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U.S. GEOLOGICAL SURVEY
Reston, Virginia 20192

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OFFICE OF SURFACE WATER TECHNICAL MEMORANDUM 2013.01

SUBJECT: Computation of Annual Exceedance Probability (AEP) for Characterization of Observed Flood Peaks

USGS hydrologists are often called upon to comment on the rarity of an observed flood. In many cases the observed flood peak is compared to flood quantiles from either a published or a newly derived flood-frequency distribution and the Annual Exceedance Probability (AEP) is inferred from the comparison. There are pitfalls associated with this practice and judgments are required to provide a reasonable and consistent result. The need for careful judgment is even greater for streams that are heavily regulated to control or moderate flood flows. The purpose of this memo is to provide guidance for the computations and judgments associated with estimating the AEPs for observed flood peaks on both unregulated and regulated streams.

Flood-Frequency Computations for Unregulated Streams

The starting point for estimating the AEP for an observed flood peak is the flood-frequency computation based on Bulletin 17B (U.S. Interagency Advisory Committee on Water Data, 1982, Guidelines for determining flood flow frequency, Bulletin 17-B of the Hydrology Subcommittee). Bulletin 17B procedures were designed to incorporate relevant regional and historical information into the flood-frequency computation process to stabilize and improve the accuracy of flood-frequency estimates for unregulated streams. The increase in accuracy is accomplished first by determination of a weighted skew based on both the regional and the at-site skew estimates. Next, the weighted skew is used to determine the at-site flood frequencies. Finally, the at-site estimates are weighted with independent estimates of the flood frequencies for that site, typically determined using regional regression techniques. Office of Surface Water Technical Memorandum 2010.05 (SW 2010.05) describes the method for weighting the two estimates.

Flood-Frequency Computations for Regulated Streams

In contrast to unregulated streams, computations of flood-frequency estimates for regulated streams depend on the unique details of the regulation, as well as at-site flood history. Accordingly, here are some general guidelines for flood-frequency computations for regulated systems:

1) Select the period of record (POR) that is appropriate for the purpose of the computations.

a) If the purpose is to ascertain the AEP of an observed flood under regulated conditions, the POR associated with the regulation is usually the appropriate base period for both the computations and comparisons. However, in some cases, the flood-control reservoir may be full during the flood such that the system is in a "run-of-the-river" situation. In such cases, the regulation may not significantly affect the magnitude of the peak flows. If an observed flood occurred during such conditions, the appropriate reference period would likely include the pre-regulated period, the observed current flood, along with any other floods during the regulated period which occurred under "run-of-the-river" conditions. Ultimately, the selection of the appropriate POR is a matter of judgment that should be documented with the computations and release of the AEPs.
b) Use data drawn from a consistent POR. Except for run-of-the-river situations as described in item "a" above, flood-frequency estimates generally should be computed separately for the regulated and unregulated periods. Combining the records would inappropriately mix together two different populations. In addition, the analyst should determine that there were no major changes in operating rules during the regulated period. If major changes did occur, the POR for the regulated period would likely be further reduced and may not be long enough to support flood frequency estimates.
c) Explain and document the selection of the flood data and similarity or differences in flood conditions, especially the effects of regulation on individual floods, including the observed flood for which the AEP estimate is sought. Include commentary and documentation of discussions with personnel from agencies responsible for the regulation.

2) Use only the station skew. The regional (generalized) skew (for example, Bulletin 17B, plate 1) and the weighted skew (which is calculated using the regional skew) are based on skews observed at streamgages with unregulated flow. As a result they are not applicable for a streamgage with regulated flow.
3) Use the "systematic record" output from PeakFQ (figure 1). In almost all cases involving regulated flow, the systematic results from PeakFQ are the estimates that should be used. Since the Bulletin 17B estimates include the weighted skew, the "systematic record estimate" should be used.
4) For a final estimate of the flood frequency relation, do not weight the at-site flood-frequency estimates with flood-frequency estimates from regional regression equations. Weighting techniques should be used only when the site is a valid location where the regional regression equations apply. Since the equations do not generally apply to regulated streams, use of the weighted estimates, as described in SW Technical Memorandum 2010.05, is not appropriate.

