Report for 2004RI23B: Stream Stability and Scour Potential for Rhode Island Bridges

There are no reported publications resulting from this project.

Report Follows

Note: This is a draft.

1. Scour Models

In this section two scour models that will be used in the study are described. These include the widely used in the United States HEC model and an alternative recent Chinese model. It is noted that the HEC model uses one equation to predict scour whereas the Chinese model utilizes two equations depending on the flow conditions. In subsequent sections these two models will be compared based on a reliability analysis using existing scour data in order to determine the best model to be used in evaluating effects of changing land use.

1.1 HEC Equation

The approach which is used most often in the USA is the "HEC-Equation". The name is originated in the fact that the procedure is outlined in the Hydraulic Engineering Circular #18. It has been developed at the Colorado State University which is the reason why sometimes it is also called "Colorado-Equation". According to the HEC model the expected scour depth can be calculated as the following:

$$y_s = 2 * (K_1 * K_2 * K_3 * K_4) * (\frac{b}{y_1})^{0.65} * Fr^{0.43} \tag{1}$$

In this Equation the following definitions are used:

- y_s = predicted scour depth
- K_1, K_2, K_3, K_4 = correction factors due to the appearance of several pier shapes, flow attack angles, bed conditions and armouring of the bed material; see below for details
- b = pier width
- y_1 = flow depth measured directly upstream of the site
- Fr = Froude number, [William Froude, Englishman, 1810-1879]; see below for details

The *Froude number* is of great importance in various areas of hydrodynamic science. It is a dimensionless quantity defined as the ratio of inertial force and the gravitational force in a specific hydrodynamic system. The Froude number is expressed as:

$$Fr = \frac{v}{\sqrt{g * h}} = \frac{v}{c} \qquad (2)$$

where:

- v = stream velocity
- g = acceleration of gravity
- $h = y_1$ (flow depth measured directly upstream of the site)
- c = wave velocity

Based on the value of the Froude number a flow can be categorized into one of three cases:

1. $Fr < 1 \rightarrow$ drifty flow
2. $Fr = 1 \rightarrow$ critical flow
3. $Fr > 1 \rightarrow$ supercritical flow

The following picture visualizes the meaning of the different areas of the Froude number. It shows a river right after a stone is thrown into it:

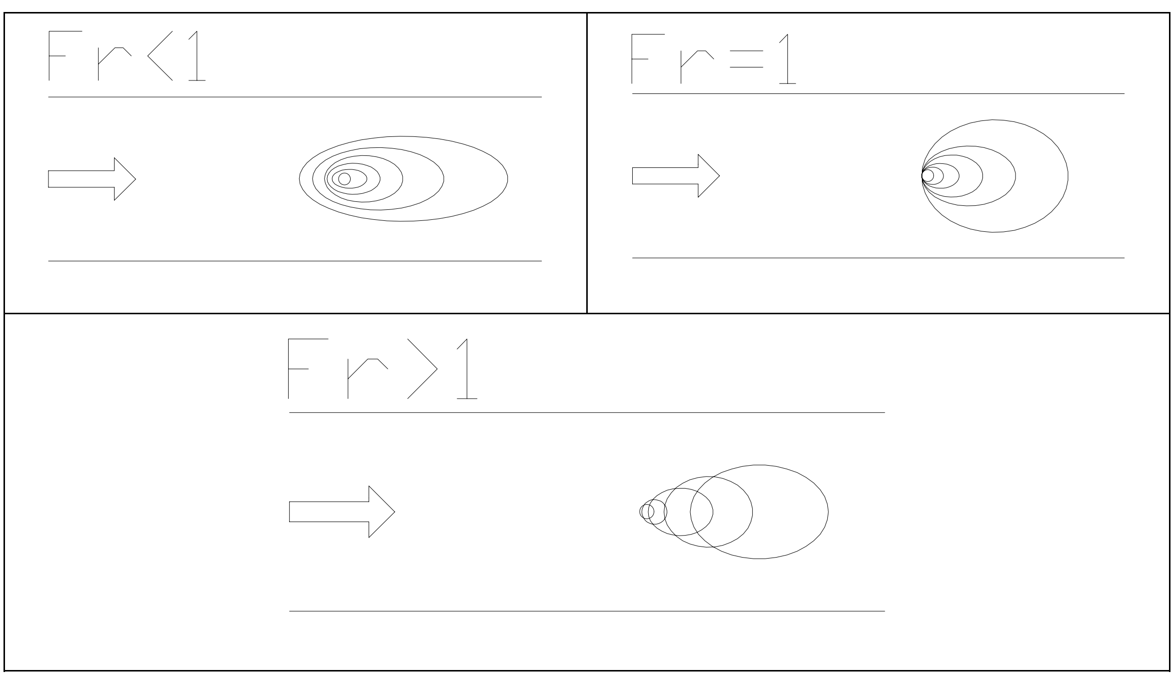


Fig. 1 Illustration of Froude number areas

Note the analogy which exists between the Froude number and the Mach number [Ernst Mach, Czech, 1838-1916] which is used in aviation.

$$Ma = \frac{v}{c_s} \tag{3}$$

Where:

- v = velocity of the specific compound
- c_s = sonic speed

In the HEC equation (Eq. 1), four correction factors appear. The first, K_1 accounts for the different pier shapes which are used most often for bridge piers. The following table illustrates those:

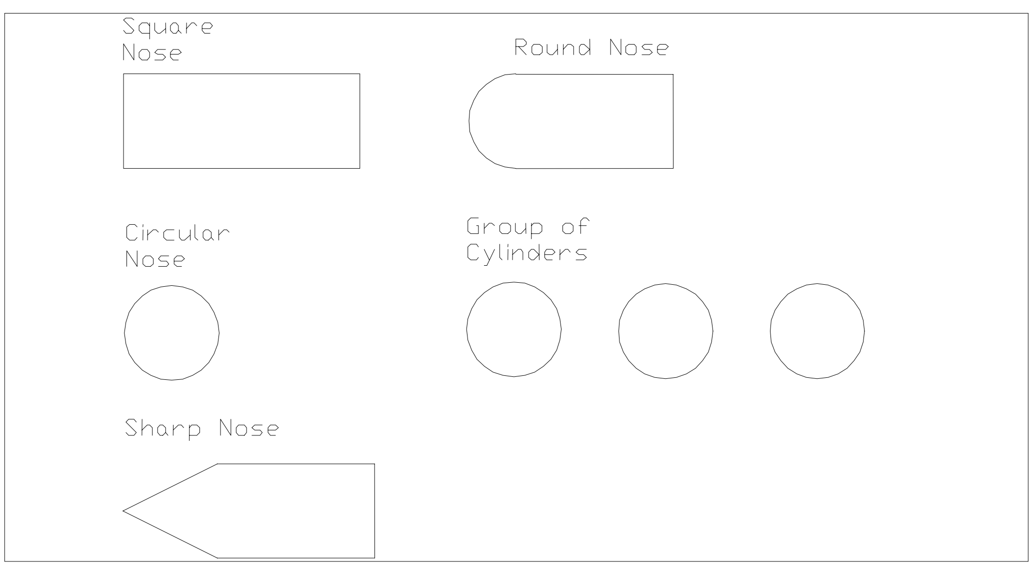


Fig. 2 Typical pier shapes

Values for K_1 :

square nose	1.1
round nose	1.0
circular nose	1.0
group of cylinders	1.0
sharp nose	0.9

Note: K_1=1.0 if the angle of flow attack exceeds 5°.

K_2 is a function of the angle of flow attack θ and the ratio of the pier length to the pier width $\frac{L}{b}$. It can be determined by:

$$K_2 = (\cos\theta + \frac{L}{b}\sin\theta)^{0.65} \tag{4}$$

Typical values for K_2 are as follows:

θ, [°]	L/b=4	L/b=8	L/b=12
0	1.0	1.0	1.0
15	1.5	2.0	2.5
30	2.0	2.8	3.5
45	2.3	3.3	4.3
90	2.5	3.9	5.0

Depending on the size of the underwater dunes and on the sediment transport conditions, K_3 can be found as:

Bed Conditions	Dune Height, [m]	K3
clear-water	N/A	1.1
plane bed and anti-dune flow	N/A	1.1
small dunes	0.6 to 3	1.1
medium dunes	3 to 9	1.1 to 1.2
large dunes	≥9	1.3

For the definition of the last correction factor, *K₄*, the soil particle size D is needed. The size of the medium-sized soil particle is called D_m and has length units. Another name is D_{50} where the "50" is the percentage of soil particles which have already passed a grid once this size is met. Grading curves show the distribution of sizes and give information about the soil. Other characteristic particle sizes are D_{16}, D_{84} and D_{95}.

