A STUDY OF METHODS USED IN

MEASUREMENT AND ANALYSIS OF SEDIMENT LOADS IN STREAMS

SUBCOMMITTEE ON SEDIMENTATION

INTER-AGENCY COMMITTEE ON WATER RESOURCES

REPORT NO. 12

SOME FUNDAMENTALS OF PARTICLE SIZE ANALYSIS

DECEMBER 1957

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A Study of Methods Used in

MEASUREMENT AND ANALYSIS OF SEDIMENT LOADS IN STREAMS

Cooperative Project

Sponsored by the

Subcommittee on Sedimentation
Inter-Agency Committee on Water Resources
(Formerly Federal Inter-Agency River Basin Committee)

Corps of Engineers	**	Geological Survey
Soil Conservation Service	**	Bureau of Reclamation
Agricultural Research Service	**	Coast and Geodetic Survey
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Report No. 1
FIELD PRACTICE AND EQUIPMENT USED IN SAMPLING SUSPENDED SEDIMENT

Report No. 2
EQUIPMENT USED FOR SAMPLING BED-LOAD AND BED MATERIAL

Report No. 3
ANALYTICAL STUDY OF METHODS OF SAMPLING SUSPENDED SEDIMENT

Report No. 4
METHODS OF ANALYZING SEDIMENT SAMPLES

Report No. 5
LABORATORY INVESTIGATIONS OF SUSPENDED-SEDIMENT SAMPLERS

Report No. 6
THE DESIGN OF IMPROVED TYPES OF SUSPENDED-SEDIMENT SAMPLERS

Report No. 7
A STUDY OF NEW METHODS FOR SIZE ANALYSIS OF SUSPENDED-SEDIMENT SAMPLES

Report No. 8
MEASUREMENT OF THE SEDIMENT DISCHARGE OF STREAMS

Report No. 9
DENSITY OF SEDIMENTS DEPOSITED IN RESERVOIRS

Report No. 10
ACCURACY OF SEDIMENT SIZE ANALYSES MADE BY THE BOTTOM-WITHDRAWAL-TUBE METHOD

Report No. 11
THE DEVELOPMENT AND CALIBRATION OF THE VISUAL-ACCUMULATION TUBE

Report No. 12
SOME FUNDAMENTALS OF PARTICLE SIZE ANALYSIS

REPORTS -- LETTERED SERIES

Report A -- PRELIMINARY FIELD TESTS OF THE US SEDIMENT-SAMPLING EQUIPMENT IN THE COLORADO RIVER BASIN — APRIL 1944

Report B -- FIELD CONFERENCES ON SUSPENDED-SEDIMENT SAMPLING — SEPTEMBER 1944

Report C -- COMPARATIVE FIELD TESTS ON SUSPENDED-SEDIMENT SAMPLERS PROGRESS REPORT — DECEMBER 1944

Report D ** -- COMPARATIVE FIELD TESTS ON SUSPENDED-SEDIMENT SAMPLERS PROGRESS REPORT - - - - AS OF JANUARY 1946

Report E -- STUDY OF METHODS USED IN MEASUREMENT AND ANALYSIS OF SEDIMENT LOADS IN STREAMS — JULY 1946
(Paper presented at ASCE convention, Spokane, Washington)

Report F -- FIELD TESTS ON SUSPENDED-SEDIMENT SAMPLERS, COLORADO RIVER AT BRIGHT ANGEL CREEK NEAR GRAND CANYON, ARIZONA — AUGUST 1951

Report G ** -- PRELIMINARY REPORT ON US DH-48 (HAND) SUSPENDED-SEDIMENT SAMPLER
(Out of print--Superseded by material in Report No. 6)

Report H ** -- INVESTIGATION OF INTAKE CHARACTERISTICS OF DEPTH-INTEGRATING SUSPENDED-SEDIMENT SAMPLERS AT THE DAVID TAYLOR MODEL BASIN — NOVEMBER 1954

Report I -- OPERATION AND MAINTENANCE OF US P-46 SUSPENDED SEDIMENT SAMPLER

Report J -- OPERATING INSTRUCTIONS SUSPENDED-SEDIMENT HAND SAMPLER, US DH-48

Report K -- OPERATOR'S MANUAL (PRELIMINARY) THE VISUAL-ACCUMULATION-TUBE METHOD FOR SEDIMENTATION ANALYSIS OF SANDS

Report L -- VISUAL-ACCUMULATION TUBE FOR SIZE ANALYSIS OF SANDS
(Paper presented at ASCE convention, Austin, Texas) — SEPTEMBER 1954

** Very limited distribution

SYNOPSIS

Some of the basic concepts, definitions, and relations essential to particle size analysis are discussed. The relations between various measures of particle size are illustrated with data for sand sizes of sediment.

Fall velocity is emphasized both in importance and in relation to physical size. Data on the effect of concentration on fall velocity are presented, but the available information is insufficient for adequate coverage of the problem.

A method of preparing sand samples having a known fall-velocity distribution is outlined. Such samples may be used to study the effect of concentration on fall-velocity and to calibrate methods and equipment for sedimentation-size analysis.

TABLE OF CONTENTS

Section Page

I. INTRODUCTION

II. DEFINITIONS AND BASIC CONCEPTS

III. BASIC FALL-VELOCITY RELATIONS

IV. RELATIONS BETWEEN PARTICLE SIZE SYSTEMS

LIST OF ILLUSTRATIONS

Figure Page

LIST OF TABLES

Table Page

SOME FUNDAMENTALS OF PARTICLE SIZE ANALYSIS

I. INTRODUCTION

1. Scope of the general study--This report is part of the general project, "A Study of Methods Used in Measurement and Analysis of Sediment Loads in Streams", which has been sponsored by cooperating Federal agencies since 1939. The objective of the project is to gather basic engineering data and information on the characteristics and behavior of sedimentary materials transported by natural streams in order to gain a better knowledge of the fluvial-sediment problem and its solution as related to the development of water resources for industrial, agricultural, commercial, and domestic purposes. The various aspects of the problem that have been investigated are indicated by the following titles and brief abstracts of previously published reports:

Report No. 1--"Field Practice and Equipment Used in Sampling Suspended Sediment" is a detailed review of the equipment and methods used in suspended-sediment sampling from the earliest known investigations to the present, with discussions of the advantages and disadvantages of the various methods and instruments used. The requirements of a sampler that would satisfy all field conditions are set forth.

Report No. 2--"Equipment Used for Sampling Bed-Load and Bed Material" reviews the equipment and methods used in bed-load and bed-material sampling in a manner similar to that in which Report No. 1 covers suspended sediment.

Report No. 3--"Analytical Study of Methods of Sampling Suspended Sediment" covers an investigation of the accuracy of various methods of sampling suspended sediment in a vertical section of a stream. Analytical study is based on the application of turbulence theories to sediment transportation.

Report No. 4--"Methods of Analyzing Sediment Samples" describes many methods developed for determining the size of small particles and for establishing the particle-size gradation and the total concentration of sediment in samples. Detailed instructions are given for many of the common methods that have been developed and used by agencies doing extensive work in sedimentation.

Report No. 5--"Laboratory Investigations of Suspended-Sediment Samplers" reports the effects of intake conditions on the representativeness of sediment samples and on the filling characteristics of slow filling samplers.

Report No. 6--"The Design of Improved Types of Suspended-Sediment Samplers" describes the development of various integrating samplers suitable for taking vertically depth-integrated samples in flowing streams and others suitable for taking time-integrated samples at a fixed point. Details of the adopted types are given.

Report No. 7--"A Study of New Methods for Size Analysis of Suspended-Sediment Samples" reports on research to develop methods of size analysis suitable for most suspended-sediment investigations and describes a new apparatus and technique, the bottom-withdrawal-tube method.

Report No. 8--"Measurement of the Sediment Discharge of Streams" describes methods and equipment for use in making sediment measurements under the diverse conditions that are encountered in streams.

Report No. 9--"Density of Sediments Deposited in Reservoirs" presents data on the apparent density of sediment deposited in various existing reservoirs. The results are summarized, and certain conclusions useful in engineering studies are given.

Report No. 10--"Accuracy of Sediment Size Analyses Made by the Bottom-Withdrawal-Tube Method" recounts detailed and extensive tests made to evaluate the accuracy of the bottom-withdrawal-tube method. Glass spheres of sand sizes were used as the sediments.

Report No. 11--"The Development and Calibration of the Visual-Accumulation Tube" describes a simple and accurate method for rapid size analysis of sediments of sand sizes.

2. Introduction--An investigation undertaken to determine the best method for making sedimentation analyses of sand samples required a study of many sedimentation methods and fundamental concepts. The nomenclature and definitions in literature on sedimentation methods of size analysis were unstandardized and the accuracy of sedimentation-size analysis methods had seldom been evaluated; consequently, extensive orientation was required to organize sedimentation data into a usable and consistent form.

This report is primarily an attempt to record in simple terms the definitions, basic theories, illustrative comparisons, and analytical processes that were essential to the developmental problem and to an understanding of some major conflicts in reports on sedimentation methods of size analysis. The secondary purpose of the report is to indicate some of the techniques and methods which may be useful in further studies.

The scope of this report is somewhat limited. Although terms, definitions, and concepts have been made equally applicable to all sizes of sediments, the report reflects its origin as a by-product of an investigation of the size analysis of sands. In keeping with present emphasis on the problem of the transport of sediment in streams, the sedimentation size or fall velocity is emphasized.

The report was prepared by Byrnon C. Colby with the cooperation of Russell P. Christensen and under the general supervision of Martin E. Nelson and Paul C. Benedict, who also reviewed the report.

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II. DEFINITIONS AND BASIC CONCEPTS

4. Definitions--Some basic concepts in the field of sedimentation are inherent in the definitions that are used. Also many sedimentation terms do not have universally accepted definitions. Hence, several definitions are listed before the basic concepts are discussed. The definitions published by the American Geophysical Union subcommittee on sediment terminology have been carefully considered and to some extent have been used as a guide. The following definitions are used in this report and are recommended for general acceptance.

The NOMINAL DIAMETER of a particle is the diameter of a sphere that has the same volume as the particle.[1]*

The SIEVE DIAMETER of a particle is the length of the side of the smallest square opening through which the given particle will pass.

The STANDARD FALL VELOCITY of a particle is the average rate of fall that the particle would finally attain if falling alone in quiescent distilled water of infinite extent and at a temperature of 24°C.

The STANDARD FALL DIAMETER, or simply FALL DIAMETER, of a particle is the diameter of a sphere that has a specific gravity of 2.65 and has the same standard fall velocity as the particle.

The SEDIMENTATION DIAMETER of a particle is the diameter of a sphere that has the same specific gravity and has the same terminal uniform settling velocity as the given particle in the same sedimentation fluid.[1] (However, this settling velocity may be in any fluid, at any temperature.)

The STANDARD SEDIMENTATION DIAMETER of a particle is the diameter of a sphere that has the same specific gravity and has the same standard fall velocity as the given particle.

SIZE-DISTRIBUTION, or simply DISTRIBUTION, when applied in relation to any of the size concepts, will specify frequency by weight rather than by particle count.

FALL VELOCITY and SETTLING VELOCITY are general terms which may apply to any rate of fall or settling as distinguished from standard fall velocity. In this paper fall velocity will designate rate of fall of a single particle in water.

SAND SIZES are particle sizes from .0625 to 2mm (62 to 2000 microns).

QUARTZ, as the term is used herein, indicates a specific gravity of 2.65 and does not imply special mineral content or crystalline structure.

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* Numbers in brackets indicate references listed on pages 51 and 52

5. <u>Physical size of sediment particles</u>--The basic concept of "size" of a sediment particle is best expressed in terms of volume, and the "true nominal diameter" defined as the diameter of the sphere of the same volume as the particle is a convenient evaluation of the size.[1,2] The expression "true nominal diameter" will be shortened to "nominal diameter" throughout this discussion. When some consideration of the density or mass of the particle is required, the specific gravity will be used.

6. <u>Sieve size of sediment particles</u>--Sieves are frequently used for the separation of sediments into "size" grades. However, as has been pointed out by Mitscherlich, sieves do not grade particles entirely by size but partly by shape as well.[3] Actually, many irregular particles with nominal diameters much greater than the sieve openings will pass a sieve. Ideally, a size distribution obtained by sieving would show the relative quantities of the sample which could pass sieves with stated sizes of uniform square openings.

The sieve diameter of a particle may be defined as the length of the side of the smallest square opening through which the given particle will pass. Therefore, the nominal and sieve diameters of a sphere are always equal.

When a group of particles is analyzed by sieving, the particles are sorted only at the sizes of sieves actually used; consequently, the size distribution of the grains retained between any two sieves is not determined. The average, mean, and median diameters of a group of particles depend not only on the range of sizes within the group but also on the frequency distribution of particle sizes within the range. Therefore, no average, mean, or median sieve diameter of the particles in a sieve fraction can be deduced directly from the openings of either the passing or retaining sieve or both. The arithmetic average or the geometric mean of the sizes of passing and retaining sieves sometimes is a useful approximation for the sieve diameter of the particles in a sieve fraction.

7. <u>Triaxial size of sediment particles</u>--Particle size may be based on the lengths a, b, and c of three axes of the particle. Writers generally define a, b, and c thus:

a = the longest (or major) axis of the particle,
b = the intermediate axis of the particle,
c = the shortest (or minor) axis of the particle,
with all axes mutually perpendicular.

The definition is not precise and differences in interpretation, and variations in measuring techniques are common. For example, because the axes are perpendicular, measurements of a, b, and c for a given particle depend on which axis is chosen first. No attempt will be made to standardize or evaluate the microscopic-size definitions and techniques. For present purposes, measurements of a, b, and c have been accepted as determined by various investigators. Averages of several such measurements are undoubtedly consistent enough to determine a usable shape factor and to indicate the general relation of the length of the b axis to the nominal diameter of the particle, and these are the only characteristics to be used in this discussion.

Because of the smallness of the particles, the axis measurements in this paper were determined with a microscope.

8. Fall velocity of a particle--The physical size of a sediment particle is not an adequate measure of the behavior of the particle in motion in a fluid. Studies of the transport of sediments in streams require a knowledge of the dynamic properties of the particles. Writers are generally agreed that the velocity of fall of the individual particle in quiet water is the most fundamental hydraulic characteristic which can be measured. Various terms such as "settling velocity", "fall velocity", "velocity of settling", or "velocity of fall" have been applied to this fundamental characteristic.

Two recent statements on "fall velocity" follow: "The fundamental property governing the motion of a sediment particle in a fluid is its fall velocity, a function of its volume, shape, density, and the viscosity and density of the fluid. As research in sediment transportation becomes more refined, it is necessary that sediments be classified on this basis."[4] "The term 'fall velocity', a term denoting the velocity of fall of an individual sediment particle in water, has gained general acceptance in hydraulic engineering and is presently conceded the most significant measurement of particle size."[5] These statements express our own understanding that the fall velocity is the most desirable basis for sedimentation size analyses.[6]

In common usage the term "fall velocity" has had a general meaning but not a precise definition. If data of various investigators are to be directly comparable, some definite standard for measuring fall velocity is necessary--for example, "standard fall velocity". The standard fall velocity of a particle may be defined as the average rate of fall that the particle would finally attain if falling alone in quiescent distilled water of infinite extent and at a temperature of 24°C. In this definition rate of fall is rate of change in altitude relative to a datum in the fluid. In an unlimited time a particle is assumed to reach its most stable orientation and the standard fall velocity will represent average terminal rate of fall in that position.

9. Fall diameter of a particle--The direct use of fall velocity as the basis for classification of sediments seems simple and logical. However, the size concept is so thoroughly embedded in sedimentation thinking that some measure of "radius" or "diameter" is strongly demanded. Also a designation of size, although only an approximation of physical size, is a convenience in some studies of bed shear and bed load. The term hydraulischer Werth (hydraulic value) was used by Schone in 1868 to define the diameter of a quartz sphere having the same settling velocity as a given particle in water.[3] Fall diameter, a comparable term, is used in this discussion. It is defined as the diameter of a sphere that has a specific gravity of 2.65 and has the same standard fall velocity as the particle.

The relation between standard fall velocity and fall diameter depends only on the well known and readily available relation between the velocity of fall and the diameter of a sphere of specific gravity 2.65.[6,7,8] A given particle has only one fall diameter and that diameter is independent of the type of material with which the particle is associated, the concentration in which it is found or analyzed, and the method of analysis. The conversion of standard fall velocity to fall diameter supplies a linear size by which the hydraulic size of the particle may be visualized readily.

10. Fall-diameter size distribution of a sample--The "size" distribution of a sample may be advantageously expressed in terms of fall diameter. The basic difficulty encountered in sedimentation analyses of samples derives from the generally recognized fact that a particle settling as one of a group of particles frequently falls with a velocity very different from that for the particle settling alone.[9,10] However, the fall diameter is based on the standard fall velocity of the individual particle, and the fall-diameter distribution of a sample is the distribution which would result if each particle in the sample were to fall with the same velocity that it would have if falling alone in an infinite extent of distilled water at a temperature of 24°C. Any sample has only one fall-diameter distribution. This fall-diameter distribution may also be described as the distribution if each particle were dropped separately and a summation were made of the fall diameters of the particles. The distribution considered here is based on weight of material and fall diameter, and the concept of a summation of individual particle fall diameters would necessarily include determining the distribution on the basis of weight. The size distribution by weight expressed in terms of standard fall velocity or fall diameter is believed to be the most basic and desirable expression for the sedimentation analysis of a sample.