Estimating the AEP of Observed Flood Peak on Unregulated and Regulated Streams

When estimating the AEP of a recent flood, a new flow frequency analysis should be conducted that *includes* the observed flood peak for which the AEP is sought. The AEP can be determined through log-linear interpolation of PeakFQ flood-frequency output using equation 1:

$$AEP_{Observed\ Flood} = AEP_1 + \frac{(AEP_2 - AEP_1) \times (log10(Q_{Observed}) - log10(Q_1))}{log10(Q_2) - log10(Q_1)}, \quad (1)$$

where:

AEP_1 and AEP_2 are taken from column of AEP values associated with the flows Q_1 and Q_2 that bracket the observed flood. For example, given an observed peak flow of 20,000 cubic feet per second (cfs) and the flood-frequency quantiles listed in the PeakFQ results shown in figure 1 (columns A and B for unregulated streams and columns A and C for regulated streams), the AEP is computed as:

$$\text{AEP} = 0.02 + \frac{(.01 - .02) * (log10(20{,}000) - log10(18{,}760))}{log10(22{,}810) - log10(18{,}760)}$$
$$= 0.017$$

If the final flood frequency estimates are obtained by weighting the at-site estimates with regional regression estimates, then the interpolation should be performed on the weighted estimated (from WIE).

Reporting and presenting the AEP of Observed Flood Peaks

Flood-frequency concepts including those found in Bulletin 17B were developed to provide reasonably precise estimates of the flood magnitudes associated with a set of AEPs. However, the converse of that operation, assigning an AEP to an observed flood, is associated with more uncertainty. To communicate the uncertainty, AEP estimates must be accompanied with a confidence internal likely to include the true AEP.

The computation of a confidence interval for the AEP could be performed by interpolation of the upper and lower confidence interval information contained in PeakFQ output in a manner similar to interpolation process described above. However, the flood-frequency distribution computed by PeakFQ is based on the computed mean, standard deviation, and skew and an assumption that these statistics represent their true values and the true flood distribution, assumptions that may be flawed. While the interpolation process is reasonable for the estimation of the AEP, it fails to account for the uncertainty of the distributional assumptions and the increased uncertainty arising for the lack of reference data for extreme flood being assessed. AEP confidence intervals that are less susceptible to such assumptions can be computed using nonparametric methods. OSW recommends that the confidence interval of the AEP be calculated using the procedure based on the Beta distribution, which can be implemented in Excel using the cell formula:

AEP_CL = 1-BETAINV(Prob, Alpha, Beta, lowerbound, upperbound)

where:

AEP_CL= the confidence interval (upper or lower range) of the AEP;
Prob = the probability associated with the upper or lower confidence limit (0.167 or 0.833, based on the 66.7% confidence level);
Alpha = the number of floods minus the rank plus 1 (i.e., N-Rank+1);
Beta = the rank of the observed flood;
lowerbound = 0; and,
upperbound = 1.

While the use of the 66.7-percent confidence level is unusual, the use of confidence intervals based on the 90- or 95-precent level (which are more typical) result in ranges that are extremely large and tend to be misunderstood by lay audiences. There is some precedent for use of the 66.7-percent confidence level. The Intergovernmental Panel on Climate Change uses it to describe events it considers "likely".

As an example, a streamgage with 19 years of record experiences a peak of record in the current (20^{th}) year of record. Using Excel and the BETA.INV statistical function, the formula arguments for the upper confidence limit would be:

AEP_CL (upper) = 1 - BETA.INV(0.167, 20, 1, 0, 1) = 1 - 0.914

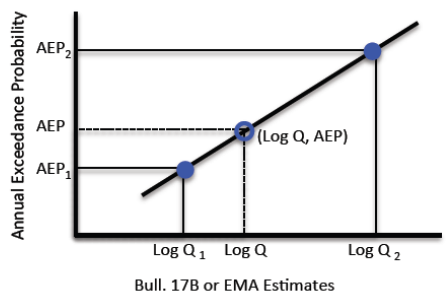
producing a result of 0.086. This corresponds to the upper CL value in Table 1 for a streamgage with 20 years of record and an observed flood rank of 1, and a confidence interval of 66.7 percent.

Table 1 provides upper and lower confidence limits at the 66.7-percent level computed from the Excel Beta function for selected combinations of lengths of POR and flood ranks. Table 1 is the preferred source for ranges of the AEP of an observed flood. In order to use it, an analyst need only determine the number of floods for which the conditions (regulated, unregulated, or 'run-of-the-river") are comparable to the observed flood and the rank of the observed flood relative to those data.

Note that use of the non-parametric confidence intervals calculated using either the Excel Beta function or Table 1 provide the AEP range which is expected for a flood of a given rank within a specified period of record and that these confidence intervals are not centered around the interpolated AEP estimate. Consequently, the AEP estimate will occasionally be outside of the range of the confidence interval. To explain this result, we suggest the following language:

We estimate the exceedance probability corresponding to this particular flood at 2%. However, the uncertainty in this statistic is large, and, in general, a 66.7% confidence interval for the true exceedance probability of the largest flood in 47 years extends from 0.4% to 4%.

