P6 = 10^6                            ' ROUNDOFF CONSTANT
1190   I = 0                               ' INDEX
1200  Y0 = .05                             ' SAMPLER DEPTH, IN FEET
1210  KS = .045275                         ' LUMPED P-61 DRAG COEFF.
1220  KW = .01519                          ' LUMPED WIRE DRAG COEFF.
1230  WS = 87!                             ' P-61 WT. (SUBMERGED), LBS
1240  WC = .0223                           ' SUB. WT. OF WIRE, LBS/FT
1250  KM = .0057594                        ' LUMPED COEFF. IN FORCE EQU.
1260  DX = 0!                              ' DOWNSTREAM DRIFT, IN FEET
1270  CL = 0!                              ' CABLE LENGTH, IN FEET
1280   Y = Y0
1290  TMAX = 0!                            ' MAX. CABLE TENSION, LBS
1300  VK = 9.521*MN*(D^-.166667)           ' LUMPED VELOCITY COEFF.
1310  IF Y0 > 1!  THEN 1360                ' TEST FOR SAMPLER ABOVE 1 FT
1320     V0 = Y0*VBAR*(1+VK*(1+LOG(1/D)))        ' VELOCITY AT SAMPLER
1330 '  RATIO (1/D) SETS VELOCITY AT 1 FT DEPTH, BUT MULTIPLYING
1340 '   BY Y0 LINEARIZES VELOCITY CURVE BELOW ONE FOOT
1350  GOTO  1370                           ' JUMP NEXT STEP
1360  V0 = VBAR*(1+VK*(1+LOG(Y/D)))        ' VELOCITY ABOVE 1 FT
1370  FS = KS*V0*V0                        ' HORIZONTAL FORCE ON SAMPLER
1380  TS = SQR(FS*FS+WS*WS)                ' TENSION FORCE DUE TO SAMPLER
1390  A0 = ATN(FS/WS)                      ' ANGLE OF TENSION FORCE
1400  A1 = A0   :      TEN = TS            :      AZ = A0*57.2958
1410  PRINT                                ' PRINT INITIAL VALUES
1420  PRINT"Y0= ";Y0;"  FS= ";FS;"  TS= ";TS;"  A0= ";AZ;"  V= ";V0
1430  IF Y0 > .05  THEN 1450               ' RESET SAMPLER ELEVATION
1440     Y0 =0!
1450  IF Y > .5   THEN 1480                ' SET CABLE SEG. ELEV.TO MIDPT.
1460     Y = .5
1470  GOTO 1490
1480  Y = Y + 1                            ' INCREMENT ELEVATION BY 1 FT
1490   V = VBAR*(1+VK*(1+LOG(Y/D)))        ' MIDPOINT VELOCITY
1500   VSQ = V*V                           ' SQUARE VELOCITY
1510  CA = COS(A1)
```

```
1520  FX = KW*VSQ*CA*CA+TEN*SIN(A0)      ' HORIZ. FORCE ON WIRE SEGMENT
1530  FY = -KW*VSQ*CA*SIN(A1)+TEN*COS(A0)+WC/CA         ' VERT. FORCE
1540  A1 = ATN(FX/FY)                    ' FIRST ESTIMATE OF SEG. ANGLE
1550 '                 SECANT METHOD OF ESTIMATING ROOTS
1560  D1 =  .99995 * A1                  ' TWO ANGLES VERY NEAR THE
1570  D2 = 1.00005 * A1                  '     FIRST ESTIMATE
1580  F1 = KM*VSQ*COS(D1)-.5*WC*SIN(D1)-TEN*SIN(D1-A0)  ' D1 FORCE
1590  F2 = KM*VSQ*COS(D2)-.5*WC*SIN(D2)-TEN*SIN(D2-A0)  ' D2 FORCE
1600  F3 = F2*(D1-D2)/(F1-F2)            ' ESTIMATE OF SEGMENT ANGLE
1610  A2 = A1 - F3                       ' DIFFERENCE
1620  TR = F3 / A2                       ' TEST RATIO
1630  TR = INT(TR*P6+.5)/P6              ' ROUND TO NEAREST 0.000001
1640  IF TR = 0!  THEN 1680              ' TEST FOR ZERO
1650    D1 = D2   :  D2 = A2             ' SET FOR NEXT ITERATION
1660    A1 = A2   :  F1 = F2
1670  GOTO 1590
1680  A0 = A2      :  CA = COS(A1)       ' RECOMPUTE FINAL SEG. VALUES
1690  FX = KW*VSQ*CA*CA+TEN*SIN(A0)
1700  FY = -KW*VSQ*CA*SIN(A1)+TEN*COS(A0)+WC/CA
1710  CL = (1/COS(A2)) + CL              ' ACCUMULATE CABLE LENGTH