11. Sedimentation diameter--Sedimentation diameter is defined as "the diameter of a sphere of the same specific gravity and the same terminal uniform settling velocity as the given particle in the same sedimentation fluid".[1] The need for a concept like fall diameter has been so great that the specific definition of sedimentation diameter has been commonly disregarded, and in general use sedimentation diameter has been considered equivalent to fall diameter as defined in this paper except that the water temperature was not designated. The relation of sedimentation diameter to settling velocity depends on the specific gravity of the particle. The definition of sedimentation diameter does not specify the fluid, or its temperature, in which the settling velocity may be determined. The settling velocity for any particle of a sediment sample varies depending on the characteristics of the sample, the concentration, the fluid, and the apparatus in which it is analyzed. Any one of the possible settling velocities is commonly used with a determination or estimate of specific gravity to establish sedimentation diameter. Sedimentation diameter has so many possible meanings that it is ambiguous unless specially defined or qualified in respect to all these possible variables.

Some of the conditions that affect the settling velocity of a particle obviously affect the sedimentation diameter. Because the density and viscosity of the fluid in which the settling velocity is determined are both part of the size-velocity relation the effect of the fluid on the sedimentation diameter may not be as obvious. If a quartz particle that has a nominal diameter of 1 mm and a shape factor of 0.7 falls in quiescent distilled water of infinite extent, the sedimentation diameter will depend on the water temperature thus:

Water temp. (°C)	Fall velocity (Table 1) (cm/sec)	Sedimentation diameter (Table 2) (mm)
0	10.4	0.820
24	12.3	0.761
40	13.0	0.725

Therefore, the sedimentation diameter varies with water temperature under relatively ideal conditions. If sedimentation diameters were compared for a particle in water and molasses, or if the particle were of another specific gravity, the disparity could be either greater or less.

Table 5 shows the errors if the fall velocity at 24°C is computed by the velocity-temperature relation for quartz spheres from the fall velocity measured at other water temperatures. Table 5 shows that sedimentation diameter must vary with water temperature.

12. Standard sedimentation diameter--Sedimentation diameter may be made more precise by the addition of the word standard. The standard sedimentation diameter of a particle may be defined as the diameter of a sphere that has the same specific gravity and has the same standard fall velocity as the given particle. So defined, the standard sedimentation diameter depends only on the volume and shape of the particle, and the relation of nominal diameter to standard sedimentation diameter becomes a measure of the effect of shape and roughness on the settling velocity of the particle in water at 24°C. There is only one standard sedimentation diameter for a particle and that figure is useful for comparing the effect of shape on the relations between nominal diameters or sieve diameters and diameters which depend on fall velocity.

For particles with a specific gravity of 2.65, fall diameter is the same as standard sedimentation diameter, and standard sedimentation diameter may be easily and accurately determined from standard fall velocity. For other specific gravities, conversion of velocity to either sedimentation diameter or standard sedimentation diameter is not as readily and accurately made unless tables or curves have been prepared for those specific gravities. If standard fall velocity is the fundamental hydraulic property of a sediment particle, the sedimentation diameter is not a direct measure of the fundamental property. Because of a difference in specific gravity, particle A may have a faster standard fall velocity than particle B, but B may have the larger sedimentation diameter.

13. Practical limitations--Several measures of particle size have been defined as a basis for simple, accurate, and practical determination and expression of sediment sizes. Because certain measures of particle size are sometimes difficult to evaluate, some are seldom determined and others are determined only approximately. Precise standard units of measure are desirable even though ideal conditions and results are seldom attained in routine analyses.

The nominal diameter of an irregular particle is difficult to measure accurately especially for particles of sand sizes and smaller. Hence, sieve diameter has sometimes been used as a substitute. Actually, for most irregular particles the nominal diameter is larger than the sieve diameter and the relative difference is generally greater at the smaller particle sizes. (See Fig. 5 which will be discussed later.)

Sieving a sand sample does not divide the individual particles precisely according to their sieve diameters for at least three reasons: (1) inaccuracies in size and shape of sieve openings, (2) sieving can seldom be continued until all particles which might pass a given sieve have had the necessary opportunity to do so, and (3) significant percentages of very fine material may be retained on a sieve because the fine material is clinging to larger particles.

The standard fall velocity of a particle of sand size or larger is not hard to determine with satisfactory accuracy because the fall of the particle may be followed visually while the particle settles in distilled water at 24°C in a transparent tube sufficiently large to make wall effects negligible. The fall of particles somewhat smaller than sand sizes may be traced with special lighting and photographic techniques, but for particles of clay and of the finer silt sizes the determination of the standard fall velocity of the individual particle may be very difficult if not impossible. For some particles unstable orientation may make a single determination of fall velocity unrepresentative, but this is seldom an important problem with natural sand particles.

The standard fall-velocity distribution for a sample may be determined accurately from the standard fall velocities of many individual particles if sufficient time and care are used. However, routine determinations of standard fall velocity for a sample or group of particles are usually based on settling rates of the particles falling in mass, even though the method generally requires calibration and the accuracy depends on the apparatus and techniques.

Standard fall velocity may be converted accurately to fall diameter by the relation between the diameter and fall velocity of quartz spheres, and the fall velocity of an individual particle in quiescent distilled water of sufficient extent to avoid wall effects may be converted to accurate fall diameter if the water temperature was close to 24°C. (See Tables 1 to 5 which will be discussed later.)

An accurate determination of sedimentation diameter requires an accurate specific gravity and, except for a specific gravity of 2.65, the conversion from settling velocity is not easily made because solution of a C_D vs R_e curve is required.

To minimize temperature corrections to fall velocity, a standard water temperature of 24°C has been adopted for the determination of standard fall velocity. Actual laboratory temperatures should seldom be far from this figure. However, not all determinations of fall velocity will be made at 24°C, and fall velocities at other temperatures will frequently be converted to fall velocities at 24°C. Computation of fall velocity at one temperature from that at another involves uncertainties which increase with increasing temperature differences.

III. BASIC FALL-VELOCITY RELATIONS

14. Drag coefficient and Reynolds number--The net gravitational force, F, of a particle in a fluid is its buoyant weight

F = volume x mass density difference x acceleration of gravity

$$= (\pi/6)\, d_n^3\, (\rho_s - \rho_f)\, g \qquad (1)$$

where d_n = diameter of the sphere that has the same volume as the particle (nominal diameter), in cm

ρ_s & ρ_f = mass density of the particle and fluid, respectively, in $gm\text{-}sec^2/cm^4$

g = acceleration owing to gravity (980.7 cm/sec^2)

An expression for F', the force resisting the fall of a particle through a fluid, was first developed by Sir Isaac Newton as

$F' = C_D A \rho_f v^2/2$ and, although the original derivation was based on inertial forces only, the general applicability of the relation has been substantiated by experimental data. In this equation

A = the projected area of the particle in a plane normal to the direction of motion, in cm^2

v = the terminal fall velocity (free from side-wall and mass-fall effects), in cm/sec

C_D = a drag coefficient, which has been found to vary with the particle geometry and the Reynolds number. For a given shape of particle, C_D varies only with the Reynolds number, which expresses the relative effect of inertial and viscous forces. The variation of C_D with Reynolds number allows the equation to apply to particles acted on by viscous as well as inertial forces.

When a particle falls at terminal velocity, $F = F'$,

and
$$C_D = \frac{(\pi/6)\, d_n^3\, (\rho_s - \rho_f)\, g}{A \rho_f v^2/2} \qquad (2)$$

if $A = (\pi/4)\, d_n^2$ as for spheres

$$C_D = \frac{4\, d_n\, (\rho_s - \rho_f)\, g}{3\, \rho_f\, v^2} \qquad (3)$$

For spheres and other particles for which $(\pi/4)\, d_n^2$ represents the projected area in a plane normal to the direction of motion, equation 3 is as universal as equation 2 and either equation provides a basis for direct comparison even between particles of different shapes. For all particles of any one shape, C_D will conform to a single curve of C_D vs Reynolds number, R_e, for which R_e expresses the relative effect of inertial and viscous forces. The Reynolds number can be expressed as

$$R_e = \frac{d\ v}{\nu}$$

d = a characteristic physical length or diameter of the particle (not necessarily the nominal diameter), in cm

ν = kinematic viscosity of the fluid, in cm^2/sec.

The relation between C_D and R_e has been established empirically for spheres and discs, and the relation for spheres is widely used.[8,11,13] From such a dimensionless plot one may find either the fall velocity or the nominal diameter if the other variables are known. Although the direct C_D versus R_e relation can be solved only by trial and error, modified plottings such as C_S and C_W versus R_e permit simpler computations of d and v.[8,11,13]

$$C_S = F/\rho_f \nu^2 = \frac{\pi d_n^3}{6} \frac{(\rho_s - \rho_f)g}{\rho_f \nu^2}$$ and also equals $(\pi/8) C_D R_e^2$

if C_D is computed from equation 3.

$$C_W = \frac{(\rho_s - \rho_f)\ g \nu}{\rho_f \quad v^3}$$ and also equals $\frac{3\ C_D}{4\ R_e}$ if C_D is computed

from equation 3.

Because velocity appears to the first power in the numerator of R_e and to the second power in the denominator of C_D, the product of C_D and R_e^2 is independent of the velocity and may be computed directly if the velocity, v, is unknown. Similarly, if C_D is divided by R_e, the result is independent of the diameter and may be computed if the diameter, d, is unknown.

The C_D versus R_e relations have been discussed on a theoretical basis. Experimental data will vary somewhat because determinations of C_D and R_e are not exact. Also R_e may be based on any one of several characteristic lengths of a particle, and C_D may be computed from the nominal diameter, or some other diameter or axis length. Sometimes numerical coefficients are dropped or changed for convenience in C_D versus R_e relations. In any comparison of data consistency must be maintained throughout.

15. Shape factor for irregular particles--If natural sediment particles were limited to a few shapes, a C_D versus R_e curve could be defined for each shape. Thus relations between particle, fluid, and fall velocity could be established empirically. Because the number of shapes is infinite, the logical approach is to choose some simple system for the classification of shape. A shape factor, S.F.[8], which appears as satisfactory as any is

$$S.F. = c / \sqrt{a\ b}$$

where a = longest axis

b = intermediate axis

c = shortest of the three mutually perpendicular axes of the particle.

If S.F. completely defined the particle shape as shape affects fall velocity, a curve of C_D versus R_e could be established for irregular particles of any given shape factor and the accuracy could be comparable to that for spheres. Because S.F. does not completely define the effect of shape, the C_D vs R_e figures for individual natural particles that have a given shape factor vary considerably from an average curve.

S.F. relates only three of a multitude of dimensions of an irregular particle and only approximately defines particle shape. There may be rounded, angular, rough, and smooth particles all with the same shape factor. To make S.F. more restrictive, especially as to roughness, data in this report are limited to "naturally worn" sediment particles. Shape factors based on roundness, sphericity, or other physical characteristics of particles might be used but they would not adequately define the shape for hydraulic studies. A determination of fall velocity is simpler than a determination of particle shape and weight, and the fall velocity expresses the hydraulic characteristics of a particle more satisfactorily.

16. Drag-coefficient curves for irregular particles--The drag coefficient, C_D, depends on the projected area, A, of a particle in a plane normal to the direction of motion, but for an irregular particle A is generally approximated, not determined directly. A particle will usually fall with the greatest projected area normal to the direction of motion so that an approximate area for the drag coefficient may be logically based on the a and b axes of the particle. Perhaps $(\pi/4)$a b is the best area on which to base C_D. For spheres equation 2 reduces to equation 3 if $(\pi/4)$a b is used. However, substitution of d_n^2 for a b tends to emphasize the effect of shape on C_D, so $(\pi/4)\ d_n^2$ will be used for the projected area in this discussion. (Fig. 4 shows that at low shape factors b is larger than d_n. Also a is larger than b by definition. The lower the shape factor the greater a b is in relation to d_n^2. When d_n^2 is used in the denominator of the equation for C_D, C_D is progressively larger for lower shape factors. If the larger--and perhaps more fundamental--a b is used in place of d_n^2, C_D is reduced and the reduction is greater at the lower shape factors. The reduction is such that at Reynolds numbers less than 50 the C_D curve based on $(\pi/4)$ a b is essentially the same for all shape factors.) The constant $\pi/4$ will be carried to conform to the generally accepted C_D for spheres and discs.

The Reynolds number requires a characteristic diameter or length, and the nominal diameter of the particle appears satisfactory.

Now if one has the shape factor, volume, density, and fall velocity of many particles and the density and viscosity of the fluid in which the fall velocity was determined, a relation of C_D to R_e may be plotted with shape as a third variable. This has been done in Fig. 1 with data for naturally worn sediments from a report entitled "Influence of shape on the fall velocity of sedimentary particles".[8] These curves are not the same as those in Fig. 14 of reference 8 for three reasons: (1) In the reference, points for spheroids, cylinders, prisms, and double cones and the curve for spheres were computed from equation 3

$$C_D = \frac{4\ d_n\ (\rho_s - \rho_f)\ g}{3\ \rho_f\ v^2}$$

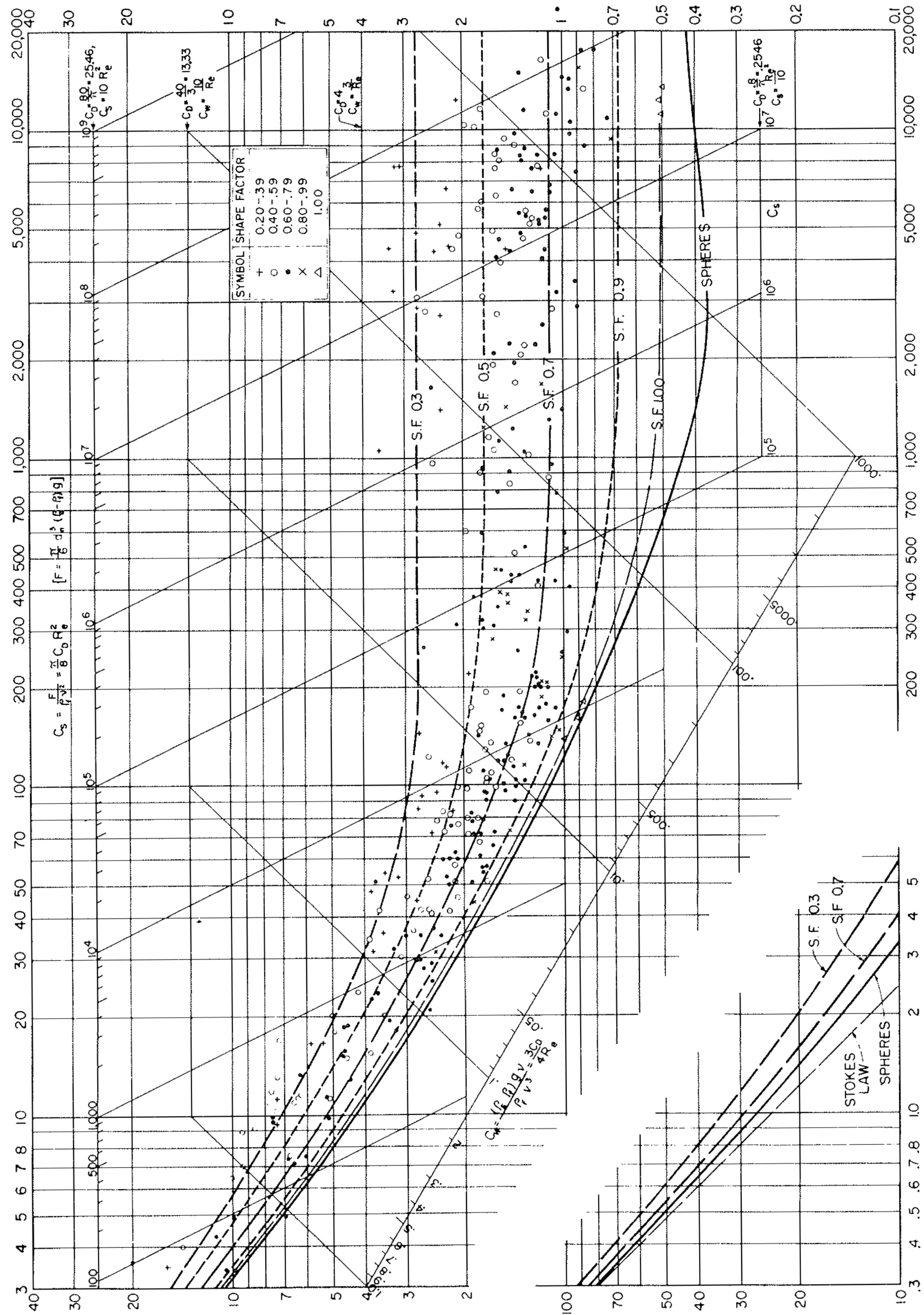


FIG. 1 - C_D vs. R_e RELATIONS FOR NATURALLY WORN SEDIMENT PARTICLES

(BASED ON DATA FROM REFERENCE NO. 8)

but the points for other particles were computed from

$$C_D = \frac{\pi d_n}{3} \frac{(\rho_s - \rho_f)\, g}{\rho_f \quad v^2}$$ that is, d_n^2 was used for the area A.

All drag coefficients plotted in Fig. 1 were computed from equation 3, and the recomputed data moved the C_D vs R_e curves upward except the curve for spheres. (2) The undulations in the curves of the reference have been smoothed partly because of a re-evaluation of the data and partly because a curve representing the average for many particles of different shapes (although of the same shape factor) probably should not show abrupt changes for small changes in Reynolds number, even though such changes might appear for individual particles or for a specific shape of particle. (3) Data for about 40 of the smallest particles were not given full weight because they were inconsistent with those for larger particles dropped in oil and the volume and specific gravity should be more accurate for the larger particles.