Finally, in an attempt to make the end user of USGS AEP estimates fully aware of the uncertainty, it is recommended that the flood quantiles and associated 95% confidence intervals for selected AEP's (for example, 4%, 2%, 1%, and 0.2%) also be presented. The 95% confidence intervals are basic output in the analysis methods used by USGS for unregulated streams and regulated streams. An example of this information in table form is presented in table 2. Note that if the AEP estimate for a flood is less than 0.2% (500-year flood), the AEP estimate should be listed simply as being less than 0.2%, without specifying the actual calculated value. The confidence intervals should still be listed.



ANNUAL FREQUENCY CURVE -- DISCHARGES AT SELECTED EXCEEDANCE PROBABILITIES

ANNUAL EXCEEDANCE PROBABILITY	BULL.17B ESTIMATE	SYSTEMATIC RECORD	'EXPECTED PROBABILITY' ESTIMATE	<-- FOR BULLETIN 17B ESTIMATES --> VARIANCE OF EST.	66% CONFIDENCE INTERVALS LOWER	UPPER
0.9950		646.2	--	--	--	--
0.9900		840.0	--	--	--	--
0.9500	2454.	1622.	2391.0	1.000	2323.0	2582.0
0.9000	2856.	2228.	2807.0	1.000	2719.0	2991.0
0.8000	3478.	3172.	3444.0	1.000	3331.0	3623.0
0.6667	4237.	4284.	4218.0	1.000	4076.0	4398.0
0.5000	5286.	5700.	5286.0	1.000	5099.0	5478.0
0.4292	5818.	6361.	5828.0	1.000	5613.0	6032.0
0.2000	8506.	9168.	8613.0	1.000	8168.0	8875.0
0.1000	11170.	11280.	11460.0	1.000	10650.0	11750.0
0.0400	15210.	13670.	15990.0	1.000	14350.0	16200.0
0.0200	18760.	15250.	20180.0	1.000	17560.0	20150.0
0.0100	22810.	16670.	25240.0	1.000	21190.0	24720.0
0.0050	27440.	17950.	31360.0	1.000	25300.0	29990.0
0.0020	34600.	19460.	41570.0	1.000	31590.0	38220.0

AEP_1 (0.0200), AEP_2 (0.0100); Q_1 (18760.), Q_2 (22810.)

Column labels =>> A B C D E F G

Figure 1. Output from PeakFQ showing "Bulletin 17B" (column B) and "Systematic" (column C) flood-frequency estimates, upper confidence limit for 66.7-percent confidence level (column F) and lower confidence limit (column G). The box encloses the input data for a hypothetical observed flood of 20,000 cubic feet per second. If the computation is for a regulated site, the "systematic record" estimates should be used.

Table 1. Lower and Upper Nonparametric AEP Confidence Limits as a Function of the Rank of Observed Flood and Length of Record for the 66.7-percent level.

Lower and Upper Nonparametric AEP Confidence Limits as a Function of the Rank of Observed Flood and Length of Record