The last correction factor, K_4, is a dimensionless factor that can be determined by:

$$K_4 = \sqrt{1 - 0.89 * (1 - V_R)^2} \tag{5}$$

Where V_R is a dimensionless velocity ratio: $V_R = (\frac{V_0 - V_i}{V_{c(D90)} - V_i})$ (6)

V_i is the incipient motion velocity meaning that at this velocity the average-sized soil particle starts floating: $V_i = 0.645 * (\frac{D_{50}}{b})^{0.053} * V_{c(D50)}$ (7)

Generally speaking, the incipient flow velocity can be determined for a soil particle of size D_n, where n again is the mass-percentage of soil which has already passed the grid. The velocity can be calculated as:

$$V_{c(Dn)} = 6.19 * y_0^{\frac{1}{6}} * D_n^{\frac{1}{3}} \tag{8}$$

The pier width *b* is being measured as shown in the following sketch irrespective to the number of pier columns:

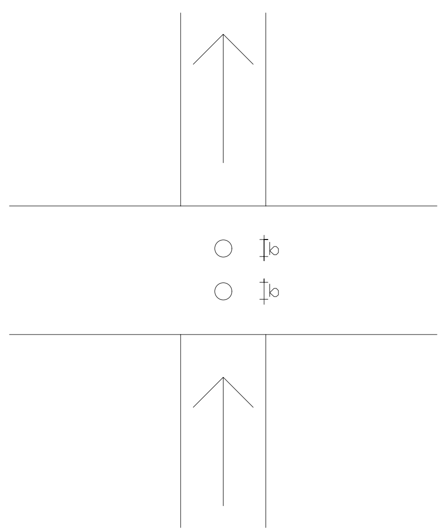


Fig. 3 Definition of pier width, *b*

1.2 Chinese Scour Model

As a result of laboratory tests as well as field data, several scour equations were investigated in China. Gao et al. [1992] finally developed a simplified pier-shape coefficient and came up with two equations, one for clear-water conditions and another for live-bed conditions:

For clear-water conditions the Chinese model is**:**

$$y_{sp} = 0.78 * K_s * b^{0.6} * y_0^{\ 0.15} * D_m^{\ -0.07} * (\frac{V_0 - V_c'}{V_c - V_c'}) \tag{9}$$

In this equation the following parameters are used:

- K_s = factor related to pier shape. It takes values of 1.0 for cylinders, 0.8 for round nosed piers and 0.66 for sharp nosed piers.
- b = pier width
- y_0 = water depth at the pier
- D_m = diameter of the average sized soil particle
- V_0 = flow velocity
- V_c = critical flow velocity; see below for details
- V_c' = approach velocity in the constricted reach; see below for details

To visualize the meaning of the critical velocity, V_c, one can consider that whether the soil particles are in motion (live-bed) or not (clear-water) depends on the ratio of actual flow velocity to critical velocity. If this ratio is greater than one, live-bed conditions are present whereas clear-water conditions can be expected if the ratio is less than one. The actual value for V_c has units of velocity and is given by:

$$V_c = (\frac{y_0}{D_m})^{0.14} * (17.6 * (\frac{\rho_s - \rho}{\rho}) * D_m + 6.05E(-7) * (\frac{10 + y_0}{D_m^{\ 0.72}}))^{0.5} \tag{10}$$

- ρ_s = density of the sediment particles of the soil
- ρ = density of water

Once the critical velocity is known, the incipient approach velocity can be calculated by:

$$V_c' = 0.645 * (\frac{D_m}{b})^{0.053} * V_c \tag{11}$$

The term $(\frac{V_0 - V_c'}{V_c - V_c'})$, inherent in both the clear-water and live-bed equations, is a dimensionless parameter characterizing the flow intensity.

For live-bed conditions, the Chinese model becomes:

$$y_{sp} = 0.65 * K_s * b^{0.6} * y_0^{\ 0.15} * D_m^{\ -0.07} * (\frac{V_0 - V_c'}{V_c - V_c'})^c \tag{12}$$

Looking at both equations (Eqs. 9 and 10) the only differences between live-bed and clear-water is the factor 0.65 which was 0.78 for clear-water and the power of c for live-bed conditions which is defined as:

$$c = (\frac{V_c}{V_0})^{9.35+2.23*\log(D_m)} \tag{13}$$

The parameter c will always be less than one, making sure that the estimated scour depth for clear-water conditions will never be less than the one for live-bed conditions.

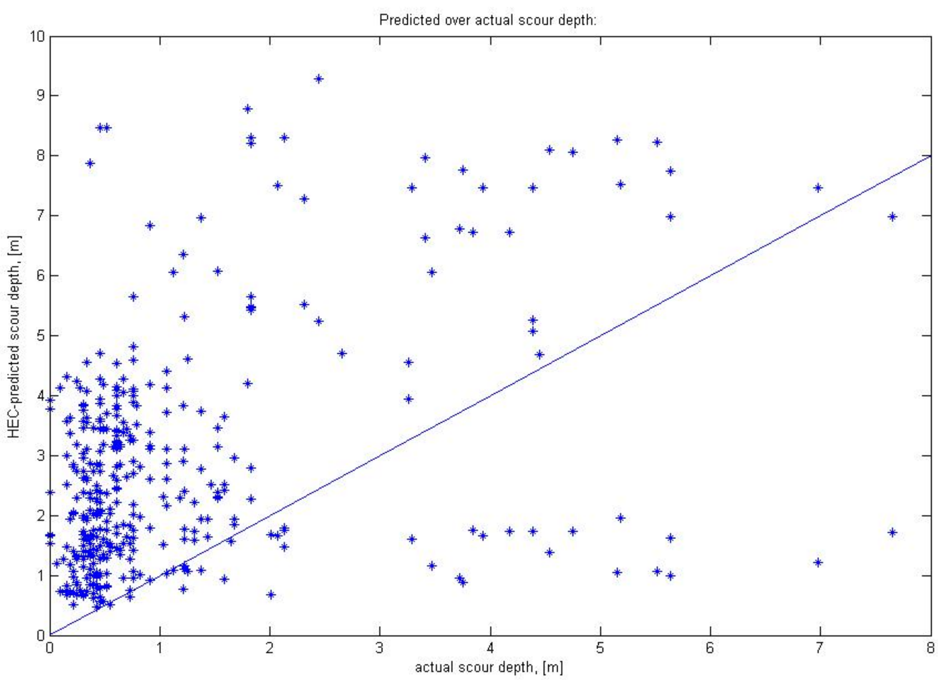
An important note is the fact that all the above equations used for the Simplified Chinese approach are dimensional and have to be used with SI-units. Therefore, lengths have to be inputted in [m] and velocities in [m/sec]. Theoretically, the densities have to be also used with SI-units [kg/m³] but in this case it does not matter since when looking at the whole term that includes the densities it is found that it is dimensionless.

2. Reliability Analysis of the HEC Equation

The ultimate test of any prediction model is how reliably does it predict the behaviour it is trying to model. In the present section a summary of the reliability procedures used to critically compare the two scour models (HEC Model and Simplified Chinese Equation Model) is provided. The first procedure is based on the use of a reliability index, β, whereas the second approach is based on regression analysis. The data used for the reliability studies are based on the report "Channel Scour at Bridges in the United States" (FHWA-RD-95-184) by Landers and Mueller.