Fig. 1 is a preliminary attempt to define drag coefficients for particles of irregular shapes with shape classified by the shape factor

$$S.F. = c / \sqrt{a\, b}.$$

The basic data were obtained by different investigators using different sands and the results are not entirely consistent. As additional data become available the C_D vs R_e curves should be re-studied and revised as necessary. Meanwhile they are a guide to basic size and fall-velocity relations for irregular particles. Except for spheres, the curves are for naturally worn sediments and although spheres have a shape factor of 1.0, not all particles with a shape factor of 1.0 are spheres. Data for naturally worn particles with a shape factor of 1.0 diverge from the relation for spheres. C_D vs R_e curves for very angular particles are somewhat different from those of Fig. 1.

17. General fall-velocity relations--Table 1 lists the fall velocities for spheres and for naturally worn sediment particles at each of four shape factors, four specific gravities, six temperatures, and six nominal diameters. These were computed from the drag-coefficient curves of Fig. 1. In Table 1 the fall velocity of sediment particles is shown for a few conditions, but by interpolation the data may be used for a very wide range of fall velocities. Because most sediments have a specific gravity near 2.65, further expansion of the data will be limited to "quartz" particles although data for other specific gravities may be expanded as readily.

18. Size, shape, and fall velocity of quartz particles--Fig. 2 shows, for shape factors of 0.5, 0.7, and 0.9 the relation of the nominal diameter of a naturally worn quartz particle to its fall velocity in quiescent distilled water of infinite extent and at temperatures of 0°, 10°, 20°, 24°, 30°, and 40°C. The shape factor of 0.7 is about average for natural sediments.

The concept of resistance to fall within the range of Stokes' Law leads to the conclusion that the shape of the particle will have little or no effect on the fall velocity within this range. This conclusion has also been verified by experiments.[3] Data of Table 1 were expanded on the basis of the curves of Fig. 1 and were extended down to 60 microns on the assumption that the relations

TABLE 1

FALL VELOCITIES OF SEDIMENT PARTICLES

[In cm/sec]

Temp. (°C)	Specific gravity 2.00					Specific gravity 2.65					Temp. (°C)
	Shape factor					Shape factor					
	0.3	0.5	0.7	0.9	Spheres	0.3	0.5	0.7	0.9	Spheres	
	Nominal diameter = 0.20 mm										
0	0.84	0.90	0.95	1.00	1.06	1.29	1.38	1.48	1.57	1.66	0
10	1.04	1.12	1.20	1.26	1.33	1.56	1.68	1.81	1.92	2.05	10
20	1.21	1.32	1.42	1.51	1.60	1.78	1.94	2.11	2.26	2.43	20
24	1.27	1.40	1.51	1.61	1.72	1.86	2.04	2.23	2.40	2.58	24
30	1.36	1.50	1.63	1.75	1.87	1.99	2.18	2.40	2.59	2.80	30
40	1.51	1.67	1.83	1.98	2.13	2.18	2.41	2.68	2.90	3.16	40
	Nominal diameter = 0.50 mm										
0	2.79	3.14	3.47	3.79	4.04	4.01	4.47	5.02	5.48	5.92	0
10	3.19	3.61	4.02	4.41	4.73	4.50	5.12	5.72	6.30	6.88	10
20	3.53	3.99	4.47	4.95	5.35	4.90	5.63	6.31	7.02	7.68	20
24	3.63	4.13	4.64	5.16	5.58	5.03	5.79	6.53	7.30	7.97	24
30	3.80	4.32	4.88	5.43	5.90	5.24	6.03	6.84	7.66	8.38	30
40	4.02	4.62	5.25	5.87	6.40	5.52	6.38	7.30	8.24	9.05	40
	Nominal diameter = 1.00 mm										
0	5.76	6.59	7.47	8.50	9.20	7.83	9.04	10.4	11.8	12.8	0
10	6.16	7.16	8.23	9.36	10.3	8.21	9.66	11.4	13.0	14.3	10
20	6.39	7.58	8.86	10.2	11.2	8.49	10.1	12.1	14.0	15.6	20
24	6.45	7.70	9.10	10.5	11.6	8.57	10.2	12.3	14.3	16.0	24
30	6.54	7.88	9.38	10.9	12.1	8.66	10.4	12.6	14.8	16.6	30
40	6.65	8.09	9.80	11.4	12.9	8.77	10.6	13.0	15.6	17.5	40
	Nominal diameter = 2.00 mm										
0	9.50	11.4	13.8	16.3	18.1	12.4	14.9	18.4	22.1	25.2	0
10	9.66	11.7	14.4	17.4	19.8	12.5	15.3	19.0	23.1	27.3	10
20	9.73	11.9	14.8	18.1	21.1	12.5	15.5	19.3	23.9	28.9	20
24	9.76	12.0	14.9	18.3	21.6	12.6	15.6	19.4	24.0	29.4	24
30	9.79	12.1	15.1	18.7	22.2	12.6	15.7	19.5	24.3	30.1	30
40	9.83	12.3	15.3	19.0	23.1	12.6	15.8	19.7	24.7	31.0	40
	Nominal diameter = 4.00 mm										
0	13.8	17.2	21.4	26.8	32.9	17.7	22.3	27.8	34.9	43.8	0
10	13.8	17.3	21.6	27.3	34.6	17.7	22.4	27.9	35.3	45.8	10
20	13.8	17.4	21.8	27.5	35.9	17.8	22.4	28.0	35.6	46.9	20
24	13.8	17.5	21.9	27.6	36.3	17.8	22.4	28.1	35.7	47.2	24
30	13.9	17.5	21.9	27.8	36.8	17.8	22.5	28.1	35.8	47.6	30
40	13.9	17.6	22.0	28.0	37.4	17.8	22.5	28.2	35.9	48.1	40
	Nominal diameter = 8.00 mm										
0	19.5	24.7	30.9	39.2	52.4	25.1	31.7	39.6	50.4	67.5	0
10	19.5	24.7	30.9	39.2	53.0	25.1	31.7	39.6	50.4	67.5	10
20	19.5	24.8	30.9	39.3	53.1	25.1	31.7	39.7	50.5	67.5	20
24	19.5	24.8	30.9	39.3	53.1	25.1	31.7	39.7	50.5	67.5	24
30	19.6	24.8	31.0	39.4	53.0	25.2	31.8	39.8	50.6	67.5	30
40	19.7	24.9	31.1	39.5	52.9	25.2	31.9	39.9	50.7	67.5	40

TABLE 1 (Continued)

FALL VELOCITIES OF SEDIMENT PARTICLES

[In cm/sec]

Temp. (°C)	Specific gravity 4.30					Specific gravity 7.50					Temp. (°C)
	Shape factor					Shape factor					
	0.3	0.5	0.7	0.9	Spheres	0.3	0.5	0.7	0.9	Spheres	
	Nominal diameter = 0.20 mm										
0	2.21	2.42	2.62	2.76	2.94	3.73	4.08	4.44	4.76	5.16	0
10	2.60	2.87	3.14	3.36	3.61	4.30	4.75	5.28	5.75	6.19	10
20	2.95	3.27	3.62	3.90	4.22	4.80	5.35	6.01	6.58	7.10	20
24	3.09	3.42	3.80	4.11	4.45	5.00	5.58	6.28	6.88	7.44	24
30	3.27	3.64	4.06	4.40	4.78	5.25	5.90	6.64	7.30	7.92	30
40	3.57	3.98	4.48	4.87	5.29	5.65	6.40	7.19	7.93	8.67	40
	Nominal diameter = 0.50 mm										
0	6.41	7.26	8.19	9.06	9.90	10.0	11.5	13.0	14.5	15.9	0
10	7.10	8.15	9.22	10.3	11.3	10.8	12.6	14.4	16.2	17.8	10
20	7.62	8.79	10.1	11.3	12.4	11.4	13.4	15.5	17.7	19.5	20
24	7.79	9.04	10.4	11.7	12.8	11.6	13.7	15.9	18.3	20.1	24
30	7.99	9.32	10.8	12.2	13.4	11.8	14.0	16.5	19.0	21.0	30
40	8.24	9.76	11.4	13.0	14.3	12.0	14.4	17.3	20.1	22.3	40
	Nominal diameter = 1.00 mm										
0	11.7	13.8	16.2	18.5	20.4	17.0	20.3	24.5	28.6	31.8	0
10	12.1	14.4	17.3	20.2	22.5	17.3	20.9	25.7	30.7	34.8	10
20	12.3	14.8	18.1	21.5	24.3	17.5	21.3	26.4	32.0	37.1	20
24	12.4	14.9	18.4	21.9	25.0	17.5	21.4	26.6	32.5	37.9	24
30	12.4	15.1	18.7	22.5	25.8	17.6	21.6	26.9	33.0	39.1	30
40	12.5	15.3	19.1	23.2	27.1	17.7	21.8	27.3	33.8	40.9	40
	Nominal diameter = 2.00 mm										
0	17.7	21.7	26.9	32.8	38.3	24.9	30.7	38.4	47.7	58.4	0
10	17.7	22.0	27.4	33.9	41.2	24.9	31.2	38.9	48.8	61.2	10
20	17.7	22.2	27.7	34.6	43.4	24.9	31.4	39.2	49.4	63.4	20
24	17.7	22.3	27.8	34.8	44.2	24.9	31.5	39.3	49.6	64.0	24
30	17.8	22.4	27.9	35.1	45.1	24.9	31.5	39.4	49.8	65.0	30
40	17.8	22.5	28.0	35.4	46.4	25.0	31.6	39.5	50.1	66.2	40
	Nominal diameter = 4.00 mm										
0	25.1	31.7	39.5	49.9	65.5	35.2	44.5	55.6	70.6	94.1	0
10	25.1	31.7	39.6	50.2	66.8	35.2	44.5	55.6	70.7	95.2	10
20	25.1	31.7	39.7	50.4	67.7	35.2	44.5	55.7	70.8	95.7	20
24	25.1	31.7	39.7	50.5	68.0	35.2	44.5	55.7	70.8	95.7	24
30	25.1	31.8	39.7	50.5	68.3	35.2	44.6	55.7	70.9	95.6	30
40	25.2	31.8	39.8	50.6	68.6	35.3	44.7	55.8	71.0	95.4	40
	Nominal diameter = 8.00 mm										
0	35.4	44.8	56.0	71.3	96.3	49.7	62.9	78.6	100.0	134.3	0
10	35.4	44.8	56.0	71.3	95.4	49.7	62.9	78.6	100.0	132.1	10
20	35.5	44.8	56.1	71.4	94.5	49.8	62.9	78.7	100.0	130.3	20
24	35.5	44.9	56.1	71.4	94.1	49.8	63.0	78.7	100.1	129.7	24
30	35.5	44.9	56.2	71.5	93.6	49.8	63.0	78.8	100.2	128.9	30
40	35.6	45.0	56.3	71.6	92.8	49.9	63.1	79.0	100.4	127.8	40

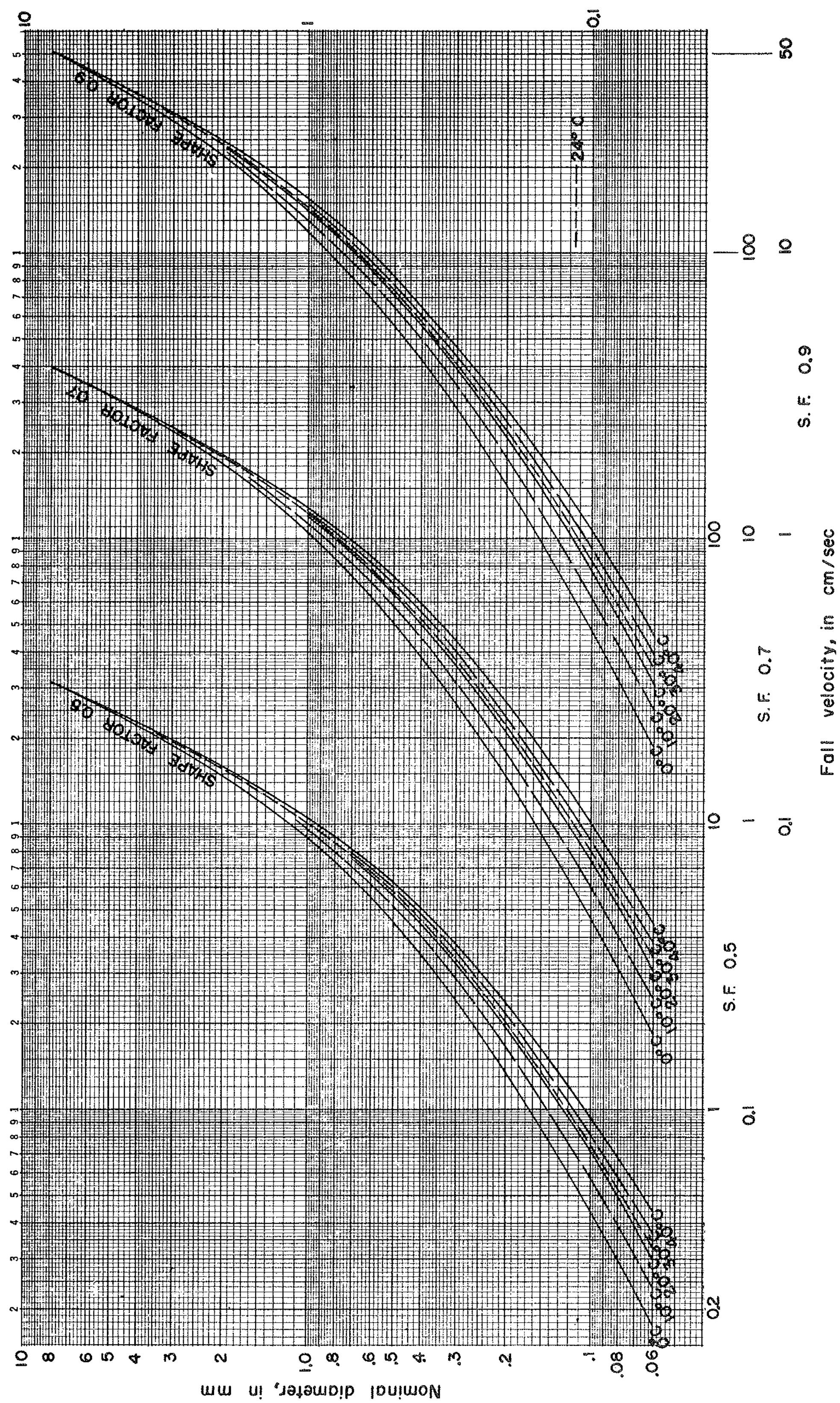


FIG. 2—RELATION OF NOMINAL DIAMETER AND FALL VELOCITY FOR NATURALLY WORN QUARTZ PARTICLES FALLING ALONE IN QUIESCENT DISTILLED WATER OF INFINITE EXTENT

of size and fall velocity would approach those for spheres.

The relations of diameter and fall velocity for quartz spheres can be obtained from Fig. 1. Because the data are readily available in graphical form [6,7,8], the presentation here is in a tabular form that is convenient for accurate computations. (See Table 2.)

19. Change in fall velocity with temperature--Tables 3 and 4 show the change in fall velocity with temperature for quartz spheres and for naturally worn quartz particles with a shape factor of 0.7. Table 4 was computed from the curves of Fig. 2. Table 3 was computed from similar data for quartz spheres. Except for Table 4, tables corresponding to Tables 2 and 3 for spheres were not prepared for other shapes because the additional work probably is not warranted until more basic data have been obtained. For most natural sediments Table 4 should give more accurate temperature corrections to fall velocity than Table 3, especially for extreme temperature ranges.

Primarily, Tables 3 and 4 are intended to simplify the computation of fall velocity in distilled water at 24°C from the fall velocity at another temperature. For example, if the fall velocity of a quartz sphere in water at 28°C is 1.10 cm per sec, Table 3 shows that the fall velocity at 24°C is 1.10 + (-0.077) or 1.02 cm per sec.

If one has the fall velocity at 24°C, the tables may also be used to determine the fall velocity at other temperatures. To determine from Table 3 the fall velocity at 0°C of the quartz sphere having a fall velocity of 1.02 cm per sec at 24°C, find the combination of velocity and change of velocity at 0°C that will show a fall velocity of 1.02 cm per sec at 24°C. A velocity of 0.50 and change of +0.40 show 0.90 cm per sec and a velocity of 0.60 and change of +0.46 show 1.06 cm per sec at 24°C. By interpolation, a velocity of 1.02 at 24°C would be equivalent to 0.58 cm per sec at 0°C.

Table 5 shows the error if, for a given particle, the velocity at 24°C was computed from the velocity at each other condition of Table 1 by the relation (Table 3) for quartz spheres falling in water, which is the relation usually used to compute the effect of temperature on fall velocity. Fall velocities corrected to 24°C from temperatures between 20° and 30°C were generally accurate within 2 percent for shape factors of 1.0 to 0.5 and at extreme shape factors of 0.3 the corrected velocities from Table 3 were within 4 percent. Average corrections for a given temperature range would be even more accurate. However, corrections over larger temperature ranges involve greater errors. (Some percentage error figures could not be computed for Table 5 because the relationships had not been defined for high velocities.)