1720  DX = TAN(A2)+ DX                   '   SAME FOR DOWNSTREAM DRIFT
1730  TEN= SQR(FX*FX+FY*FY)
1740  IF TEN<TMAX GOTO 1780              ' TEST FOR NEW MAXIMUM TENSION
1750    TMAX = TEN
1760    EY=Y
1770    SE=Y0
1780  Y = Y+1                            ' INCREMENT Y
1790  IF Y > D THEN 1810                 ' TEST FOR CABLE AT SURFACE
1800    GOTO 1490                        '  IF NOT, REPEAT COMPUTATION
1810  PRINT"Y= ";Y-1;"   CL= ";CL;"   DX= ";DX;"   THETA= ";A2*57.2958
1820  PRINT
1830  I = I+1                            ' INCREMENT INDEX, STORE VALUES
1840    Y0(I) = Y0      : V0(I) = V0     : CL(I) = CL    :  DX(I) = DX
1850    AN(I) = A2*57.2958               ' STORES ANGLE IN DEGREES
1860    DX = 0!         : CL = 0!        ' RESET VARIABLES
1870    Y0 = Y0+1                        ' RAISE SAMPLER ONE FOOT
1880  IF Y0 = D  THEN 1910               ' TEST FOR SAMPLER AT SURFACE
1890    Y = Y0                           ' RESET Y TO SAMPLER
1900    GOTO  1310
1910  PRINT CHR$(7)                      ' CLEAR SCREEN
1920  OPEN"O",1,D$                       ' OPEN DISK FILE
1930   WRITE#1,"           ",I,"              ",D$
1940    PRINT
1950  FOR N = 1 TO I
1960    PRINT#1,USING"#####.#####     ";Y0(N);V0(N);CL(N);DX(N);AN(N)
1970  NEXT N
1980  CLOSE#1
1990  PRINT     :  PRINT "MAX. CABLE TENSION IS  ";TMAX
2000  PRINT TAB(10);"THE CABLE SEGMENT ELEVATION WAS  ";EY;" FEET, "
2010  PRINT TAB(10);"AND THE SAMPLER WAS AT ";SE;" FEET"
2020  PRINT     :  PRINT
2030  END
```

The segment elevation is incremented one foot and the new elevation is tested. If the segment is not above the water surface, then the process repeats for the next segment above starting at line 1490. If the segment is above the surface, the values are printed (line 1810) and stored (lines 1830-1850). The downstream drift and cable length are reset to zero, and the sampler elevation is incremented by one foot. The new elevation is tested to see if the sampler is at the water surface and, if not, the process repeats beginning at line 1310. If the sampler is at the surface, the entire data array is transferred to the disk file.

The program has been modified extensively over the years. Most all of the modifications were to the output format. However, the part of the program dealing with the velocity at the sampler, sounding weight, or mid-point of a cable segment has been changed in several ways. The early work used the 'universal' logarithmic defect equation (Vanoni, ed., 1975, p. 75) which requires an estimate of Manning's n and the mean velocity. In testing Shenehon's wet-line correction table, a file of the depth and velocities was made up for both stations so that point velocities could be interpolated. Testing the tow data given by Coon and Futrell (1986) only required inserting a constant velocity.

APPENDIX II. Determination of true depth and downstream drift by means of two transducers

The difficulties of measuring the depth of a sounding weight with a fathometer while under way was described earlier in this report. Coon and Futrell (1986) alluded to some of the problems. The scheme discussed here was discovered while studying methods of collecting field data to corroborate the present analysis.