Reliability estimates using "lamda" values

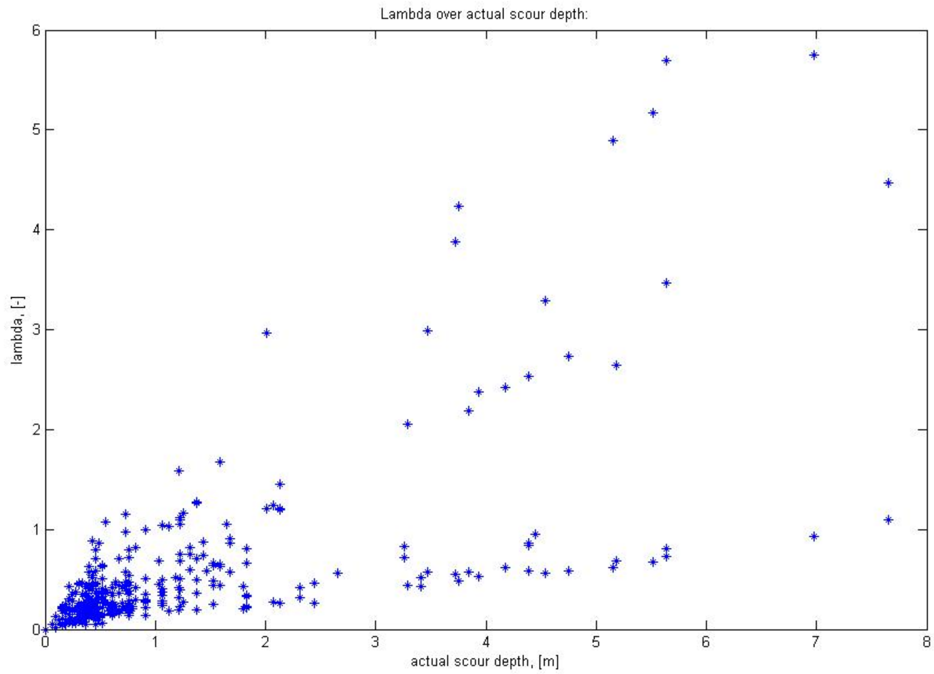
As mentioned before the scour predicting equation which is used most frequently in the area of the USA is the HEC-18-equation. Taking the measurements from the Landers data as an input for this equation, assuming that the correction factor K_3 = 1.15 for all live-bed conditions (medium height dunes) enables to compare the HEC results to the actual measured scour depths:



The data points are sorted by increasing actual scour depth and the continuous even symbolizes the optimal prediction where actual scour depth = predicted scour depth. The vast majority (88.6%) of actual scour depths are below 2.5 meters (8.2ft) and for this range there are hardly underestimations. Unfortunately this does not necessarily mean that the approach is quite close to the actual scour depth. As it can be seen in the picture above, overestimations are quite enormous and up to 9 times the actual scour depth. An advantage of the HEC-equation is that statistically speaking there are not many bridge sites where scour depths deeper than 2.5 meters can be expected and therefore it should be a conservative approach for the majority of sites.

But does the hope just to overestimate satisfy completely?

As per definition $\lambda = \frac{actual}{predicted}$ scour depth, it can be said that as the scour depth increases, the variation of the HEC equation increases as well. At the point of maximum actual scour depth in the Landers data the underestimation is up to 500% and at another point of about the same actual scour depth the prediction meets the actual scour depth quite close as $\lambda \approx 1.0$. When comparing the numbers of points which are under- and overestimated it comes out that 89.9% of all HEC-predictions provide overestimations, i.e. $\lambda < 0$.



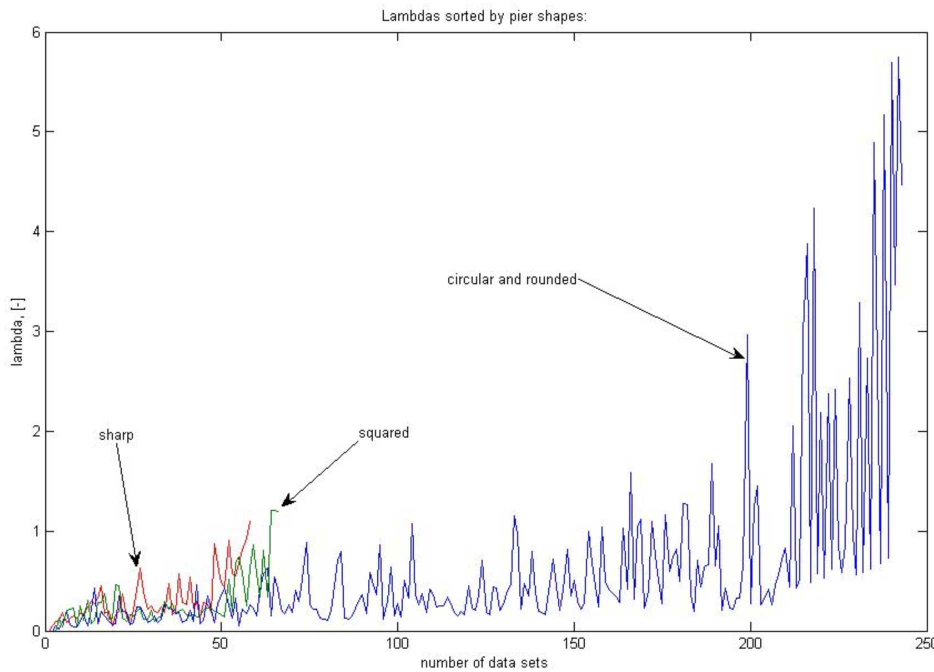
When determining the reliability index β, its quality essentially depends on the performance of the lambda table. In the case of bridge scour there are many parameters which are used and for this reason there are many options to sort the data sets.

Examples are

- pier-shape,
- conditions (live-bed or clear-water),
- cohesive or non-cohesive bed material,
- bed material size,

- Froude number,
- Reynolds number,
- etc.

As the pier shape is a factor which is used by most of the existing scour equations, the following picture illustrates the relation of the lambda values for data sets with the same pier shapes:



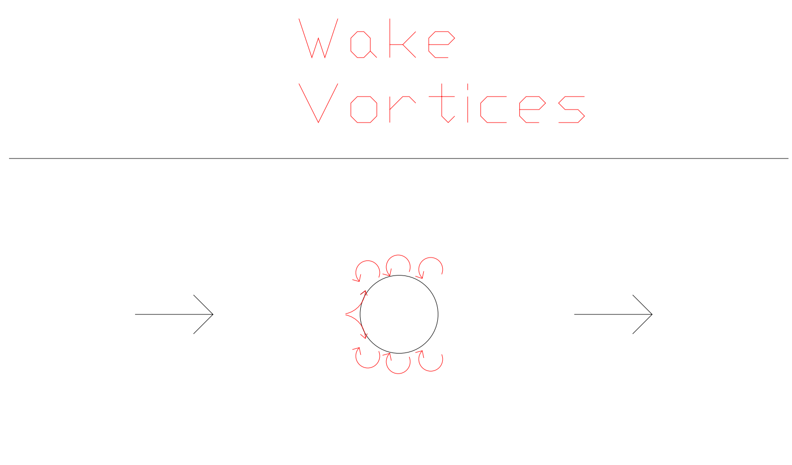
The lambda values for sharp nosed (58 data sets) and for squared (66 data sets) piers are quite similar respectively. The latter have an COV of 7.6%, the sharp nosed sets of 6.1%. When looking at the 243 data sets for circular and rounded piers it comes out that there is a huge variation ranging from lambda values of 0.02 to 5.75 and a COV of 87.3% indicating that a sorting only by pier-shapes is not sufficient yet. Especially taking one group for all the circular and rounded piers and therefore getting a lot of data sets (243) is not pertinent.

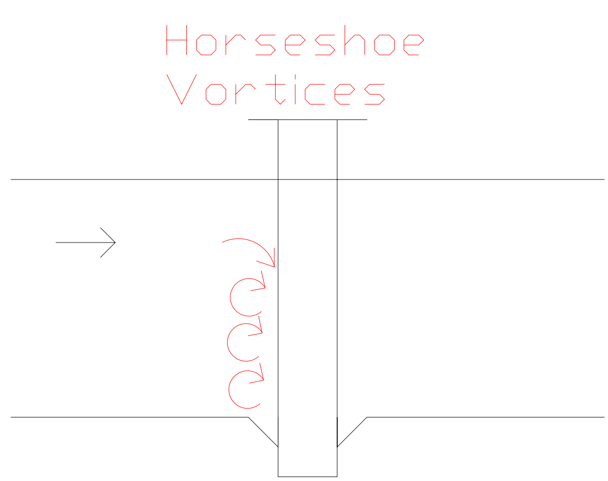
To approve one will have to sort by another characteristic parameter additionally.