	Rank of Observed Flood									
	1		2		3		5		10	
Length of Record (in Years)	Lower CL	Upper CL	Lower CL	Upper CL	Lower CL	Upper CL	Lower CL	Upper CL	Lower CL	Upper CI
10	0.018	0.164	0.074	0.289	0.145	0.400	0.309	0.600	0.836	0.982
11	0.016	0.150	0.067	0.266	0.131	0.368	0.279	0.554	0.734	0.933
12	0.015	0.139	0.062	0.246	0.120	0.341	0.254	0.515	0.659	0.880
13	0.014	0.129	0.057	0.228	0.110	0.318	0.234	0.480	0.599	0.830
14	0.013	0.120	0.053	0.213	0.102	0.297	0.216	0.450	0.550	0.784
15	0.012	0.113	0.049	0.200	0.095	0.279	0.201	0.424	0.508	0.742
16	0.011	0.106	0.046	0.189	0.089	0.263	0.188	0.400	0.473	0.704
17	0.011	0.100	0.043	0.178	0.084	0.249	0.176	0.379	0.442	0.669
18	0.010	0.095	0.041	0.169	0.079	0.236	0.166	0.360	0.415	0.637
19	0.010	0.090	0.039	0.161	0.075	0.224	0.157	0.342	0.392	0.608
20	0.009	0.086	0.037	0.153	0.071	0.214	0.149	0.327	0.371	0.582
25	0.007	0.069	0.029	0.124	0.057	0.173	0.118	0.266	0.292	0.477
30	0.006	0.058	0.024	0.104	0.047	0.146	0.098	0.224	0.241	0.403
35	0.005	0.050	0.021	0.090	0.040	0.126	0.084	0.193	0.206	0.349
40	0.005	0.044	0.018	0.079	0.035	0.110	0.073	0.170	0.179	0.308
45	0.004	0.039	0.016	0.070	0.031	0.099	0.065	0.152	0.159	0.276
50	0.004	0.035	0.015	0.063	0.028	0.089	0.058	0.137	0.142	0.249
55	0.0033	0.032	0.013	0.058	0.026	0.081	0.053	0.125	0.129	0.227
60	0.0030	0.029	0.012	0.053	0.023	0.074	0.049	0.115	0.118	0.209
65	0.0028	0.027	0.011	0.049	0.022	0.069	0.045	0.106	0.109	0.194
70	0.0026	0.025	0.010	0.045	0.020	0.064	0.042	0.099	0.101	0.180
75	0.0024	0.024	0.010	0.042	0.019	0.060	0.039	0.092	0.094	0.169
80	0.0023	0.022	0.009	0.040	0.018	0.056	0.036	0.087	0.088	0.158
85	0.0021	0.021	0.009	0.038	0.017	0.053	0.034	0.082	0.083	0.149
90	0.0020	0.020	0.008	0.036	0.016	0.050	0.032	0.077	0.078	0.141
95	0.0019	0.019	0.008	0.034	0.015	0.047	0.031	0.073	0.074	0.134
100	0.0018	0.018	0.007	0.032	0.014	0.045	0.029	0.070	0.070	0.127
105	0.0017	0.017	0.007	0.030	0.013	0.043	0.028	0.066	0.067	0.121
110	0.0017	0.016	0.007	0.029	0.013	0.041	0.026	0.063	0.064	0.116
115	0.0016	0.015	0.006	0.028	0.012	0.039	0.025	0.061	0.061	0.111
120	0.0015	0.015	0.006	0.027	0.012	0.038	0.024	0.058	0.059	0.106

Table 2. Example presentation of observed flood peak data along with AEP estimates

	Flood data					AEP for observed 2011 flood			Expected peak streamflows for selected AEP with 95-percent confidence intervals (ft^3/s)								
									10-percent AEP (10-year recurrence)			1-percent AEP (100-year recurrence)			0.2-percent AEP (500-year recurrence)		
							66.7% Confidence Interval			95% Confidence Interval			95% Confidence Interval			95% Confidence Interval	
Station number	Rank	# of Annual Peaks in analysis	Date of Peak Streamflow	Peak Stage (ft)	Peak Streamflow (ft^3/s)	Estimate	Lower	Upper	Estimate	Lower	Upper	Estimate	Lower	Upper	Estimate	Lower	Upper
02473460	4	44	3/9/2011	22.69	12,700	**8.5%**	4.9%	12.8%	**11,900**	9,480	14,900	**21,000**	14,600	30,100	**28,200**	17,600	45,200
02473480	1	47	3/9/2011	18.15	8,000	**1.7%**	0.4%	3.7%	**4,440**	3,400	5,790	**8,940**	5,970	13,400	**12,400**	7,380	20,900
02477330	2	47	3/9/2011	22.90	9,850	**4.6%**	1.6%	6.7%	**7,240**	5,560	9,450	**14,900**	9,820	22,500	**21,100**	12,400	36,100
02483000	2	75	9/6/2011	31.10	39,500	**1.6%**	1.0%	4.2%	**22,600**	18,900	27,000	**43,600**	32,200	59,100	**61,000**	40,600	91,700
02484600	3	39	9/6/2011	16.59	8,460	**7.4%**	3.6%	11.3%	**7,430**	5,820	9,490	**14,100**	9,490	21,000	**19,600**	11,700	32,900
03291780	1	42	4/19/2011	11.60	8,300	**2.2%**	0.4%	4.2%	**6,290**	5,470	7,230	**9,350**	7,550	11,600	**11,300**	8,760	15,500
03303280	2	33	4/28/2011	49.30	648,000	**5.8%**	2.2%	9.5%	**610,000**	568,000	698,000	**742,000**	664,000	1,070,000	**827,000**	713,000	1,370,000
03381500	1	72	5/3/2011	36.42	55,300	**0.7%**	0.3%	2.5%	**29,800**	25,800	36,100	**49,400**	39,400	79,500	**64,200**	46,800	128,000
03333333	1	111	5/28/2011	15.20	151,000	< **0.2%**	0.16%	1.6%	**83,800**	75,600	93,000	**120,000**	99,800	145,000	**144,000**	112,000	186,000

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Distribution: GS-W All

Additional Claification and Guidance for this memo can be found at:
https://xcollaboration.usgs.gov/wg/osw/OSWNotes/Shared%20Documents/OSW%20Note%202013.21%20Additional%20Clarification-Guidance%20OSW%20TM%202013.01.pdf