TABLE 2

RELATION OF DIAMETER TO FALL VELOCITY
FOR QUARTZ SPHERES
[Fall diameter in microns]

Velocity (cm/sec)	Temperature in degrees Centigrade 0°	10°	20°	21°	22°	23°	24°	25°	26°	27°	28°	29°	30°	40°	Velocity (cm/sec)
0.10	45	38	33.4	33.0	32.6	32.2	31.9	31.5	31.2	30.9	30.6	30.3	30.0	27	0.10
0.20	63	54	47.4	46.8	46.2	45.6	45.0	44.5	44.0	43.5	43.1	42.6	42.2	38	0.20
0.30	77	66	58.5	57.6	56.8	56.0	55.2	54.5	53.8	53.2	52.7	52.2	51.8	47	0.30
0.40	90	76	67.8	66.8	65.8	64.9	64.0	63.2	62.4	61.7	61.0	60.4	59.8	55	0.40
0.50	101	86	76.5	75.3	74.1	73.0	72.0	71.1	70.2	69.4	68.6	67.9	67.2	62	0.50
0.70	121	104	92.0	90.4	89.1	87.9	86.8	85.8	84.8	83.8	82.8	81.9	81.0	75	0.70
0.90	140	120	106.7	105.1	103.6	102.2	100.9	99.7	98.5	97.4	96.3	95.3	94.3	87	0.90
1.10	157	135	120.8	119.0	117.4	115.9	114.5	113.2	111.9	110.7	109.5	108.3	107.2	98	1.10
1.30	174	150	134.4	132.4	130.7	129.1	127.6	126.1	124.7	123.3	121.9	120.6	119.3	108	1.30
1.50	190	164	147.5	145.3	143.4	141.7	140.1	138.5	137.0	135.5	134.0	132.6	131.2	118	1.50
1.70	206	178	160.1	157.8	155.8	153.9	152.2	150.6	149.0	147.4	145.9	144.4	142.9	129	1.70
1.90	222	191	172.3	169.7	167.5	165.6	163.9	162.2	160.6	159.0	157.4	155.8	154.3	139	1.90
2.10	237	204	184.1	181.6	179.4	177.4	175.5	173.7	171.9	170.2	168.5	166.9	165.3	149	2.10
2.30	252	217	195.6	193.1	190.9	188.9	187.0	185.1	183.2	181.4	179.6	177.9	176.2	159	2.30
2.50	267	230	206.9	204.4	202.2	200.1	198.1	196.1	194.2	192.3	190.4	188.5	186.7	169	2.50
3.00	304	262	234.9	232.4	230.1	227.8	225.6	223.5	221.4	219.3	217.2	215.1	213.0	193	3.00
3.50	340	294	263.1	260.5	258.0	255.6	253.2	250.8	248.4	246.0	243.6	241.3	239.0	217	3.50
4.00	375	326	291.4	288.7	286.0	283.4	280.8	278.2	275.6	273.0	270.4	267.8	265.2	242	4.00
4.50	409	357	319.8	316.9	314.0	311.2	308.4	305.6	302.8	300.0	297.2	294.4	291.6	267	4.50
5.00	443	389	348.4	345.3	342.2	339.2	336.2	333.2	330.2	327.2	324.2	321.2	318.2	292	5.00
6.00	512	452	406.0	402.5	399.0	395.6	392.2	388.8	385.4	382.0	378.6	375.3	372.0	342	6.00
7.00	581	513	463.6	459.8	456.0	452.3	448.6	444.9	441.2	437.6	434.0	430.4	426.8	394	7.00
8.00	650	577	522.0	518.0	514.0	510.0	506.0	502.0	498.0	494.0	490.0	486.0	482.0	447	8.00
9.00	720	641	581.0	576.7	572.4	568.1	563.8	559.5	555.2	550.9	546.6	542.4	538.2	501	9.00
10.00	791	706	641.0	636.0	631.4	626.6	622.0	617.5	613.0	608.6	604.2	599.8	595.4	556	10.00
11.00	864	773	702	696	691	686	681	676	671	667	662	657	653	612	11.00
12.00	938	841	765	759	753	747	742	737	732	727	722	717	712	668	12.00
13.00	1014	910	829	823	817	811	805	799	793	788	783	778	773	725	13.00
14.00	1090	980	893	887	881	875	869	863	857	851	845	840	835	784	14.00
15.00	1170	1050	958	951	945	939	933	927	921	915	919	903	898	844	15.00
16.00	1250	1120	1026	1018	1011	1004	998	992	986	980	974	968	962	906	16.00
17.00	1330	1192	1094	1086	1078	1070	1063	1056	1050	1044	1038	1032	1026	969	17.00
18.00	1410	1264	1162	1153	1144	1136	1129	1122	1115	1108	1102	1196	1091	1033	18.00
19.00	1490	1336	1230	1221	1213	1205	1197	1190	1183	1176	1169	1163	1157	1098	19.00
20.00	1570	1410	1300	1291	1282	1274	1266	1259	1252	1245	1238	1231	1225	1165	20.00
22.00	1730	1560	1444	1434	1425	1416	1408	1400	1393	1386	1379	1372	1365	1302	22.00
24.00	1900	1710	1592	1581	1571	1561	1552	1544	1537	1530	1523	1516	1509	1444	24.00
26.00	2080	1870	1742	1731	1721	1711	1702	1694	1686	1678	1671	1664	1657	1590	26.00
28.00	2280	2040	1906	1894	1883	1872	1862	1853	1845	1837	1829	1821	1813	1742	28.00
30.00	2480	2230	2079	2067	2055	2043	2032	2022	2013	2014	1996	1988	1980	1904	30.00
35.00	2990	2730	2556	2544	2532	2521	2510	2501	2492	2484	2476	2468	2460	2380	35.00
40.00	3510	3250	3074	3062	3051	3040	3030	3021	3012	3004	2996	2988	2980	2900	40.00
45.00	4080	3830	3654	3742	3731	3720	3610	3601	3592	3584	3576	3568	3560	3490	45.00
50.00	4700	4470	4342	4331	4320	4310	4300	4293	4286	4279	4272	4266	4260	4190	50.00
60.00	6500	6320	6230	6222	6214	6207	6200	6193	6186	6179	6172	6166	6160	6100	60.00

TABLE 3

CHANGE OF FALL VELOCITY WITH WATER TEMPERATURE

FOR QUARTZ SPHERES

[Fall velocity changes in cm/sec]

Velocity	Temperature in degrees Centigrade														Velocity
(cm/sec)	0°	10°	20°	21°	22°	23°	24°	25°	26°	27°	28°	29°	30°	40°	(cm/sec)
0.10	+0.10	+0.042	+.010	+.007	+.004	+.002	0	-.002	-.005	-.007	-.009	-.011	-.014	-.030	0.10
0.20	+0.19	+0.085	+.022	+.016	+.010	+.005	0	-.005	-.009	-.014	-.018	-.022	-.026	-.056	0.20
0.30	+0.27	+0.125	+.032	+.024	+.016	+.008	0	-.007	-.014	-.020	-.026	-.032	-.038	-.080	0.30
0.40	+0.34	+0.16	+.041	+.030	+.020	+.010	0	-.009	-.018	-.026	-.034	-.042	-.050	-.10	0.40
0.50	+0.40	+0.19	+.050	+.037	+.024	+.012	0	-.011	-.022	-.032	-.042	-.051	-.060	-.12	0.50
0.60	+0.46	+0.22	+.058	+.042	+.027	+.013	0	-.012	-.024	-.036	-.048	-.059	-.070	-.14	0.60
0.70	+0.51	+0.24	+.066	+.048	+.031	+.015	0	-.014	-.028	-.041	-.054	-.067	-.079	-.16	0.70
0.80	+0.56	+0.27	+.074	+.052	+.033	+.016	0	-.016	-.031	-.046	-.061	-.075	-.088	-.18	0.80
0.90	+0.60	+0.29	+.081	+.058	+.037	+.018	0	-.017	-.034	-.050	-.066	-.082	-.097	-.21	0.90
1.00	+0.65	+0.32	+.088	+.063	+.040	+.019	0	-.019	-.037	-.055	-.072	-.089	-.105	-.23	1.00
1.10	+0.69	+0.34	+.095	+.067	+.043	+.021	0	-.020	-.039	-.058	-.077	-.095	-.113	-.25	1.10
1.20	+0.73	+0.36	+.102	+.072	+.046	+.022	0	-.021	-.041	-.062	-.082	-.102	-.121	-.27	1.20
1.30	+0.77	+0.38	+.108	+.076	+.049	+.024	0	-.023	-.045	-.067	-.088	-.108	-.128	-.29	1.30
1.40	+0.81	+0.40	+.114	+.081	+.052	+.025	0	-.024	-.047	-.070	-.092	-.114	-.136	-.32	1.40
1.50	+0.85	+0.42	+.120	+.085	+.054	+.026	0	-.025	-.050	-.074	-.098	-.122	-.143	-.34	1.50
1.60	+0.89	+0.44	+.126	+.090	+.058	+.028	0	-.026	-.052	-.077	-.102	-.126	-.150	-.36	1.60
1.70	+0.93	+0.46	+.132	+.095	+.061	+.029	0	-.027	-.054	-.080	-.106	-.132	-.157	-.38	1.70
1.80	+0.98	+0.48	+.138	+.100	+.063	+.030	0	-.028	-.056	-.083	-.110	-.137	-.163	-.40	1.80
1.90	+1.02	+0.50	+.144	+.102	+.065	+.031	0	-.029	-.058	-.086	-.114	-.142	-.169	-.41	1.90
2.00	+1.06	+0.52	+.150	+.106	+.067	+.032	0	-.030	-.060	-.089	-.116	-.146	-.174	-.43	2.00
3.00	+1.42	+0.69	+.18	+.13	+.08	+.04	0	-.04	-.08	-.12	-.15	-.19	-.23	-.57	3.00
4.00	+1.66	+0.83	+.21	+.15	+.10	+.05	0	-.05	-.10	-.14	-.19	-.24	-.28	-.69	4.00
5.00	+1.87	+0.94	+.23	+.17	+.11	+.06	0	-.06	-.12	-.17	-.23	-.28	-.33	-.80	5.00
6.00	+2.06	+1.04	+.25	+.18	+.12	+.06	0	-.06	-.12	-.18	-.24	-.30	-.36	-.89	6.00
7.00	+2.25	+1.12	+.27	+.20	+.13	+.07	0	-.07	-.13	-.19	-.26	-.32	-.38	-.97	7.00
8.00	+2.44	+1.21	+.29	+.21	+.14	+.07	0	-.07	-.14	-.21	-.28	-.34	-.41	-1.04	8.00
9.00	+2.62	+1.29	+.31	+.23	+.15	+.08	0	-.07	-.14	-.22	-.29	-.36	-.43	-1.11	9.00
10.00	+2.80	+1.36	+.33	+.25	+.16	+.08	0	-.08	-.16	-.24	-.32	-.39	-.45	-1.17	10.00
15.00	+3.60	+1.75	+.43	+.32	+.21	+.10	0	-.10	-.19	-.28	-.37	-.46	-.55	-1.40	15.00
20.00	+4.20	+2.00	+.50	+.37	+.24	+.12	0	-.11	-.21	-.31	-.41	-.51	-.60	-1.50	20.00
25.00	+4.30	+2.10	+.55	+.40	+.26	+.13	0	-.11	-.21	-.31	-.41	-.51	-.60	-1.50	25.00
30.00	+4.40	+2.10	+.50	+.37	+.25	+.12	0	-.11	-.21	-.31	-.41	-.51	-.60	-1.50	30.00
35.00	+4.30	+2.00	+.45	+.33	+.22	+.11	0	-.10	-.19	-.28	-.37	-.46	-.55	-1.40	35.00
40.00	+4.10	+1.80	+.40	+.30	+.20	+.11	0	-.09	-.18	-.25	-.33	-.41	-.50	-1.30	40.00
45.00	+3.40	+1.50	+.35	+.27	+.18	+.09	0	-.07	-.14	-.21	-.27	-.33	-.40	-1.05	45.00
50.00	+2.30	+1.00	+.25	+.18	+.12	+.06	0	-.05	-.10	-.15	-.20	-.25	-.30	-0.80	50.00
60.00	+1.10	+0.50	+.12	+.09	+.06	+.03	0	-.03	-.06	-.10	-.13	-.16	-.20	-0.50	60.00

NOTE: To the velocity at a given temperature add the change from this table to obtain the fall velocity for the same quartz sphere in distilled water at 24°C.

TABLE 4

CHANGE OF FALL VELOCITY WITH WATER TEMPERATURE

NATURALLY WORN QUARTZ PARTICLES HAVING SHAPE FACTOR OF 0.7

[Fall velocity changes in cm/sec]

Velocity (cm/sec)	Temperature in degrees Centigrade														Velocity (cm/sec)
	0°	10°	20°	21°	22°	23°	24°	25°	26°	27°	28°	29°	30°	40°	
0.10	+0.10	+.040	+.010	+.007	+.004	+.002	0	-.002	-.004	-.007	-.009	-.011	-.013	-.029	0.10
0.20	+0.19	+.080	+.020	+.015	+.009	+.004	0	-.004	-.009	-.013	-.017	-.021	-.025	-.054	0.20
0.30	+0.26	+.12	+.030	+.022	+.014	+.007	0	-.006	-.013	-.019	-.025	-.030	-.035	-.076	0.30
0.40	+0.32	+.15	+.039	+.028	+.018	+.009	0	-.008	-.017	-.025	-.032	-.039	-.046	-.094	0.40
0.50	+0.37	+.18	+.047	+.034	+.022	+.011	0	-.010	-.020	-.030	-.039	-.048	-.056	-.110	0.50
0.60	+0.43	+.21	+.055	+.039	+.025	+.012	0	-.012	-.023	-.034	-.045	-.055	-.065	-.13	0.60
0.70	+0.48	+.23	+.063	+.045	+.029	+.014	0	-.013	-.026	-.039	-.051	-.062	-.073	-.15	0.70
0.80	+0.53	+.26	+.070	+.049	+.032	+.015	0	-.015	-.029	-.043	-.056	-.068	-.080	-.16	0.80
0.90	+0.57	+.28	+.076	+.054	+.035	+.017	0	-.016	-.032	-.047	-.061	-.075	-.086	-.18	0.90
1.00	+0.61	+.31	+.083	+.059	+.038	+.018	0	-.018	-.035	-.051	-.067	-.081	-.092	-.22	1.00
1.10	+0.65	+.32	+.089	+.063	+.041	+.020	0	-.019	-.038	-.055	-.071	-.086	-.098	-.22	1.10
1.20	+0.68	+.34	+.095	+.068	+.044	+.021	0	-.020	-.040	-.059	-.076	-.091	-.104	-.24	1.20
1.30	+0.72	+.36	+.100	+.072	+.047	+.023	0	-.022	-.043	-.062	-.080	-.096	-.110	-.25	1.30
1.40	+0.75	+.38	+.105	+.076	+.049	+.024	0	-.023	-.045	-.065	-.083	-.100	-.115	-.27	1.40
1.50	+0.78	+.39	+.110	+.080	+.052	+.025	0	-.024	-.047	-.068	-.087	-.104	-.120	-.28	1.50
1.60	+0.81	+.41	+.114	+.083	+.054	+.026	0	-.025	-.048	-.070	-.090	-.108	-.125	-.30	1.60
1.70	+0.84	+.43	+.118	+.086	+.056	+.027	0	-.026	-.050	-.073	-.094	-.113	-.130	-.32	1.70
1.80	+0.87	+.44	+.122	+.089	+.058	+.028	0	-.027	-.052	-.076	-.098	-.117	-.135	-.33	1.80
1.90	+0.90	+.46	+.126	+.092	+.060	+.029	0	-.028	-.054	-.079	-.102	-.122	-.140	-.34	1.90
2.00	+0.93	+.48	+.130	+.095	+.062	+.030	0	-.028	-.055	-.081	-.105	-.126	-.145	-.36	2.00
2.50	+1.06	+.56	+.15	+.11	+.07	+.03	0	-.03	-.06	-.09	-.12	-.15	-.17	-.42	2.50
3.00	+1.18	+.62	+.16	+.12	+.08	+.04	0	-.04	-.07	-.10	-.14	-.17	-.19	-.47	3.00
3.50	+1.29	+.68	+.18	+.13	+.08	+.04	0	-.04	-.08	-.11	-.15	-.18	-.21	-.52	3.50
4.00	+1.39	+.73	+.19	+.14	+.09	+.04	0	-.04	-.09	-.12	-.16	-.20	-.23	-.57	4.00
4.50	+1.48	+.78	+.20	+.14	+.09	+.04	0	-.04	-.09	-.13	-.18	-.22	-.25	-.62	4.50
5.00	+1.56	+.82	+.21	+.15	+.10	+.05	0	-.05	-.10	-.14	-.19	-.23	-.27	-.66	5.00
6.00	+1.70	+.90	+.22	+.16	+.11	+.05	0	-.05	-.11	-.16	-.21	-.26	-.31	-.73	6.00
7.00	+1.82	+.95	+.23	+.17	+.11	+.06	0	-.06	-.11	-.17	-.22	-.28	-.33	-.78	7.00
8.00	+1.91	+1.00	+.24	+.18	+.12	+.06	0	-.06	-.12	-.18	-.24	-.29	-.34	-.80	8.00
9.00	+1.98	+1.02	+.24	+.18	+.12	+.06	0	-.06	-.12	-.18	-.24	-.29	-.34	-.80	9.00
10.00	+1.98	+1.02	+.24	+.18	+.12	+.06	0	-.06	-.12	-.18	-.24	-.29	-.34	-.80	10.00
15.00	+1.70	+.85	+.20	+.15	+.10	+.05	0	-.05	-.10	-.15	-.20	-.25	-.30	-.68	15.00
20.00	+1.10	+.55	+.14	+.10	+.07	+.03	0	-.04	-.07	-.11	-.14	-.17	-.20	-.45	20.00
25.00	+0.50	+.25	+.08	+.06	+.04	+.02	0	-.02	-.03	-.05	-.06	-.08	-.09	-.20	25.00
30.00	+0.20	+.10	+.03	+.02	+.01	+.01	0	-.01	-.01	-.02	-.03	-.03	-.04	-.08	30.00
35.00	+0.13	+.06	+.02	+.01	+.01	0	0	0	-.01	-.01	-.02	-.02	-.02	-.05	35.00
40.00	+0.06	+.03	+.01	+.01	0	0	0	0	0	0	-.01	-.01	-.01	-.02	40.00

NOTE: To the velocity at a given temperature add the change from this table to obtain the fall velocity for the same quartz particle in distilled water at 24°C.