Shen et al. [1969] found the so called *Reynolds number* [Osborne Reynolds, Irishman, 1842-1912] to be important:

“Since the horseshoe vortex system is the system of local scour and the strength of the horseshoe vortex system is a function of the pier Reynolds number, the equilibrium depth of scour should be functionally related to the pier Reynolds number”

Horseshoe vortices are produced by the flow which streams against a barrier, i.e. a bridge pier. They are normal to the water surface while the so called wake vortices are those existing parallel to the water surface.





The dimensionless Reynolds number Re is defined as:

$$\mathrm{Re} = \frac{\rho * \upsilon * L}{\eta} = \frac{\upsilon * L}{\nu} \tag{1}$$

Where:

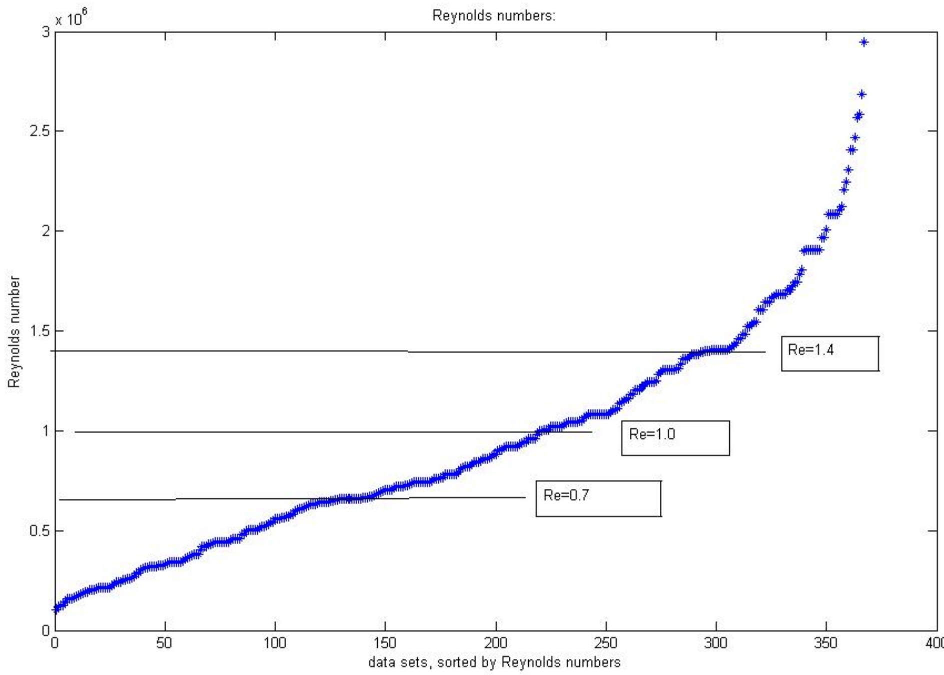
- ρ = characteristic density of the medium, [kg/m^3)
- □ = absolute value of the characteristic velocity, [m/s]
- L = characteristic length, in our case: pier width, [m]
- η = characteristic dynamic viscosity, [kg/(m*s)]
- ν = characteristic kinematical viscosity, [m^2/s]

Once the Reynolds number exceeds a specific critical value the flow turns from a laminar to a tumultuous or also known as turbulent behaviour. Generally speaking the Reynolds number can be applied for any media like fluids and gas. It is widely used in the fluid mechanics. Commonly applied for pipes it is obvious that for a river usually we will have laminar flow but still the Reynolds number has an important meaning. Assuming that we have a constant kinematical viscosity at an average water temperature the Reynolds number is somewhat related to the discharge rate Q. As $Q = V * A = V * b * y_0$, the Reynolds number can be found as:

$$\mathrm{Re} = \frac{Q}{\nu * b} \tag{2}$$

Neglecting the fact that ν is a function of the water temperature, the Reynolds number is the discharge rate per unit pier width.

For water at a temperature of 5°C (41°F) the kinematical viscosity ν can be found to be $1.52004 * 10^{-6}$ [m²/s].

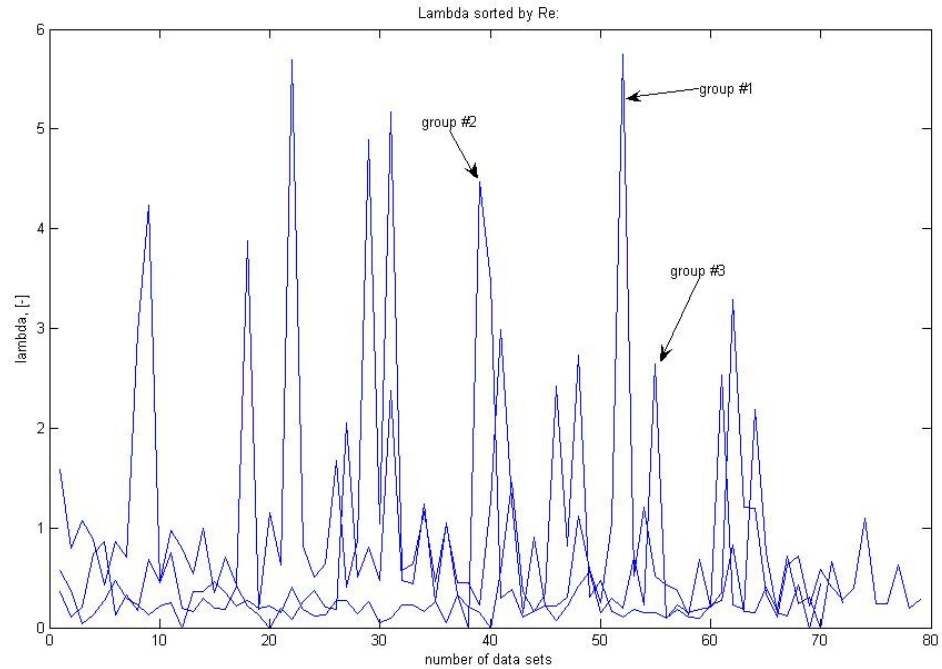


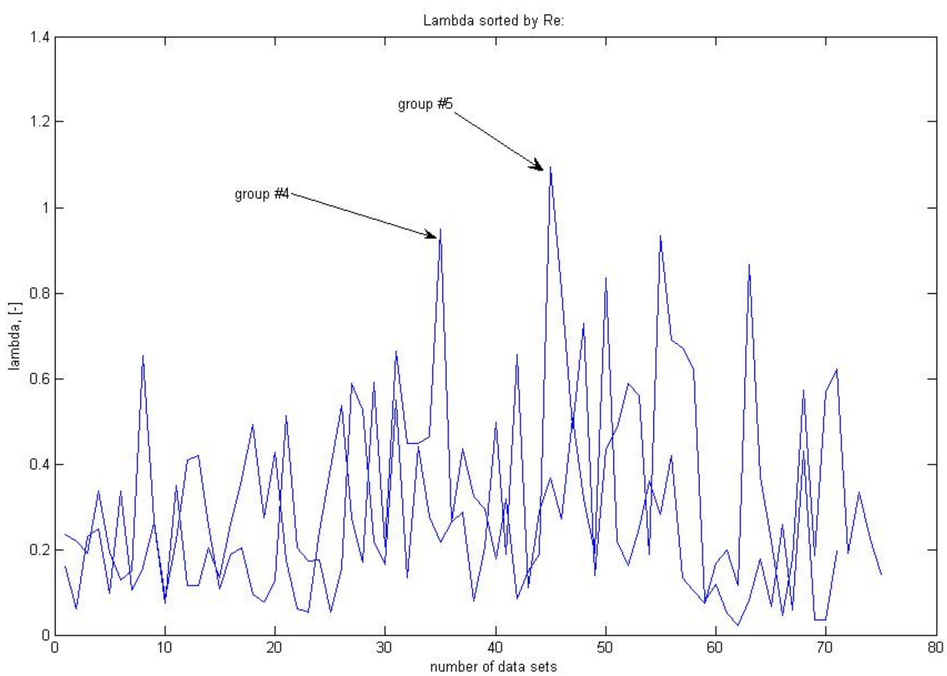
The Reynolds numbers for the whole Landers data ranges from approximately 100,000 to 3,000,000.