Figure IIa illustrates the location of devices and defines some terms. Transducer A is active; that is, it transmits and receives the signal. The distance from transducer A to the weight, W, is d_1. Time t_1 is the time required for the signal to travel from A to W and return. Half of this time is needed to travel one way. So the travel time from A to W is:

$$t_{AW} = t_1 / 2. \qquad \text{(II-1)}$$

Transducer B is passive; that is, it only receives a signal. A small reflector below A reflects a portion of the transmitted signal directly to B, which is held a fixed, known distance, d_f, behind A. The transmission time from A to B is designated t_f. B also receives the signal reflected from the sounding weight. The total travel time from A to W to B is t_2. The time for just the distance d_2 from (W to B) is:

$$t_{WB} = t_2 - (t_1/2). \qquad \text{(II-2)}$$

Each of the distances is the product of the velocity of sound in water and the travel time for the distance. The velocity is known because the distance d_f is known. All of the times are known because they are measured directly. Therefore, the angles may be computed

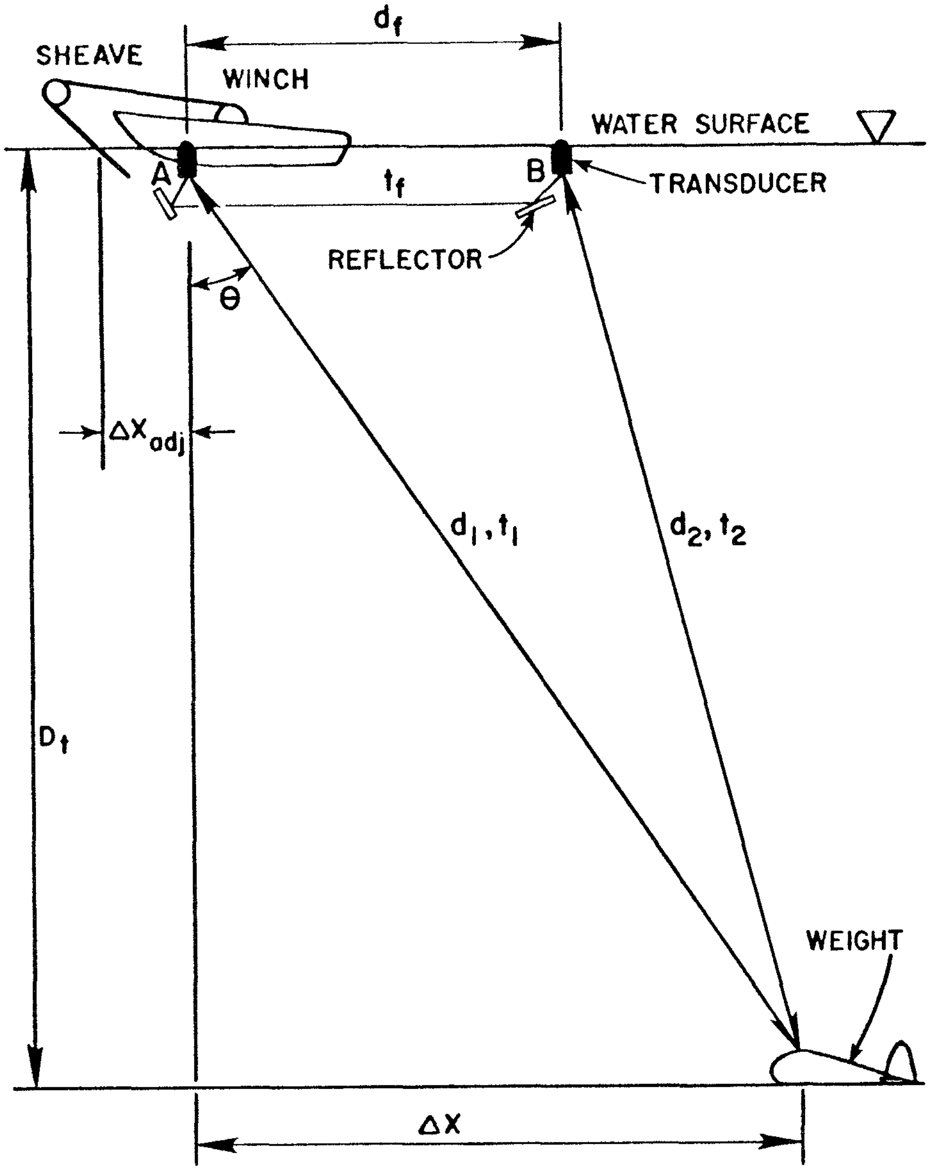

Figure IIa.--A two-transducer scheme for determining true depth, D_t, and downstream drift, ΔX, from the three time intervals: t_f, t_1, and t_2. Transducer B is passive and is a fixed, known distance behind A. Two small reflectors transmit a small portion of the energy directly from A to B, arriving after interval t_f. Time t_1 is the interval from initial transmission until receipt at A of the signal reflected from the weight. Time t_2 is the interval from initial transmission until receipt at B of the signal reflected from the weight.

because all three sides of the triangle are known:

$$\theta = 90° - \arccos((d_1^2 + d_f^2 - d_2^2) / (2\ d_1\ d_f)), \tag{II-3}$$

which expands to

$$\theta = 90° - \arccos(((V_w^2 t_1^2/4) + V_w^2 t_f^2 - V_w^2(t_2 - (t_1/2))^2) / (2\ V_w(t_1/2) V_w\ t_f)), \tag{II-4}$$

and which reduces to

$$\theta = 90° - \arccos((t_f^2 - t_2^2 + t_1\ t_2) / (t_1\ t_f)). \tag{II-5}$$

Notice that only measured times are needed to compute θ. True depth and downstream drift require the distance d_1,

$$d_1 = V_w\ (t_1\ /\ 2), \tag{II-6}$$

which converts, after substituting $V_w = d_f/t_f$, to

$$d_1 = d_f\ t_1\ /\ 2\ t_f. \tag{II-7}$$

Once we have d_1,

$$D_t = d_1 \cos\theta, \text{ and} \tag{II-8}$$

$$\Delta x = d_1 \sin\theta. \tag{II-9}$$

Only two assumptions are needed to make this scheme work. The first is that the speed of sound in the stream is equal in all directions. This implies a uniform temperature and conductivity (uniform density). The second assumption is that the speed of electrical transmission may be neglected. This assumption is for convenience only, because an adjustment is possible if needed. The speed of electrical transmission in wire is about 200,000 times faster than the speed of sound in water.

The location (depth and downstream drift) of the sampler or sounding weight is thus uniquely determined when the distance d_f is known and the three times (t_f, t_1, and t_2) are measured.