TABLE 5

PERCENTAGE ERRORS IF FALL VELOCITY AT OTHER TEMPERATURES IS ADJUSTED TO 24°C BY THE RELATION FOR QUARTZ SPHERES

[Actual fall velocities are shown in parentheses at 24°C because errors are zero]

	Specific gravity 2.00					Specific gravity 2.65					
Temp. (°C)	Shape factor 0.3	0.5	0.7	0.9	Spheres	Shape factor 0.3	0.5	0.7	0.9	Spheres	Temp. (°C)
	Nominal diameter = 0.20 mm										
0	+12	+ 8	+ 5	+ 2	+ 1	+12	+ 7	+ 4	+ 2	0	0
10	+ 8	+ 4	+ 3	+ 1	0	+ 7	+ 5	+ 3	+ 1	0	10
20	+ 3	+ 2	+ 2	+ 1	0	+ 3	+ 2	+ 1	+ 1	0	20
24	(1.27)	(1.40)	(1.51)	(1.61)	(1.72)	(1.86)	(2.04)	(2.23)	(2.40)	(2.58)	24
30	- 3	- 3	- 2	- 1	- 1	- 2	- 2	- 1	- 1	0	30
40	- 8	- 7	- 5	- 4	- 2	- 7	- 6	- 4	- 3	0	40
	Nominal diameter = 0.50 mm										
0	+14	+11	+ 8	+ 5	+ 2	+13	+ 8	+ 5	+ 2	0	0
10	+ 8	+ 6	+ 4	+ 3	+ 1	+ 7	+ 5	+ 3	+ 1	0	10
20	+ 3	+ 2	+ 1	+ 1	0	+ 3	+ 2	+ 1	0	0	20
24	(3.63)	(4.13)	(4.64)	(5.16)	(5.58)	(5.03)	(5.79)	(6.53)	(7.30)	(7.97)	24
30	- 3	- 3	- 2	- 1	- 1	- 3	- 2	- 1	- 1	0	30
40	- 8	- 7	- 5	- 3	- 2	- 7	- 6	- 3	- 2	0	40
	Nominal diameter = 1.00 mm										
0	+20	+14	+ 8	+ 5	+ 2	+19	+14	+ 8	+ 4	0	0
10	+12	+ 8	+ 4	+ 2	+ 1	+10	+ 8	+ 5	+ 2	0	10
20	+ 3	+ 2	+ 1	0	0	+ 2	+ 2	+ 2	+ 1	0	20
24	(6.45)	(7.70)	(9.10)	(10.5)	(11.6)	(8.57)	(10.2)	(12.3)	(14.3)	(16.0)	24
30	- 4	- 3	- 2	- 1	0	- 4	- 3	- 2	0	0	30
40	-12	- 9	- 5	- 3	0	-10	- 8	- 5	- 1	0	40
	Nominal diameter = 2.00 mm										
0	+25	+21	+16	+10	+ 2	+24	+19	+15	+10	0	0
10	+13	+10	+ 8	+ 5	+ 1	+12	+ 9	+ 8	+ 5	0	10
20	+ 3	+ 2	+ 2	+ 1	0	+ 2	+ 2	+ 2	+ 2	0	20
24	(9.76)	(12.0)	(14.9)	(18.3)	(21.6)	(12.6)	(15.6)	(19.4)	(24.0)	(29.4)	24
30	- 4	- 3	- 2	- 1	0	- 4	- 3	- 3	- 1	0	30
40	-11	- 9	- 7	- 4	0	-11	- 8	- 6	- 3	0	40
	Nominal diameter = 4.00 mm										
0	+23	+20	+17	+13	+ 3	+21	+18	+14	+10	0	0
10	+12	+10	+ 8	+ 7	+ 1	+10	+ 9	+ 7	+ 5	0	10
20	+ 3	+ 2	+ 2	+ 1	0	+ 2	+ 2	+ 2	+ 1	0	20
24	(13.8)	(17.5)	(21.9)	(27.6)	(36.3)	(17.8)	(22.4)	(28.1)	(35.7)	(47.2)	24
30	- 3	- 3	- 3	- 1	0	- 3	- 2	- 2	- 1	0	30
40	- 9	- 8	- 6	- 4	- 1	- 8	- 6	- 5	- 3	0	40
	Nominal diameter = 8.00 mm										
0	+21	+17	+14	+10	+ 2	+17	+14	+10	+ 4	-	0
10	+10	+ 8	+ 6	+ 4	+ 1	+ 8	+ 7	+ 4	+ 2	-	10
20	+ 3	+ 2	+ 1	+ 1	0	+ 2	+ 2	+ 1	0	-	20
24	(19.5)	(24.8)	(30.9)	(39.3)	(53.1)	(25.1)	(31.7)	(39.7)	(50.5)	(67.5)	24
30	- 3	- 2	- 2	- 1	- 1	- 2	- 2	- 1	0	-	30
40	- 7	- 6	- 4	- 3	- 2	- 6	- 4	- 3	- 1	-	40

TABLE 5 (Continued)

PERCENTAGE ERRORS IF FALL VELOCITY AT OTHER TEMPERATURES IS ADJUSTED TO 24°C BY THE RELATION FOR QUARTZ SPHERES

[Actual fall velocities are shown in parentheses at 24°C because errors are zero]

Temp. (°C)	Specific gravity 4.30 — Shape factor 0.3	0.5	0.7	0.9	Spheres	Specific gravity 7.50 — Shape factor 0.3	0.5	0.7	0.9	Spheres	Temp. (°C)
	Nominal diameter = 0.20 mm										
0	+ 8	+ 6	+ 3	0	- 2	+ 7	+ 3	- 1	- 4	- 5	0
10	+ 4	+ 3	+ 1	0	- 1	+ 3	+ 1	0	- 2	- 3	10
20	+ 1	+ 1	0	0	0	+ 1	0	0	- 1	- 1	20
24	(3.09)	(3.42)	(3.80)	(4.11)	(4.45)	(5.00)	(5.58)	(6.28)	(6.88)	(7.44)	24
30	- 2	- 1	0	0	0	- 2	- 1	0	0	+ 1	30
40	- 5	- 4	- 2	- 1	0	- 5	- 2	- 1	0	+ 2	40
	Nominal diameter = 0.50 mm										
0	+10	+ 5	+ 2	0	- 1	+10	+ 6	+ 2	- 2	- 2	0
10	+ 6	+ 4	+ 1	0	- 1	+ 5	+ 3	+ 1	- 2	- 2	10
20	+ 1	+ 1	0	0	0	+ 1	+ 1	0	- 1	- 1	20
24	(7.79)	(9.04)	(10.4)	(11.7)	(12.8)	(11.6)	(13.7)	(15.9)	(18.3)	(20.1)	24
30	- 3	- 2	- 1	0	+ 1	- 3	- 2	0	+ 1	+ 1	30
40	- 8	- 5	- 2	0	+ 1	- 8	- 5	0	+ 2	+ 3	40
	Nominal diameter = 1.00 mm										
0	+19	+16	+ 8	+ 3	- 2	+19	+14	+ 8	+ 2	- 5	0
10	+10	+ 8	+ 4	+ 1	- 2	+10	+ 7	+ 4	+ 1	- 3	10
20	+ 2	+ 2	+ 1	0	- 1	+ 3	+ 2	+ 1	0	- 1	20
24	(12.4)	(14.9)	(18.4)	(21.9)	(25.0)	(17.5)	(21.4)	(26.6)	(32.5)	(37.9)	24
30	- 4	- 2	- 2	0	+ 1	- 3	- 2	- 1	0	+ 2	30
40	-10	- 7	- 4	- 1	+ 2	- 7	- 5	- 3	0	+ 5	40
	Nominal diameter = 2.00 mm										
0	+22	+16	+12	+ 6	- 4	+17	+11	+ 8	+ 2	-	0
10	+11	+ 8	+ 6	+ 2	- 3	+ 8	+ 6	+ 4	+ 1	-	10
20	+ 3	+ 2	+ 2	+ 1	- 1	+ 2	+ 1	+ 1	0	-	20
24	(17.7)	(22.3)	(27.8)	(34.8)	(44.2)	(24.9)	(31.5)	(39.3)	(49.6)	(64.0)	24
30	- 3	- 2	- 2	- 1	+ 1	- 2	- 2	- 1	0	-	30
40	- 8	- 6	- 5	- 2	+ 3	- 6	- 4	- 3	- 1	-	40
	Nominal diameter = 4.00 mm										
0	+17	+14	+10	+ 3	-	+12	+ 8	+ 3	-	-	0
10	+ 8	+ 7	+ 4	+ 1	-	+ 6	+ 3	+ 1	-	-	10
20	+ 2	+ 2	+ 1	0	-	+ 1	+ 1	0	-	-	20
24	(25.1)	(31.7)	(39.7)	(50.5)	(68.0)	(35.2)	(44.5)	(55.7)	(70.8)	(95.7)	24
30	- 2	- 2	- 1	- 1	-	- 2	- 1	0	-	-	30
40	- 6	- 4	- 3	- 1	-	- 4	- 2	- 1	-	-	40
	Nominal diameter = 8.00 mm										
0	+12	+ 7	+ 3	-	-	+ 4	-	-	-	-	0
10	+ 5	+ 3	+ 1	-	-	+ 2	-	-	-	-	10
20	+ 1	+ 1	0	-	-	+ 1	-	-	-	-	20
24	(35.5)	(44.9)	(56.1)	(71.4)	(94.1)	(49.8)	(63.0)	(78.7)	(100.1)	(129.7)	24
30	- 2	- 1	0	-	-	- 1	-	-	-	-	30
40	- 4	- 2	- 1	-	-	- 2	-	-	-	-	40

IV. RELATIONS BETWEEN PARTICLE SIZE SYSTEMS

20. Nominal diameter and fall diameter of quartz particles--The relation between the nominal diameter (physical size) and the settling velocity of a particle depends on the density, volume, and shape of the particle and on the density and viscosity of the fluid. Within the limits of definition of Fig. 1, the settling velocity of any naturally worn sediment particle may be computed if the above characteristics of particle and fluid are known. Because the shape factor only approximates the effective shape of an individual particle, the relation of fall velocity and fall diameter for an irregular particle may differ considerably from that shown by the average curves of Fig. 1.

For simplicity the following discussion will be limited to quartz particles. For a quartz particle the nominal diameter and shape of a particle determine its fall diameter, which by definition depends on fall velocity in distilled water at 24°C. (See Fig. 3, which was computed from the fall velocity at 24°C, shown in Fig. 2 for shape factors of 0.5, 0.7, and 0.9.) For all spherical quartz particles, the nominal diameter is equal to the fall diameter by definition. The effect of the irregular shapes of sand particles is generally such that the particles fall more slowly than would spheres of the same volume and specific gravity. That is, the fall diameter of an irregular quartz particle is less than the nominal diameter of the particle.

Fig. 3 shows the effect of particle shape on fall diameter and indicates the rapid increase in effect of particle shape with increasing size within the sand size range.

21. Nominal diameter and intermediate-axis length--For this discussion the length of the intermediate axis of a sediment particle is the "b" dimension that was determined with a microscope. Because the shape factor ($c / \sqrt{a\ b}$) is actually a flatness ratio, the relation of nominal diameter to intermediate-axis length varies with the shape factor. (See Fig. 4, which is based on data from reference 8.) The average relation is fairly well defined for each shape factor, but because of the wide variations in shapes covered by a single shape factor the individual particles vary significantly from the average. If a particle is flat, the intermediate-axis length is relatively large in comparison to the volume of the particle.

22. Nominal diameter and sieve diameter--A relation of nominal diameter and sieve diameter is plotted in Fig. 5 and is based on three sets of data: (1) A curve from reference 13 (the curve originally related "b" dimension to sieve diameter but a reduction of 3 to 4 %, as indicated by Fig. 4, has been made to the "b" dimension to obtain nominal diameter for plotting in Fig. 5); (2) data reported in reference 8 for about 200 individual particles; (3) mean nominal diameters determined from the total weight and the specific gravity of a group of several hundred counted particles from one sieve fraction of sand between 350 and 500 microns and one between 700 and 1000 microns.

The data are in general agreement. A change in the size of sieve opening does not seem to justify an abrupt change in the relation of nominal diameter to