I will introduce five groups:

1. $0 < \mathrm{Re} \leq 450{,}000$
2. $450{,}000 < \mathrm{Re} \leq 700{,}000$
3. $700{,}000 < \mathrm{Re} \leq 1{,}000{,}000$
4. $1{,}000{,}000 < \mathrm{Re} \leq 1{,}400{,}000$
5. $1{,}400{,}000 < \mathrm{Re} \leq 3{,}000{,}000$

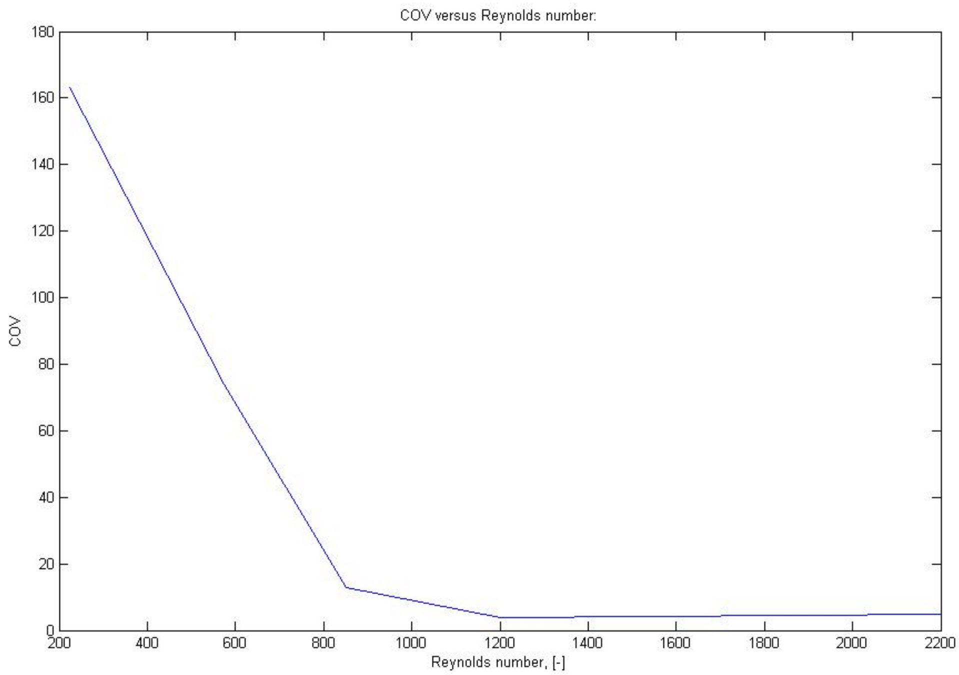
The lambda values for each of the four groups are plotted below:





The coefficients of variation of group 1 to group 5 are 163.2%, 74.4%, 12.7%, 3.9% and 5.3% respectively. The numbers of data sets for each group are 79, 70, 72, 75 and 71.

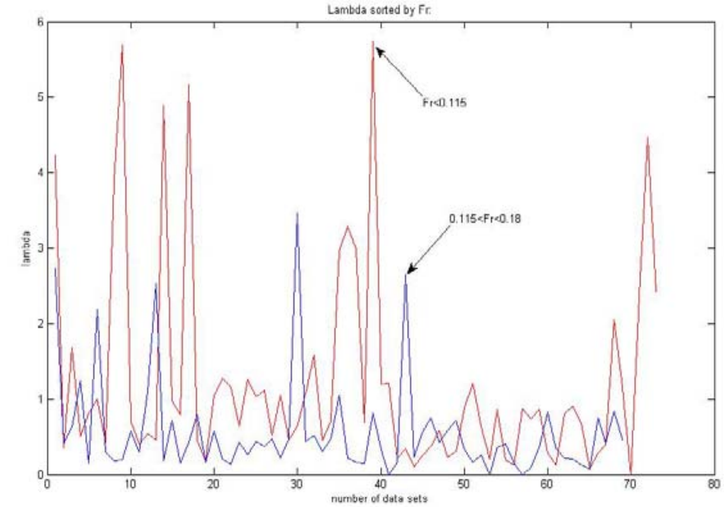
The COVs of group one and two are pretty high but indicate another interesting relation when plotting COV over average Reynolds numbers of each group:

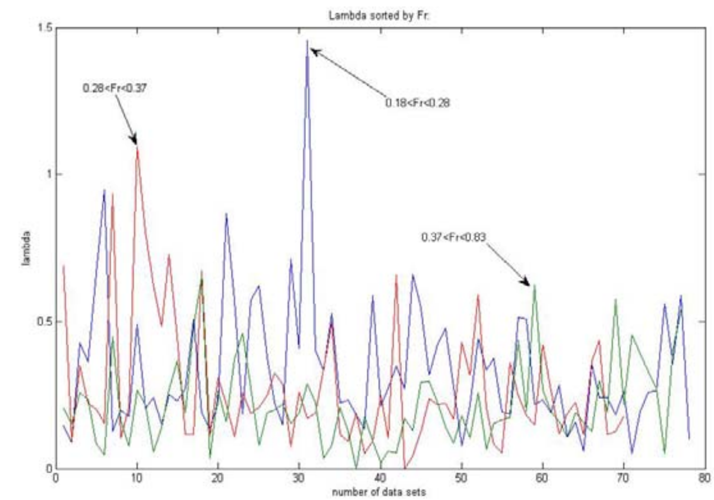


In words: The more water volume per time runs against the bridge pier, the less the variation of the lambda values produced by the HEC equation. Starting at approximately Re=1200, the COV is acceptable.

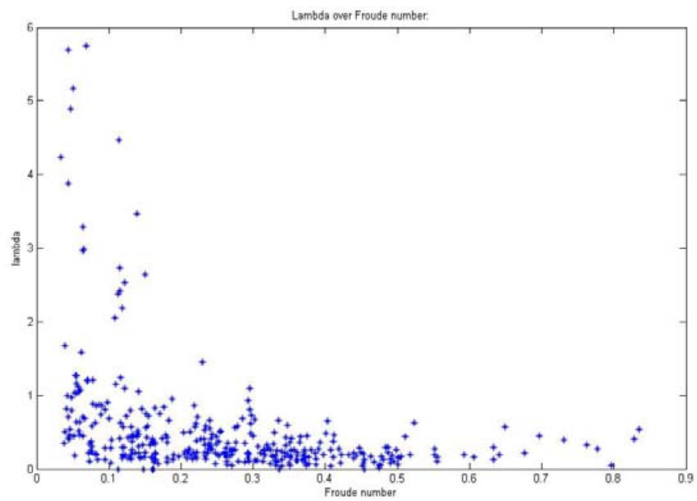
Using the Froude number introduces another important hydraulic parameter as a value to sort by. The smallest Froude number is 0.033 and the biggest was 0.835 if applied to the Landers database. Again I used five approximately equally sized groups:

1. $0 \le Fr < 0.115$
2. $0.115 \le Fr < 0.180$
3. $0.180 \le Fr < 0.280$
4. $0.280 \le Fr < 0.370$
5. $0.370 \le Fr < \infty$