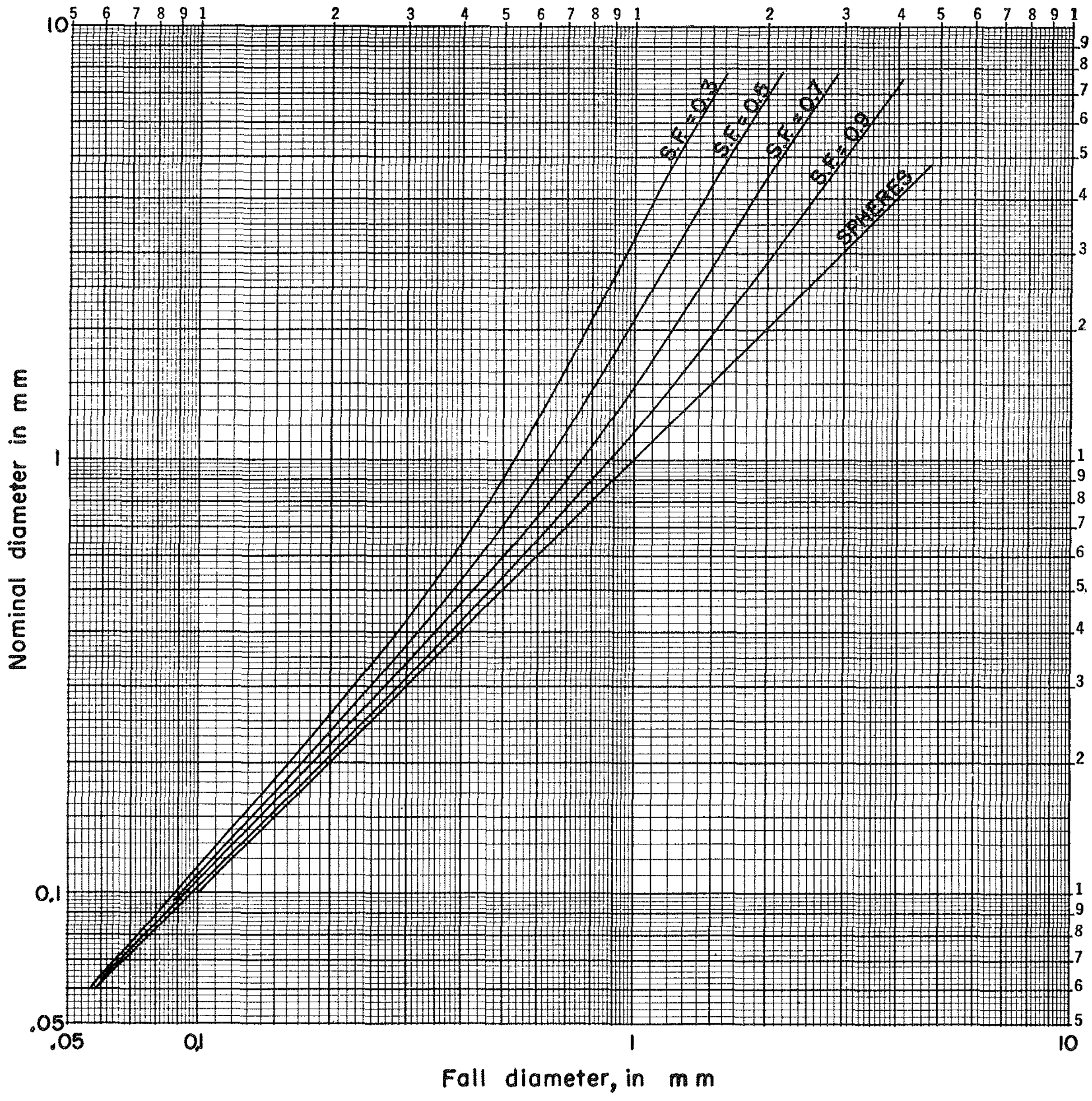


FIG. 3—RELATION OF NOMINAL DIAMETER AND FALL DIAMETER FOR NATURALLY WORN QUARTZ PARTICLES

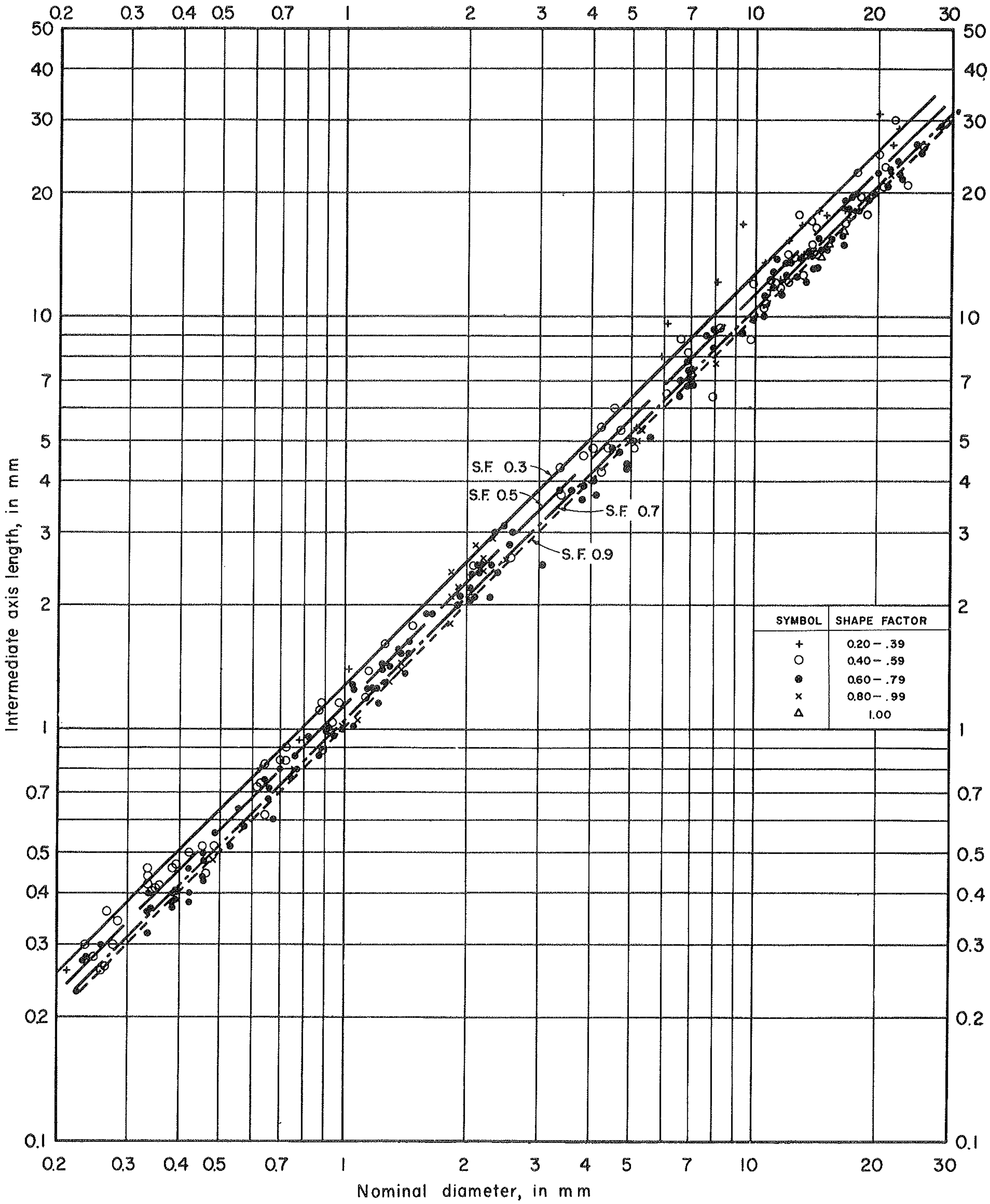


FIG. 4—RELATION OF INTERMEDIATE-AXIS LENGTH TO NOMINAL DIAMETER FOR NATURALLY WORN SEDIMENT PARTICLES

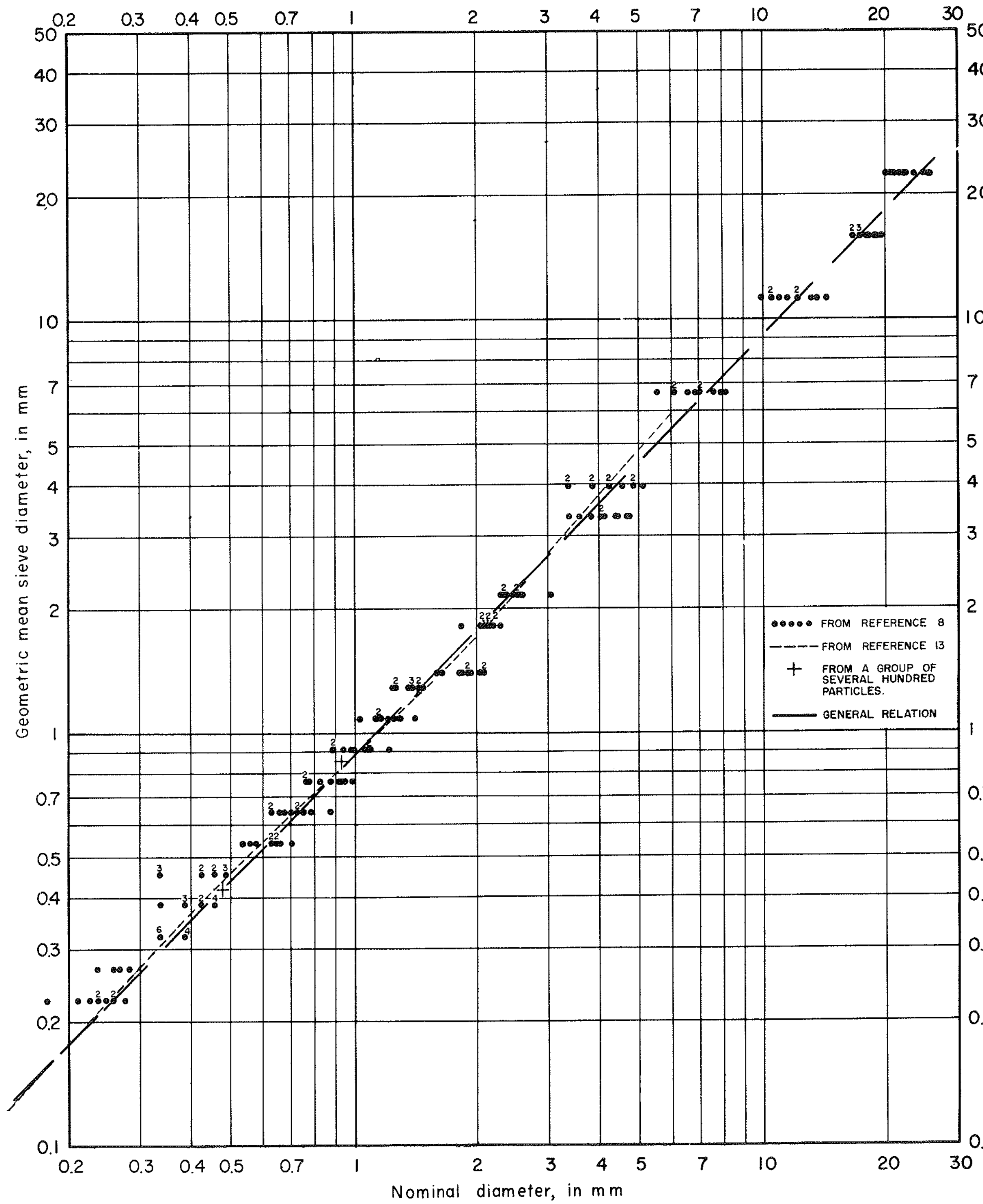


FIG. 5 -RELATION OF SIEVE DIAMETER TO NOMINAL DIAMETER FOR NATURALLY WORN SEDIMENT PARTICLES

sieve diameter so an average relation has been plotted. The ten particles from the sieve fraction with a mean diameter of about 0.38 mm had the same average nominal diameter and intermediate-axis length as those from the sieve fraction with a mean diameter of 0.46 mm; which suggests that both groups may have come from the same, the smaller, sieve fraction.

Attempts to correlate the relation of nominal diameter and sieve diameter with the shape factor were unsuccessful. For particles of low shape factor the intermediate-axis length, or "b" dimension, that can pass a given square sieve opening is longer than for particles of higher shape factors because the thinner the particle is the longer the maximum "b" dimension that can pass diagonally through a given square opening. However, the nominal diameter is generally smaller in comparison to the "b" dimension so that differences in shape factor do not change the relation of nominal diameter and sieve diameter significantly. The relation between nominal diameter and sieve diameter may vary for different sands because of different proportions of relatively long, slender particles in the sands.

23. Sieve diameter and fall velocity of quartz particles--From the average relation of nominal diameter to sieve diameter of Fig. 5 and the relation of nominal diameter and fall velocity of Fig. 2, the relation of sieve diameter and fall velocity may be determined for naturally worn quartz particles in water at various temperatures. The latter relation is shown in Fig. 6 for shape factors of 0.5, 0.7, and 0.9, and similar data for other shape factors or specific gravities could be computed from Figs. 1 and 5. Fall velocities for individual particles of irregular shapes may vary widely from the average relations of Fig. 6.

24. Sieve diameter and fall diameter of quartz particles--The relation of sieve diameter and fall diameter that is shown in Fig. 7 follows directly from Figs. 3 and 5. It is an average relation that may not be accurate for an individual particle.

The fall diameter is greater than the sieve diameter at fine sand sizes and less than the sieve diameter at coarse sand sizes. A shape factor of 0.7 is about average for naturally worn sediments. For this shape factor the fall diameter is about 10 percent larger than the sieve diameter at 60 microns; equals the sieve diameter at 250 microns; and above 250 microns is less than the sieve diameter by percentages which increase rapidly as the particle size increases. The nominal diameter of naturally worn sediment particles of sand sizes is more than 10 percent larger than the sieve diameter. For small sand particles (which fall at low Reynolds numbers) the accelerating effect owing to the nominal size being larger than the sieve size is greater than the retarding effect of shape and roughness on the fall velocity of the particle. Hence, the fall diameter is greater than the sieve diameter. At larger sand sizes, shape and roughness have a relatively greater retarding effect on the fall velocity and the fall diameter is smaller than the sieve diameter.

Even though the sieve and fall diameters agree for the median or mean diameter of a sand sample, that fact does not indicate that either a fine or coarse fraction can be removed from the sand sample by sieving with assurance that a combination of sieve and sedimentation methods of size analysis may be made smoothly

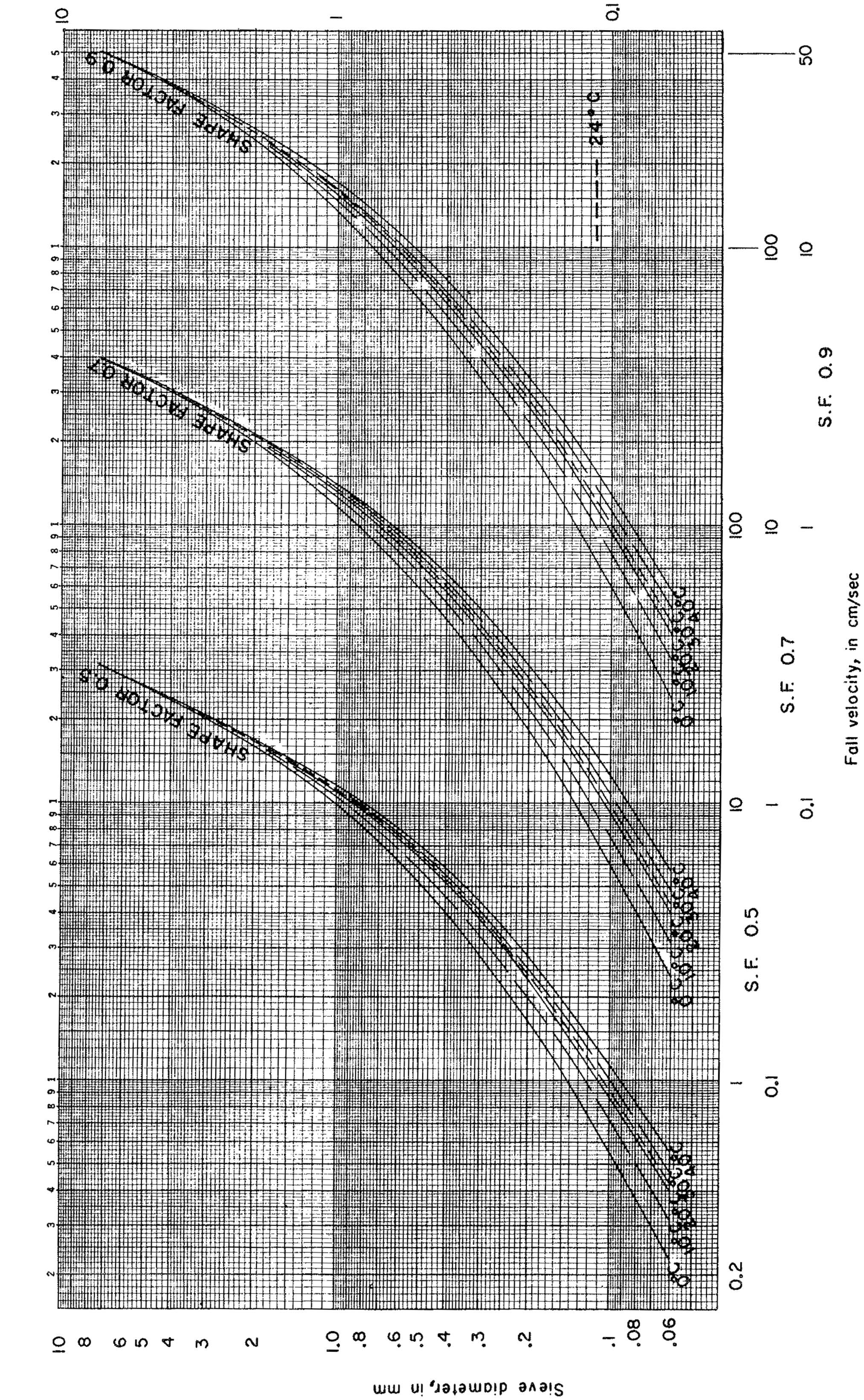


FIG. 6—RELATION OF SIEVE DIAMETER AND FALL VELOCITY FOR NATURALLY WORN QUARTZ PARTICLES
FALLING ALONE IN QUIESCENT DISTILLED WATER OF INFINITE EXTENT

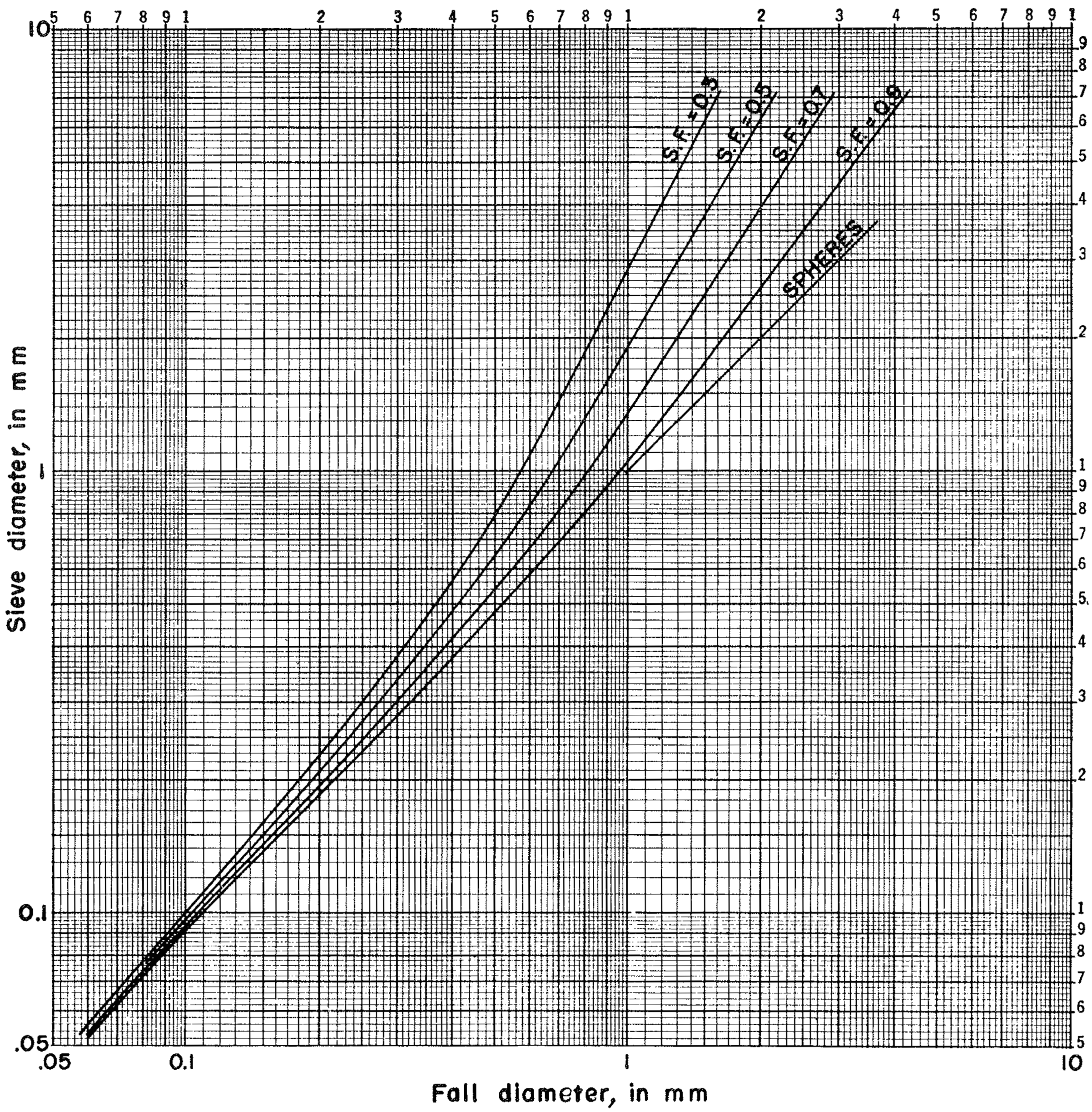


FIG. 7—RELATION OF SIEVE DIAMETER AND FALL DIAMETER FOR NATURALLY WORN QUARTZ PARTICLES

at the division point.

The relation of sieve and fall diameters will change if the specific gravity differs from 2.65.

The sieve analysis of a sand sample has characteristics of apparent simplicity, directness, and reproducibility, which may create an undue impression of accuracy. In one sense the reproducibility of such analyses is good, especially for samples containing several grams of material. Results can be duplicated closely as long as the same set of sieves are used under standard conditions. However, let the sieves become worn or clogged or be replaced by another set, the time or manner of sieving be changed, or the preparation of the sample be altered, and the results may change greatly. In terms of the sieve diameters of the particles, a given sieve has one effective size for separating spheres under standard conditions. The same sieve has a different effective size for particles of other shapes and for other times of sieving. The fall diameter analyses of sand samples express a far more fundamental property of the material, and one which is less subject to variations because of unavoidable differences in the analytical apparatus.

25. The effect of particle shape--For particles having a specific gravity of 2.65, the relations among nominal, sieve, axis length, and fall diameters are determined entirely by the effect of particle shape including, if it is to be given separate mention, the effect of particle roughness. Irregular shapes cause the differences shown in Figs. 3, 4, 5, and 7. The factor of shape may be kept in proper perspective by remembering that for smooth quartz spheres all four diameters, nominal, sieve, axis length, and fall are the same; that the relative effect of irregular shapes varies with size of particle; and that for the smaller particle sizes the effect of shape on the sieve analysis is far more significant than it is on the sedimentation analysis of the same particle.

Particle shape is important in basic sedimentation studies and research. However, the usefulness of physical shape factors is limited in routine sedimentation problems by the fact that even if the nominal, axis length, and sieve diameters are known the fall diameter may be obtained more easily and accurately by determining the fall velocity than by determining S.F. Possibly the shape factor for a sand could be estimated by microscopic comparison with other sands of known shape factor with sufficient accuracy for advantageous use under some conditions.

If the specific gravity, volume, and fall velocity of a particle are known, a shape factor may be determined from the plotting of C_D and R_e on Fig. 1. The C_D and R_e figures for a particle locate a point that will fall on a curve or between two curves so that a shape factor can be found directly or by interpolation between shape factor curves. Such a shape factor expresses the effect of shape on the hydraulic characteristics of the particle at a given Reynolds number. It is not S.F. as defined herein, but is in the same dimensionless form and could be called S_F. S_F indicates the same hydraulic effect as the average for particles with a numerically equal shape factor, S.F. The S_F figure for a particle can be used with Fig. 1 to compute the C_D, R_e relation for the same particle in other fluids or in the same fluid at other temperatures, and this may be the greatest value of Fig. 1.

C_S may be computed for any fluid condition, and if S_F is 0.7 at one Reynolds number the S.F. curve for 0.7 may be used with C_S to determine R_e and C_D . A constant relation between S_F and S.F. is assumed. The assumption is probably valid within closer limits than the accuracy of Fig. 1. Both the assumption and Fig. 1 could be verified by a study of the fall of the same particle in very different fluid conditions. However, the study would have to be repeated for many different particles to verify the whole range of Fig. 1.

V. SEDIMENT CONCENTRATION AND FALL VELOCITY

26. Individual particles--Obviously, the effect of concentration on the fall-velocity analysis of a sediment sample may be avoided if each particle of the sample is allowed to fall alone in water of sufficient extent to avoid wall effects. A complete analysis then requires the summation and classification of all the individual fall velocities on the basis of particle weights. The process has not been adapted to silt and clay sizes and is much too laborious for routine analyses of sand samples, but research on the effect of concentration on fall velocity of sands should be based on this fundamental process.

27. Very low concentrations--To speed up sedimentation analyses, particles of a sample are usually allowed to settle as a group. Sometimes an attempt is made to avoid concentration effects by keeping the concentration of particles very low so that the distance between individual settling particles is relatively large.

The photographic method [12] of size analysis uses an initially dispersed sedimentation system with a maximum concentration of about 5 ppm. At this concentration the particles presumably fall at approximately the same velocity that each would have if settling alone.

A top-introduction tube method for analysis of sands was developed by the Omaha office of the Corps of Engineers.[13] A stratified sedimentation system was used in which concentrations of material were kept low to minimize effects of concentration. The sedimentation column was 4 cm in diameter throughout most of the 168 cm length. Tests made with this apparatus on sands from 100 to 600 microns in size indicated that samples weighing up to 0.01 gm could be analyzed without significant concentration effects.

28. Normal concentrations, one size, completely dispersed--For convenience and economy, most sedimentation analyses are made on samples large enough to develop significant concentration effects. McNown and Lin [9] have reported the effect of concentration on settling velocity for one specific condition.

They used a settling tube with an inside diameter of about 11 mm; the settling velocity was measured over a length of 10 cm; and the particles of sediment had a uniform hydraulic size with a fall velocity of about 0.6 cm/sec corresponding to a nominal diameter of about 0.1 mm. The particles were uniformly distributed throughout the sedimentation column. The results are shown in Fig. 8 as "V/V_o" versus concentration. The velocity, V, is the settling velocity of the particles falling in the tube, and V_o is the velocity assigned to the particles falling alone. V_o was obtained by theoretical computations as an extrapolation of data that were subject to concentration effects.

The actual velocity obtained by dropping individual particles of sediment was 7 percent slower than the V_o used for Fig. 8, but McNown and Lin considered the velocity from individual particle drops to be less reliable than the velocity that was computed for V_o.

For particles of one uniform size that fall uniformly dispersed in a small

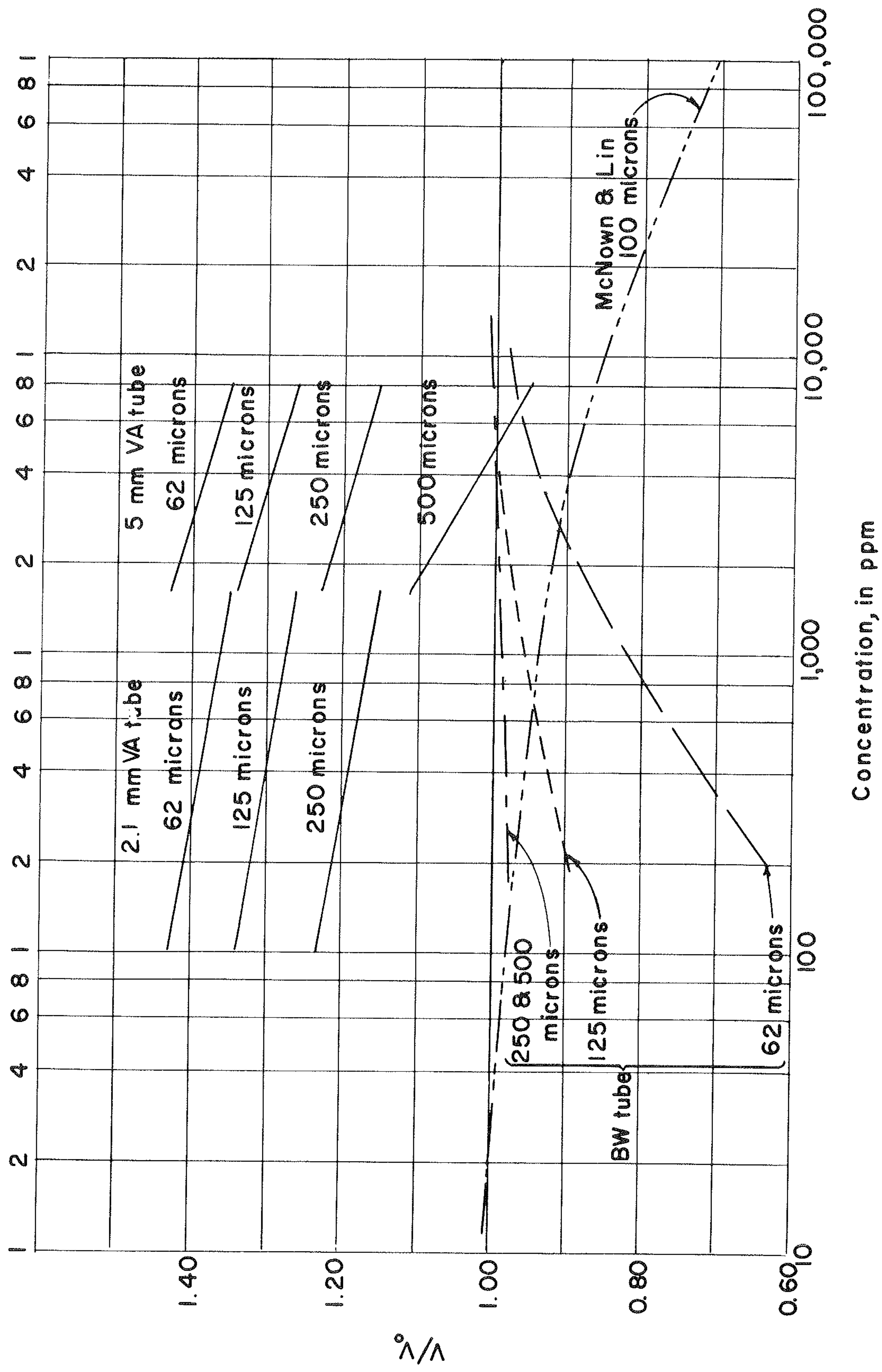


FIG. 8 — EFFECT OF CONCENTRATION ON FALL VELOCITY

sedimentation column, the report by McNown and Lin showed that: the effect of concentration increased the settling velocity slightly for concentrations below 20 ppm; for greater concentrations the effect of concentration decreased the settling velocity progressively; a concentration of 5,000 ppm of sediment in water reduced the settling velocity about 10 percent.

29. Normal concentrations, mixed sizes, nominally dispersed--The bottom-withdrawal-tube method of size analysis is another sedimentation system for which the effect of concentration on fall velocity has been determined.[14] The method was evaluated for a mixture of sizes of glass spheres in concentrations from 200 to 10,000 ppm. The spheres were dispersed throughout a sedimentation column 2.5 cm in diameter and 100 cm long. The initial dispersion was not completely uniform.

Data from analyses in the bottom-withdrawal tube were used to compute the effect of concentration on the settling velocity of glass spheres in that apparatus. The computed V/V_o relations are shown in Fig. 8 by separate curves for particle diameters of 62, 125, and 250-500 microns. The curves are approximations because the data are not entirely consistent. The size distribution within a sample influences concentration effects.

At concentrations of 200 ppm the fall velocity ratio V to V_o is less than unity and the smaller the sphere size the lower the ratio. At concentrations of 10,000 ppm the ratio V to V_o is approximately unity.

30. Normal concentrations, mixed sizes, stratified systems--The visual-accumulation tube method of size analysis is another sedimentation system for which the effect of concentration on fall velocity has been studied.[15] Samples were introduced at the top of a sedimentation column of water 120 cm in height. The diameter of the column was 2.5 cm for the top 80 cm, decreased through the next 20 cm, and was uniform at 2.1, 3.4, 5.0, or 7.0 mm through the bottom 20 cm. The accumulation of sediment with time was measured in the bottom section of the sedimentation column.

Hundreds of sand samples for which the fall-velocity distribution had been established on the basis of the fall of individual particles were analyzed in the visual-accumulation tube. The particles generally fell faster in the tube as shown for two tube sizes in Fig. 8. In each size of tube the maximum ratio of V to V_o was for the lowest concentration. Particle size has a very important effect on the V to V_o ratios. Significant effects of other factors such as the distribution of sizes within the sample were frequently apparent but these have not been separately evaluated.

Some data were obtained with mixed sizes of glass spheres. The effect of concentration on the fall velocity of glass spheres in the visual-accumulation tube has not been carefully determined. In general, the glass spheres fell faster in mass in the visual-accumulation tube than each would have fallen alone in distilled water of infinite extent. The magnitude of the acceleration effect was somewhat less than for sands.

31. <u>General concentration effects</u>--The general effect of concentration on the fall velocity of particles has been widely recognized. Usually the data have shown the effect indirectly. Emery[16] calibrated his top-introduction sedimentation tube with irregular quartz particles of known sieve-size distribution. The calibration showed that the irregular quartz particles (especially the finer sand sizes) fell much faster in the tube than quartz spheres of the same sieve size would fall alone in distilled water at the same temperature. Also some published comparisons of pipette and bottom-withdrawal tube analyses show differences that vary with concentration.[6]

In contrast, if the distribution of particles throughout a sedimentation column is uniform and the concentration is appreciable, presumably each particle falls more slowly than it would fall alone. However, the size of particle probably has an important influence on the concentration effect, and a mixture of particle sizes falling through a long sedimentation column of limited cross-section is not likely to maintain a uniform distribution. If the distribution of particles throughout the cross-section of a sedimentation column is not uniform, density currents may form and greatly increase the rate of fall of the individual particles, especially those of the smaller sizes.

If, as in the bottom-withdrawal tube, particles are only approximately dispersed throughout the sedimentation column the relation of concentration and fall velocity is too complex for theoretical analysis. In top-introduction tubes currents and eddies are set up in the sedimentation fluid which are also beyond theoretical analysis on a quantitative basis. The relations between sediment concentration and fall velocity in water appear to depend on many factors including: (1) type of sedimentation system, (2) size of sediment particle, (3) specific gravity of the particle, and (4) the shape of the particle. The effects of concentration are so widely divergent and the data available are so limited in quantity and scope compared to the number of variables that extrapolation to other sedimentation systems would be unwise. One cannot predict the effect of concentration on fall velocity for several of the sedimentation-size analyses systems common in the laboratory.

Additional research on the effect of concentration on fall velocity seems urgently needed.

VI. SAND SAMPLES OF KNOWN FALL-DIAMETER DISTRIBUTION

32. Need for a standard of accuracy--Sedimentation size-analysis procedures have been developed, accepted, and used without first establishing a specific unit for measuring sedimentation size. If the fundamental property in sedimentation size analysis is conceded to be the standard fall-velocity or fall-diameter distribution of the sample then this property should be the basis for measuring size. The accuracy of any method of sedimentation size analysis then would depend on how correctly the standard fall-velocity or fall-diameter distribution of the sample is determined by that method.

To avoid a wrong impression, some comments on early methods of size analysis seem necessary. The methods of the soil physicists for determining such characteristics as physical size, surface area of particles, voids, and permeability to fluid flow are not applicable to this problem, which is one of size in terms of settling characteristics. Those of one school of thought[7,17], set up analytical methods and calibrated them against sieve or microscopic analyses. The methods of analysis so devised were not actually sedimentation-size determinations and are therefore not directly adaptable to sedimentation problems. At least, those responsible for such developments were consistent in establishing some standard of comparison. Another approach, and perhaps the usual one[7] was to set up a distance for a sample to fall and to compute the settling velocity from distance and time of fall. Usually this type of analysis seems to have been unchecked or possibly checked only for consistency of results, or by comparison with another method equally open to question. The effects of space limitations, sample concentrations, methods of introduction and dispersion, and of other factors peculiar to the type of analysis, were sometimes mentioned but never definitely evaluated throughout the range of sand sizes. This procedure was defended on the basis that the settling velocities of the particles or sample were actually determined and therefore the sedimentation diameter distribution had been established for one set of conditions. Such reasoning minimized the fact that there could be as many answers as there were methods and apparatus of analysis; that one must know the significant features of the analysis in order to interpret the results; and that the fundamental hydraulic property, the standard fall-velocity of the individual particle, had not been determined.

If size analysis methods are to be improved and standardized, (1) some definite unit of measurement for sedimentation size is necessary (fall diameter is recommended), and (2) some accurate method of determining size distribution in samples, or of preparing samples having a known size distribution, is required. Without such a basis, the accuracy of a method of size analysis cannot be determined. Because no adequate method for satisfying item 2 for sand sizes could be found in reports on size analysis, the method presented in the following section was developed.

33. Preparation of a sand sample having a known fall-velocity distribution--It was necessary to use the fall velocity of the individual particle to make up sand samples for which the fall-velocity or fall-diameter distributions were known. The primary concept was simple and obvious and other investigators had pointed the way. Carey and Stairmand[12] had developed a photographic method

for determining the fall velocities of individual particles in samples composed of particles smaller than 100 microns. Serr[18] determined the sedimentation-diameter distribution for sands of sizes larger than about 140 microns, by an individual dropping of many representative particles from each of several sieve fractions into which the sample had been separated.

The type of analysis used by Serr was given a more rigorous mathematical treatment and was extended down to sizes of 62 microns. The essentials of the method thus developed for determining the fall-diameter distribution are as follows:[15] A bulk sample of sand is sieved, 10 grams at a time, until the desired quantity of material of each sieve fraction has been obtained. The sieve-size distribution based on the total weights of each fraction is recorded. Then each sieve fraction is carefully split and resplit until about 100 representative particles remain. These particles are dropped individually and the fall velocity of each is obtained and converted into fall diameter by use of the relation between diameter and fall velocity for quartz spheres (Table 2). The fall diameters of the particles are cubed to approximate their relative volumes and weights. A fall diameter is chosen at about the median division of a summation of the cubed diameters, arranged in order of size. A summation is made of all cubed figures larger than the cube of the chosen fall diameter and this sum is expressed as a fraction of the total of all the cubes; for example, 0.520 larger than (and 0.480 smaller than) the cube of 305 microns in the sieve fraction 250 to 350 microns. If 40.0 percent of the sample is in the sieve fractions coarser than 350 microns, and 20.0 percent in the 250 to 350 micron fraction, then 50.4 percent (40.0 plus the product of 0.520 x 20.0) of the total sample has fall diameters greater than 305 microns. Extending this process to all the sieve fractions completes the computations of fall-diameter distribution for the sample.