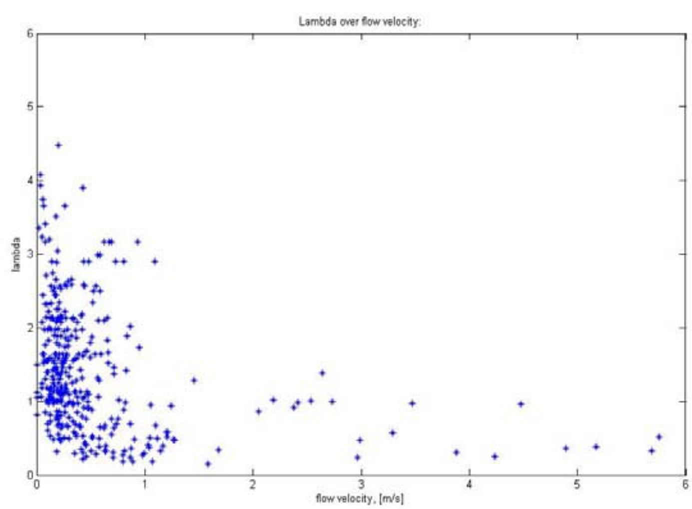




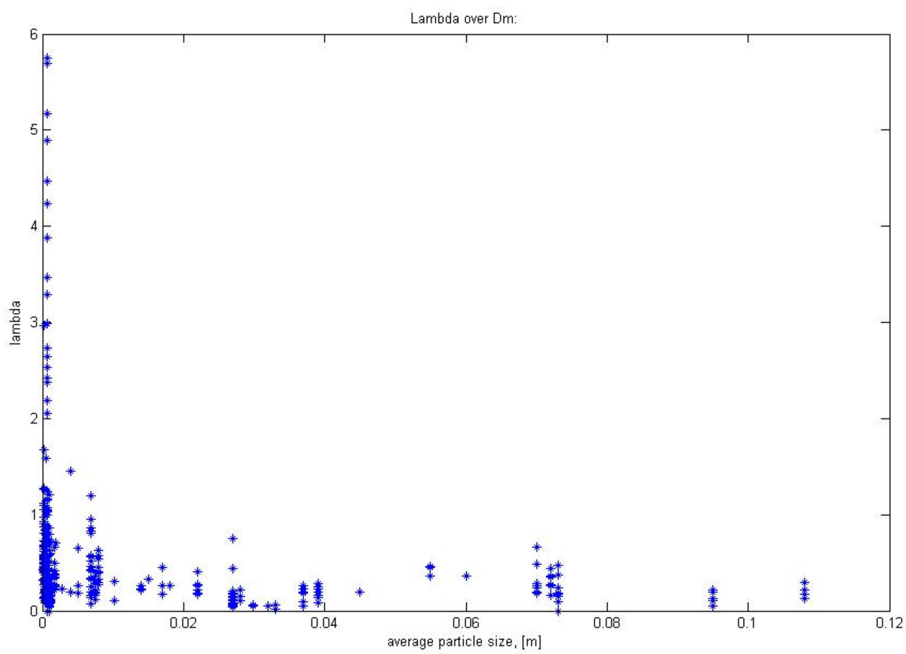
The coefficients of variation are 191.5%, 44.8%, 5.2%, 4.9% and 2.1% for groups one to five respectively. As per definition $Fr = \frac{V_0}{\sqrt{g * y_0}}$, the bigger the flow velocity and the shallower the stream, the smaller the variation of the ratio $\frac{actual}{HEC - predicted}$ scour depth:



As the flow velocity is located in the enumerator in both parameters, the Reynolds and the Froude number, it is very likely that the faster the stream goes the less variation the HEC results have. Even when using all the Landers data sets and plotting lambda over flow velocity a similarity to the plots of lambda over Froude number and accordingly Reynolds number can not be denied:



To have an improved accuracy of the predictions and a fewer COV for each group I will use two parameters in a row for the lambda table. As the flow velocity influences both, Re and Fr, I will use only one of those: The Froude number. Consequently the other parameter to take into consideration for the lambda table should not be related to the flow velocity. As mentioned above a sorting by the pier-shape is not very useful. Instead using a parameter which is not related to the flow velocity either can be achieved with the average particle size D_m.



As the picture above illustrates, the HEC results seem to become less varying as the average sized soil particle becomes bigger.

Using pier shape and the size of the average soil particle the following table of the left side was produced. For each result with a COV of the included lambdas of more or equal to 10% I subdivided those results depending on its Froude number. The results can be seen on the right side:

pier shape	median lambda, [-]	COV, [%]		Fr limits	median lambda, [-]	COV, [%]
squared	0.2902	7.58		0 to 0.12	1.4862	228.01
rounded, circular	0.6337	87.27	--->	0.12 to 0.21	0.5327	35.81
sharp	0.3279	6.09		0.21 to 0.32	0.3541	5.27
				0.32 to unlim	0.2266	2.31

Dm limits	median lambda, [-]	COV, [%]		Fr limits	median lambda, [-]	COV, [%]
0 to 0.005	0.69906	92.73	--->	0 to 0.08	1.3705	227.54
0.005 to 0.02	0.3817	5.53		0.08 to 0.15	0.8023	83.15
0.02 to 0.035	0.1559	1.92		0.15 to 0.25	0.3983	15.08
0.035 to unlim	0.2255	1.7		0.25 to unlim	0.302	5.51

As the pretty high COVs for low Froude numbers less or equal 0.15 indicates, there is still a high level of modelling uncertainties even if a two-step-division as obtained above is used. To continue a further subdivision one would need more data. This is because the smallest group used above already contains approximately only 60 data sets. For example: Rounded, circular piers and $0 \le Fr < 0.12$ includes 57 measurements.

The lambda table shown above is only one of the possible ways to sort the database. Other criteria could be chosen which again would provide different lambda tables.

Reliability estimates using regression analysis

To perform a reliability analysis several parameters have to be known and a way in finding them has to be chosen previously. The reliability index β can be calculated as the following:

$$\beta = \Phi^{-1} * P_f \qquad (1)$$

$$\beta = \frac{\mu_R - \mu_Q}{\sqrt{\sigma_R^{\ 2} + \sigma_Q^{\ 2}}} \qquad (2)$$

In eq. (1) P_f is the Probability of Failure and Φ is the function of the cumulative normal distribution. To interpret β for practical use the following table can be used which shows the values for β as a function of the Probability of Failure:

Pf	β
0.1	1.28
0.01	2.33
0.001	3.09
0.0001	3.71
0.00001	4.26
0.000001	4.75
0.0000001	5.19
0.00000001	5.62
0.000000001	5.99

In most practical situations the use of equation (2) will take place as the exact determination of P_f is linked with several difficulties and uncertainties.

Basically the idea behind is to take the input data a designer would use with the HEC-equation. Subsequently this result is being compared with the actual scour depth and its variations. Theoretically when calculating reliability indices both standard deviations have to be considered: The one from the resistance and the one from the load. In our case where the resistance is being symbolized by the designer's result using the HEC-equation, we only have one value and for this reason the standard deviation for the resistance equals zero.

Basically there are two ways to determine the mean and standard deviation of the load Q: Using a regression analysis or using a lambda table.

The way to perform a regression analysis is to put in several parameters which are considered to influence the scour depth significantly and also to put in the measured actual scour depth. Therefore to come up with a regression analysis it is necessarily to have lots of data from existing sites and accurate measurements. After that a computer can calculate an equation which approximately fits the measured scour depths. The difference between actual scour depth and the depth provided by the regression equation is quite important, of course, and its minimization is most important. The researcher has to find parameters and its pertinent limits for which the included data sets have approximately the same dependencies of input parameters and scour depth.

If the use of Lambda Tables is the way to go one takes a scour prediction equation, for example the HEC equation, the SCE or others and applies it to existing data where also the actual scour depth has to be known. After calculating the prediction results those have to be compared to the actual scour depth as ratio actual divided by predicted scour depth. As already mentioned, this ratio is known as *bias* or l. The most challenging part with the use of lambda tables is to find parameters which are not kept under consideration by the used scour equation. In other words when sorting the lambda table by parameters which are already used for the scour equation the table should provide a limited range of lambda values respectively.

Of course the above said is only true if the equation used to predict the scour depth accounts for the parameter correctly. As some parameters can be used directly (e.g. pier width) but others are hard to put into numbers (e.g. pier shape). Empirically chosen numbers may vary from site to site, soil to soil, country to country, etc.

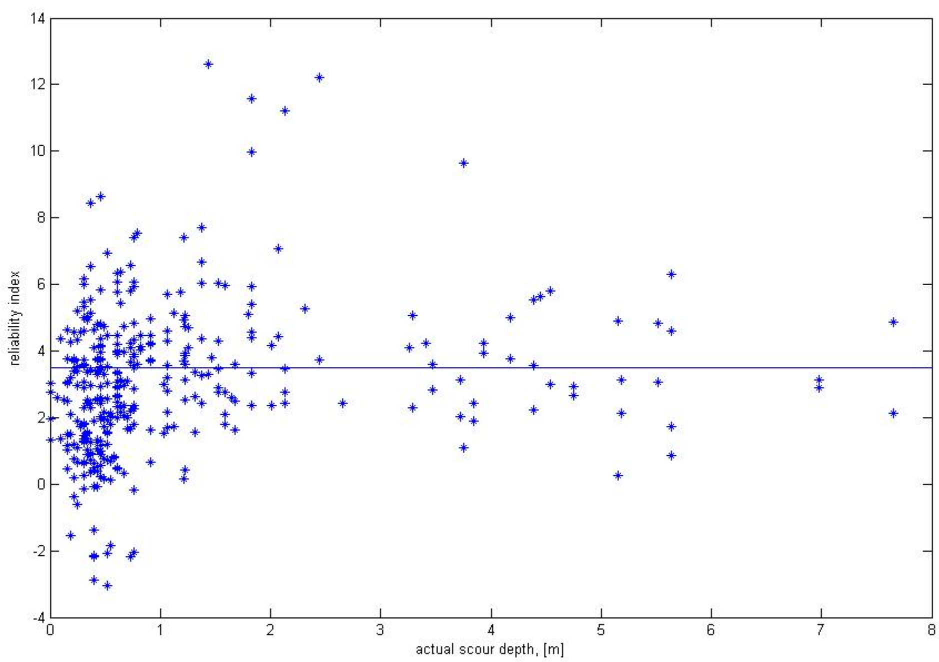
μ_Q is the prediction obtained by the Regression analysis or by the use of Lambda tables. I will use the result of a regression analysis as I expect more accurate results this way. When using one value for each parameter using the HEC-equation we get one prediction depth which automatically equals μ_R. I will produce random variables for each parameter needed as input for the regression equation. Each will be produced following its pertinent random distribution. For this reason we will have several predictions from the regression analysis to compare it with the HEC-prediction and subsequently we will have $\sigma_Q \neq 0$. The number of random variables produced for each data set of the Landers database can be chosen. I will start with 10 cycles and in the beginning I will just use one regression analysis for all Landers data. When using the pier width b, the flow velocity V_0, the flow depth y_0, the average particle size D_m, the Froude number Fr and the actual scour depth y_s as an input it turns out that the result provided by a multiple regression analysis is:

$$y_s = (-0.1155) + 0.2668 * b - 0.8078 * V_0 + 0.2452 * y_0 - 9.0795 * D_m + 3.9384 * Fr \qquad (3)$$

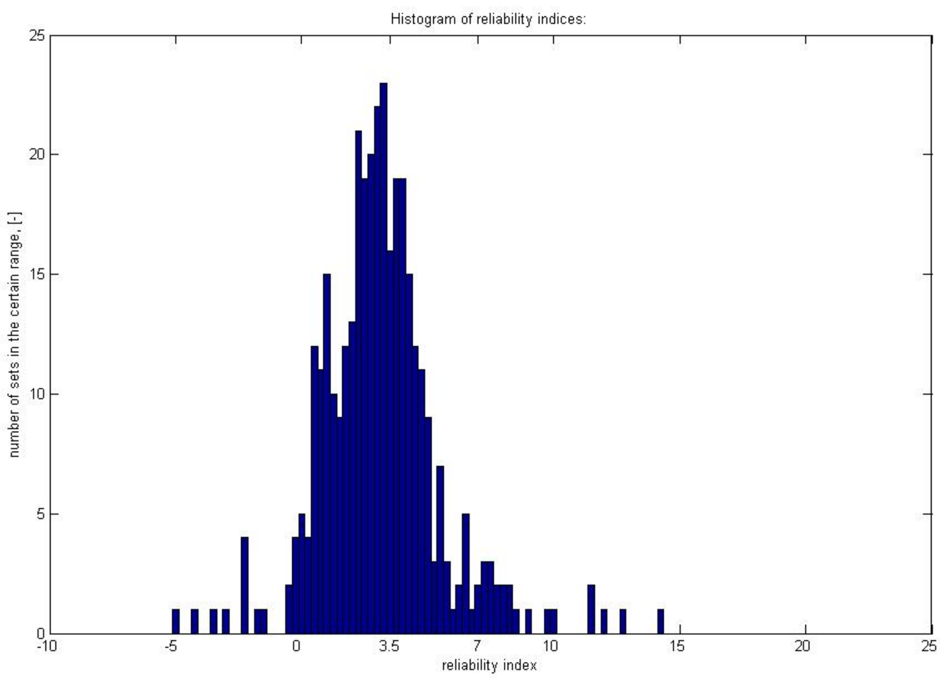
After having deleted the parameters with unknown values there were 358 data sets left to be used with the HEC-equation. As described before, the parameters used in eq.(3) are each following either normal or lognormal distributions. The following table summarizes the random distribution and the standard deviation for the several parameters respectively:

parameter name	meaning	random distribution	standard deviation
y0	flow depth	lognormal	10%*mean
b	pier width	deterministic	
V0	flow velocity	lognormal	25%*mean
Dm	average particle size	lognormal	given in database
Fr	Froude number	lognormal	16%*mean
e	residual error	normal	0.41

The calculated reliability indices are plotted over the actual scour depth where the straight even symbolizes a reliability index of 3.5 which usually is the target value:



Apparently the scattering is quite huge and when looking at the whole set of reliability indices, the obtained coefficient of variation equals 5.18. When running the procedure 10 times the average percentage of data sets which actually provide a reliability index greater than 3.5 is 37.6%. The scattering becomes less once a certain actual scour depth is exceeded. The histogram reveals that although the average of 3.01 is quite close to the desired 3.5, the majority (62%) of sets provides a beta value of less than 3.5. Approximately one third (37.15%) of sets have a beta value less than 2.33 which represents a probability of failure of only 1:100 and for 5.4% the reliability index becomes even negative.



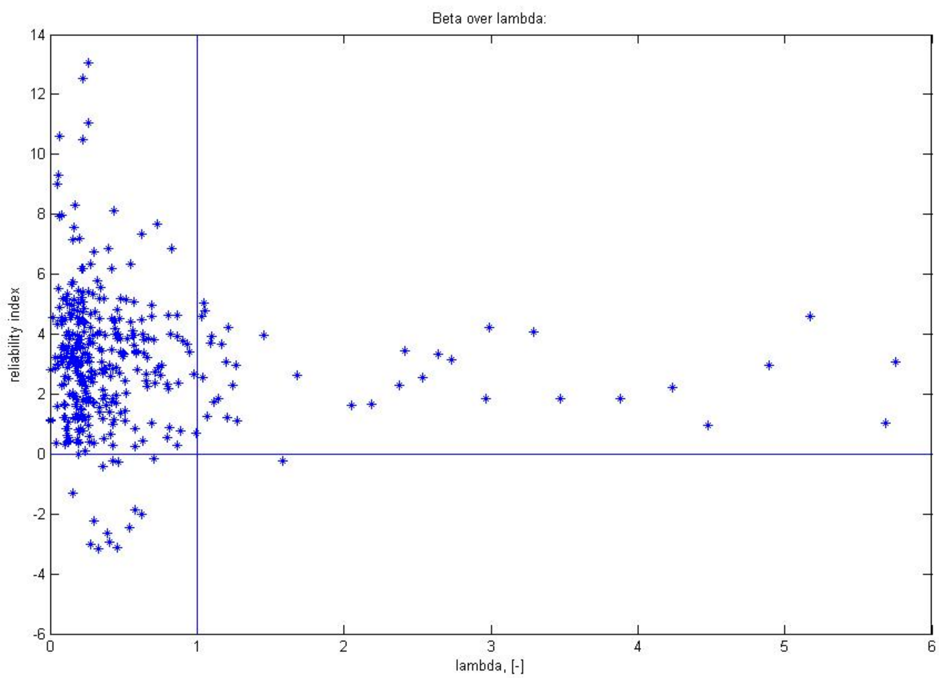
To answer the question in which cases a greater reliability index can be expected it is quite obvious to look at the equation for beta first. The enumerator includes the designer's prediction result using the HEC equation minus the average result of the regression equation:

$$\mu_R - \mu_Q$$

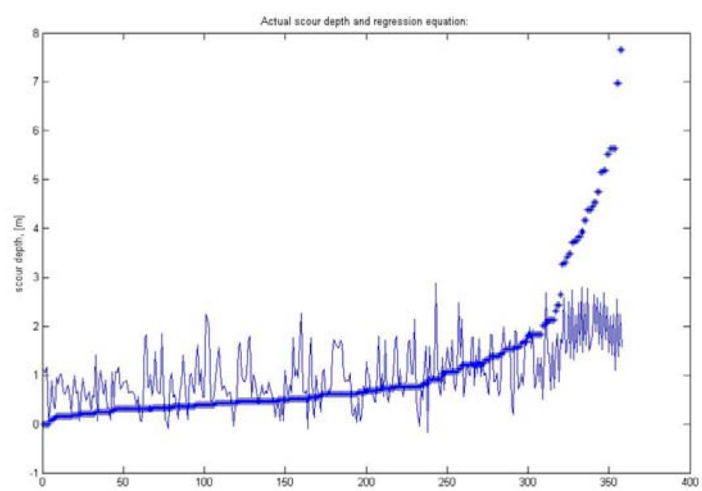
Therefore the bigger the designer's result and the smaller the regression result, the bigger the reliability gets in the end. When comparing the lambda values with the reliability indices and looking at those sets where $\lambda > 1$, one could expect beta values with negative algebraic signs as consequently $\mu_Q > \mu_R$ and as the denominator equals

$$\sqrt{\sigma_R^{\ 2} + \sigma_Q^{\ 2}}$$

it can not take any negative values.