The sieve- and fall-diameter distribution curves can be plotted for the original sand sample. One should remember that a sieve-size frequency curve is defined only at the sizes of sieves that were actually used. A fall-diameter frequency curve, obtained by the method above, is defined only at the approximate median sizes that were used to subdivide the fractions. The use of mean or average figures between points of definition or of figures based on unweighted particle counts are inexact practices.

After the fall-diameter distribution has been determined for each sieve fraction of a sand, that sand can be used to compound many samples for each of which the fall-diameter distribution can be computed. Such samples are reproducible. They also provide a basis for consistency comparisons that are independent of the accuracy of the determination of the fall-diameter distribution. The original relative weights of the different sieve fractions can be combined into samples that have the original fall-diameter distribution. Other relative weights may be chosen to provide samples that have different fall-diameter distributions, and the principles previously stated can be used to determine the fall-diameter distributions of these samples. Varying the relative quantities of the different sieve fractions does not alter the original fall-diameter distribution within each individual sieve fraction so that a graph of the fall-diameter distribution of the entire synthetic sample may not be a smooth curve.[15] Sieve fractions from two or more sands can be combined to obtain a desired size range or type of sample.

Assumptions and qualifications that pertain to the method of preparing a sample having a known fall-diameter distribution follow:

(1) The cube of the fall diameter is assumed to be proportional to the weight of the particle. The relationship is not direct, but the cube more nearly represents the volume and weight than would the first power of the fall diameters. Even the use of the first power of the fall diameter does not significantly alter the results if the range of sizes in each sieve fraction is small.

(2) The type of computations in the cited example are generally adequate. However, occasionally a significant percentage of material in the sieve fractions coarser than 350 microns has fall diameters less than 305 microns, or a significant percentage of material in the sieve fractions finer than 250 microns has fall diameters greater than 305 microns. Then, by extra computations, the weight of offending material is moved from the sieve fraction where it was originally to the proper side of the 305-micron size.

(3) A 100-particle split as the basis for determining the fall-diameter distribution for a sieve fraction is satisfactorily accurate as shown by the consistency of results throughout the size range of samples analyzed on this basis. Usually splits from about eight size fractions are used to define a curve of fall-diameter distribution for a complete sample. Because the shape of this curve is necessarily very similar to that for the sieve-diameter distribution, an inconsistent split is immediately obvious. If inconsistencies are minor, adjacent results are averaged; but if any major discrepancy is found, the split is rechecked. In the cited example, if the 52 percent larger than the cube of 305 microns should actually have been 60 percent (an extreme variation) the percentage coarser would have been changed from 50.4 to 52.0 percent which is within acceptable limits of accuracy. Errors in individual splits are independent of those for other splits, are not subject to cumulative errors, and generally apply to minor fractions of a total sample. Also inconsistencies may be observed easily and rechecked if necessary.

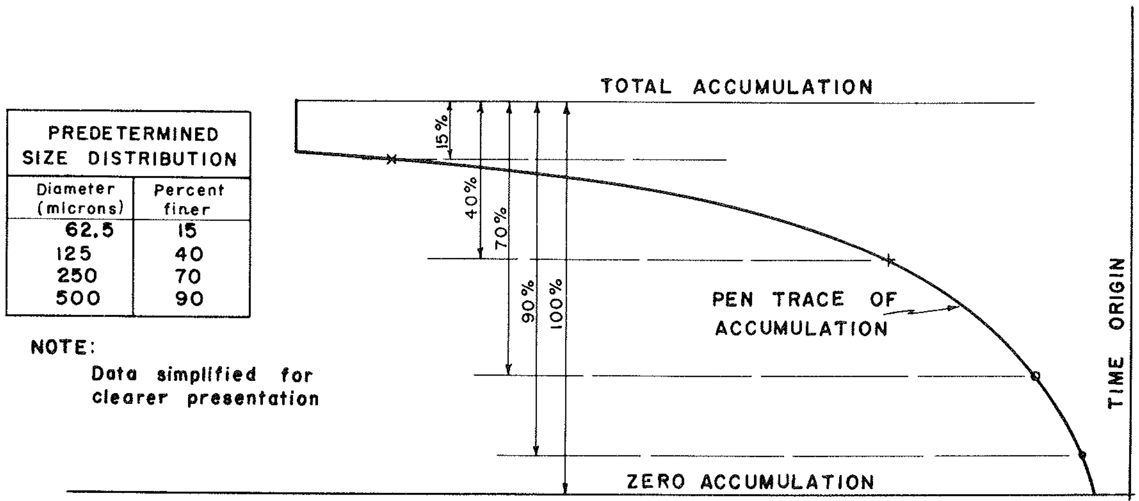
(4) Within the range of laboratory temperatures, usually from 20° to $30^{\circ}C$, the effect of temperature on the settling velocity of a particle of sediment in water is considered to be essentially the same as though the velocity were for a sphere of specific gravity 2.65. The validity of this assumption may be checked from Table 5 which shows that the errors for particles with a shape factor of 0.7 should seldom exceed two percent.

34. <u>Determination of accuracy of analysis with known samples</u>--The accuracy of a method of sedimentation analysis may be readily proved or disproved by analysis of known samples and comparison of results with the known fall-diameter distribution of the samples. Most present methods of sedimentation analysis do not yield results which check such a distribution directly. However, if any method shows consistent results a variety of samples of known distribution can be used to establish a correction coefficient or calibration for the method. Many samples would have to be analyzed to evaluate the effects of such items as concentration of material; size, shape, and specific gravity of particles; and other significant factors.

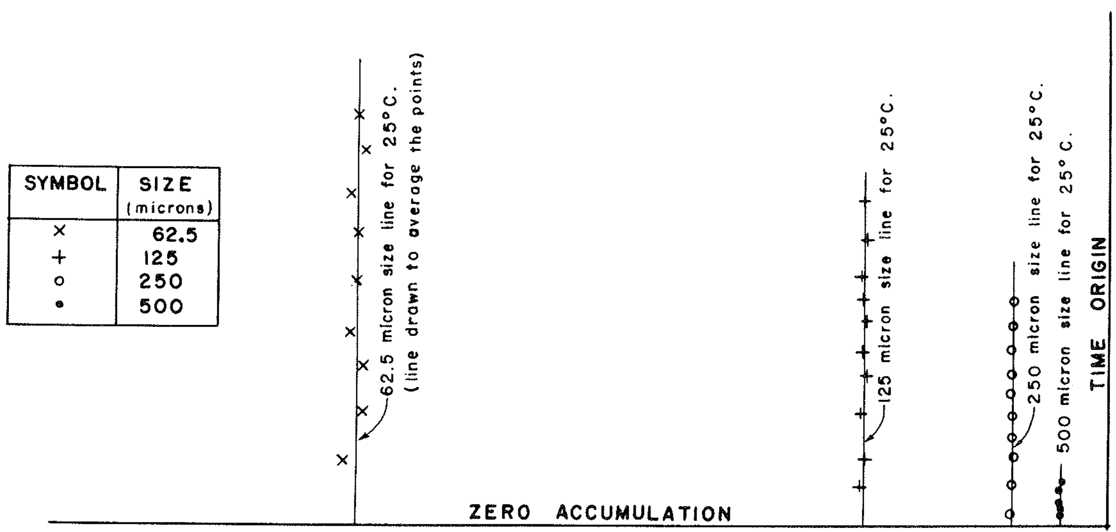
The visual-accumulation-tube method of size analysis will be used to illustrate a calibration procedure based on analysis of known samples. The visual-accumulation-tube method results in a graph of accumulation of material settled out of a stratified (top-introduction) sedimentation system with time. The calibration was accomplished by superimposing the results of the analyses of many known samples. The data have been simplified by considering only division sizes of 62.5, 125, 250, and 500 microns.

Each analysis produced a curve (Fig. 9A) of sediment accumulation with time. For each analysis, points representing the percentages of the known fall-diameter distribution for selected division sizes were marked on the curve. If 40 percent of a calibration sample has fall diameters less than 125 microns, the distance from the time origin is fixed for the 125-micron size by the intersection of the curve with the horizontal line at which 60 percent of the sample has accumulated in the bottom of the tube. Thus each analysis established a calibration point for each division size and for the water temperature in the tube. Points from several analyses were transferred to a chart, Fig. 9B. A line to represent a particular division size and water temperature was drawn through each set of points. The distance of such a line from the time origin of the chart was a measure of the time for that division size of particle to fall in the visual-accumulation tube. Analyses at other temperatures provided information for temperature adjustments.

The determination of fall-diameter distribution in terms of the fall velocity of the individual particle in water and the compounding of samples for which this distribution may be computed are laborious processes. However, both are entirely practical and require no special equipment for the range of sand sizes. These procedures make a basic standard of accuracy available to all investigators. If this standard were universally used, directly comparable size data could be obtained regardless of where, when, or how the analyses were made.



A— CALIBRATION POINTS FROM A SINGLE ANALYSIS



B— CALIBRATION POINTS FROM SEVERAL ANALYSES AT 25°C

FIG. 9 — FUNDAMENTALS OF CALIBRATION METHOD

VII. CONCLUSIONS

35. Concluding remarks--This paper does not pretend to be an organized review of the entire field of sediment-size analysis but is merely a report of investigations and concepts essential to a specific development, which related primarily to sand sizes of sediment. Although much work should still be done along similar lines, some conclusions have been reached:

1. The fundamental property governing the motion of a sediment particle in a fluid is its fall velocity. Standard fall velocity confines fall velocity to definite limits for uniformity and precision. Fall diameter is an expression for standard fall velocity that is more usable for many purposes because it does not vary with fluid temperature and provides a linear dimension to aid in visualizing size.

2. If possible, methods of sedimentation-size analysis should be calibrated or corrected to give answers in terms of the standard fall velocities, or fall diameters, of the particles, and the size-frequency distribution should be based on weight of material.

3. Methods are available for determining the fall velocity distribution of sand samples and for compounding sand samples of known distribution so that methods of particle-size analysis can be calibrated for particle sizes coarser than 60 microns.

4. Sedimentation diameter may be based on a settling velocity developed under any condition of fluid. Also the definition of sedimentation diameter has not effectively limited other conditions of analysis. Therefore, a figure for sedimentation diameter should be accompanied by a description or designation of the method used (Puri tube for example) together with a statement of specific gravity. Without these qualifications the sedimentation diameter does not express the settling characteristics of the particle or sample.

5. Standard sedimentation diameter is specifically defined so that it must be based on the standard fall velocity of the individual particle. Then the standard sedimentation diameter depends only on the volume of the particle and the effect of its shape and roughness on its fall velocity.

6. In terms of the sieve diameters of the individual particles, a standard sieve divides at different sizes depending on particle shape, time of sieving, and preparatory treatment of the sample.

7. The shape factor, S_F, based on the volume, density, and fall velocity in a fluid of known density and viscosity should be a good indication of the hydraulic characteristics of the particle in other fluids and in the same fluid at other temperatures.

8. Much that is important to an orderly development of sedimentation science and technique is still unknown and additional research is urgently needed. Some established relations have been expressed in ambiguous terms. The value of

future research depends on the degree to which data are determined and expressed in fundamental units that have precise meanings.

9. Of the many possibilities for future research that are suggested by this report two specific items are:

a. Effect of concentration on fall velocity for a variety of sedimentation systems.

b. The C_D versus R_e relation for natural sediment particles, perhaps emphasizing study of the same particle at several different Reynolds numbers.

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* These are reports of the cooperative project "A Study of Methods Used in Measurement and Analysis of Sediment Loads in Streams", of which the present paper is also a part.

NOTE: Much of the material of reference 12 also appears in Engineering, Vol. 154, 1942. Pages 141, 142, 181, 182, 221, 222, and 241, 242.

APPENDIX

36. Several possible uses of Fig. 1--Six variables are involved in the computations defining Fig. 1: fluid density and viscosity; and particle density, shape factor, nominal diameter, and fall velocity. Any one variable may be determined if the other five are known. The unknown variable may be determined from Fig. 1 as follows:

Because density appears to the same powers both above and below the line, Fig. 1 relations may be solved using mass density in gm sec^2/cm^4 or density in $grams/cm^3$ as long as consistency is maintained throughout a given computation.

A. Fluid density unknown -- Compute R_e for a known particle, a sphere if possible; determine C_D from the intersection of R_e and the shape factor curve and solve for the mass density, ρ_f, from

$$C_D = \frac{4}{3} \frac{d_n (\rho_s - \rho_f) g}{\rho_f \quad v^2} .$$ (C_S or C_W could be used instead of C_D)

B. Fluid viscosity unknown -- Compute C_D for a known small particle, a sphere that falls with a C_D high enough to be sensitive to changes in R_e is best; determine R_e from the intersection of C_D and the shape factor curve and solve $R_e = \frac{d_n v}{\nu}$ for the kinematic viscosity, ν. (C_S or C_W could be used instead of R_e.)

C. Particle density unknown -- Compute R_e and determine C_D from the intersection of R_e and the shape factor curve (interpolate between shape factor curves as necessary); solve $C_D = \frac{4}{3} \frac{d_n (\rho_s - \rho_f) g}{\rho_f \quad v^2}$ for the mass density, ρ_s. (C_S or C_W may be used instead of C_D.)

D. Shape factor unknown -- Compute R_e and C_D and the location of the intersection on Fig. 1 will indicate the shape factor. (Any two of R_e, C_D, C_S and C_W may be used.) The shape factor, S.F., is by definition $c/\sqrt{ab}$ for naturally worn sediment particles. The shape factor determined from Fig. 1 is S_F and indicates that a particle has a shape with the same hydraulic effect as the average for particles with a numerically equal shape factor, S.F.

E. Nominal diameter or volume unknown -- Compute C_W and determine R_e from the intersection of C_W and the shape factor curve; solve $R_e = \frac{d_n v}{\nu}$ for d_n. (C_S or C_D could be used instead of R_e.) Volume $= \frac{\pi}{6} d_n^3$ by definition of d_n.

F. Fall velocity unknown -- Compute C_S and determine R_e from the intersection of C_S and the shape factor curve; solve $R_e = \frac{d_n v}{\nu}$ for v. (C_D or C_W could be used instead of R_e.)

G. Special cases (rarely used)

1. The fluid is water but the temperature is unknown -- Temperature may be found by trial and error substitutions of ρ_f and ν for water at different temperatures.

2. Particle density and nominal diameter unknown, but buoyant weight, shape factor, and fall velocity known -- Compute C_S, determine R_e from the intersection of C_S and the shape factor curve and solve $R_e = \frac{d_n v}{\nu}$ for d_n. Compute the density by solving for ρ_s in buoyant weight $= \frac{\pi}{6} d_n^3 (\rho_s - \rho_f)\, g$.

The unknowns usually determined are nominal diameter and fall velocity. Specific examples follow:

1. A naturally worn sediment particle has a nominal diameter of 250 microns (.025 cm), a density of 3.00 grams per cm^3, and a shape factor of 0.7. What is the fall velocity in distilled water at 20°C?

$$C_S = \frac{\pi d_n^3 \; (\rho_s - \rho_f)\; g}{6 \quad \rho_f \quad \nu^2}$$

$$= .5236\,(.025)^3 \;\frac{(3.00 - .998)}{.998}\;\frac{981}{(.01005)^2}$$

$$= 159.0$$

From Fig. 1 if $C_S = 159.0$ and the shape factor is 0.7, $R_e = 8.20$

and $R_e = \frac{d_n v}{\nu} = 8.20 = \frac{.025v}{.01005}$ and $v = 3.30$ cm/sec.

2. Assume the same particle except that the nominal diameter is not known and is to be determined from the other five variables.

$$C_W = \frac{(\rho_s - \rho_f)\; g\; \nu}{\rho_f \quad v^3}$$

$$= \frac{(3.00 - .998)}{.998}\;\frac{981 \quad (.01005)}{(3.30)^3} = .550$$

From Fig. 1 the intersection of $C_W = .550$ and a shape factor of 0.7 shows $R_e = 8.20$

$$R_e = \frac{d_n\, v}{\nu} = 8.20 = \frac{3.30d}{.01005} \quad \text{and } d_n = .025 \text{ cm.}$$

3. The same answers could have been found by trial and error substitution in the expressions for C_D and R_e.

Thus if $d_n = .025$ cm

and $v =$	3.00	3.20	3.29	3.30	
$R_e =$	7.46	7.96	8.19	8.21	) on curve for S.F. = 0.7
$C_D =$	7.29	6.40	6.06	6.02	)

and a velocity of 3.30 cm/sec is satisfactory.

If $v = 3.30$ cm/sec

and $d_n =$	.0220	.0240	.0249	.0250	
$R_e =$	7.22	7.88	8.18	8.21	) on curve for S.F. = 0.7
$C_D =$	5.30	5.78	5.93	6.02	)

and d_n is .025 cm.

The addition of C_S and C_W scales to the usual C_D versus R_e relation allows direct solution for d_n and v because the diameter expression does not appear in C_W and the velocity does not appear in C_S .

Fig. 1 was based on data for spheres and naturally worn sediment particles and is valid only for these particles. The shape factor is $c/\sqrt{ab}$ and may not adequately define shape for individual particles. Consequently many of the computations illustrated yield answers which are only approximate for an individual particle.

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