As can be seen above, for l >1 nearly all beta values are >0. One explanation could be the lack in accuracy originated in the fact that the scour depths calculated by using the regression equation do not exactly match the actual scour depths. As a result I will perform another regression analysis using subdivided parts of the database later on to reduce the differences between actual scour depths and those



determined with the regression equation which at this point are still quite big.

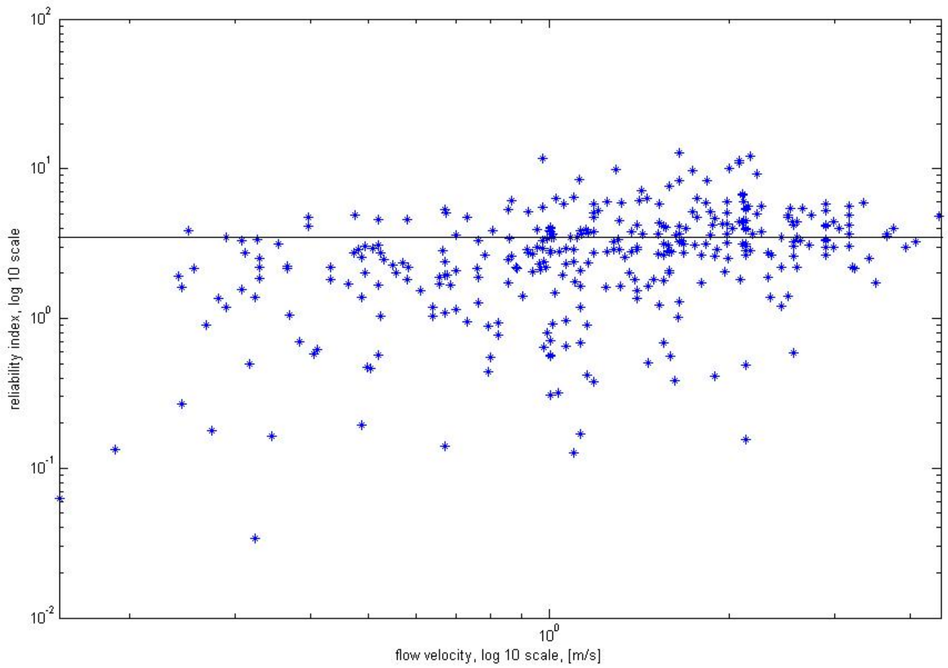
Another way to find explanations for the variety of reliability indices is to look at it as a function of other hydraulic or hydrodynamic parameters such as the flow velocity, discharge rate per flow width, discharge rate per pier, the Froude number or the flow intensity. The discharge rate per flow width can be determined as:

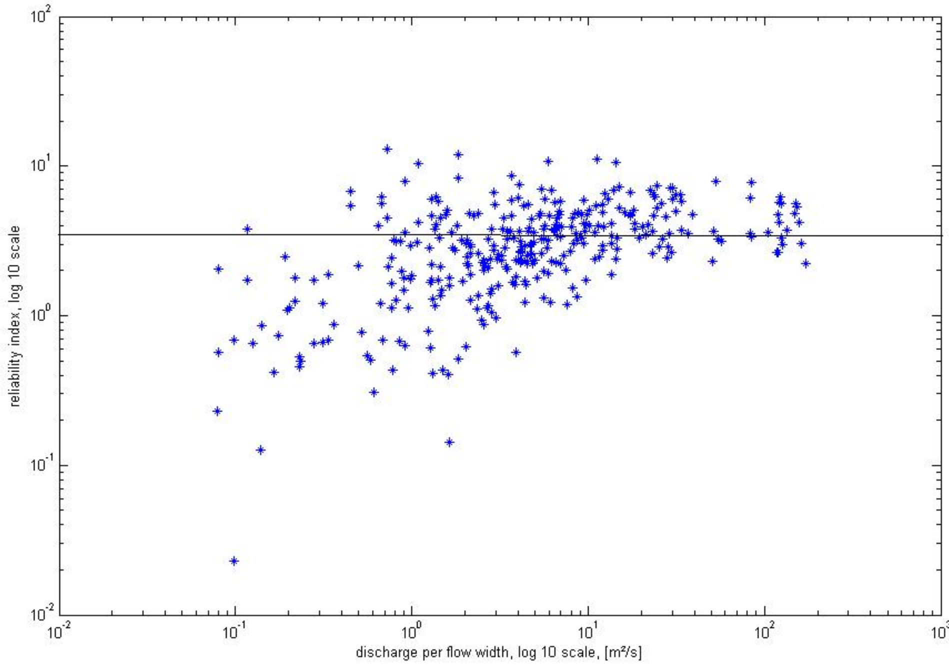
$$Q = V_0 * y_0 = [\frac{m^2}{\sec}]$$

If additionally multiplied by the pier width one gets the discharge rate per pier:

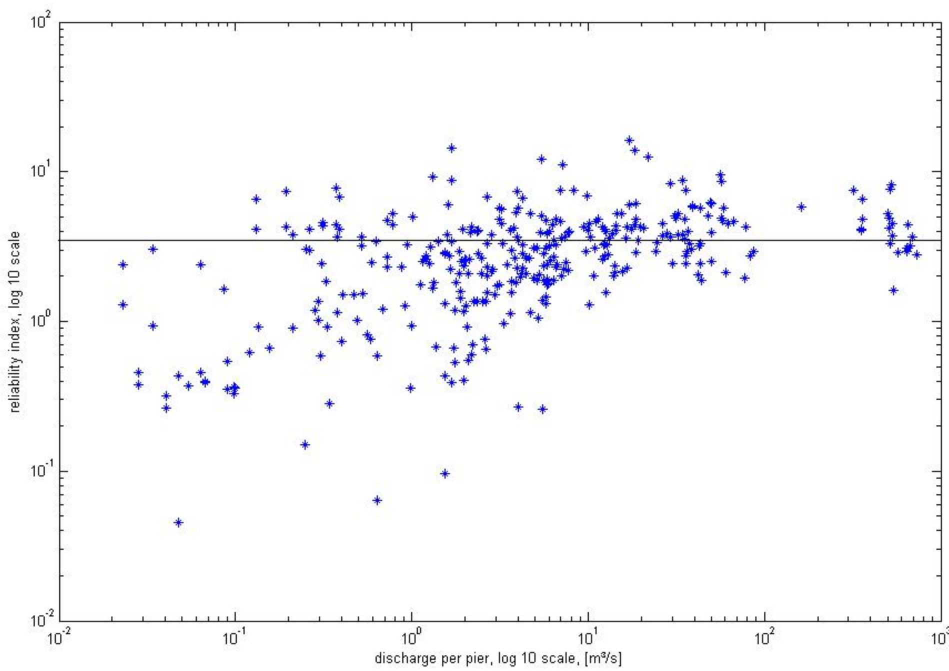
$$Q = V_0 * y_0 * b = [\frac{m^3}{\sec}]$$

The following plots can be used to interpret the reliability indices β as a function of the several parameters. Again the straight even symbolizes a desired β of 3.5.

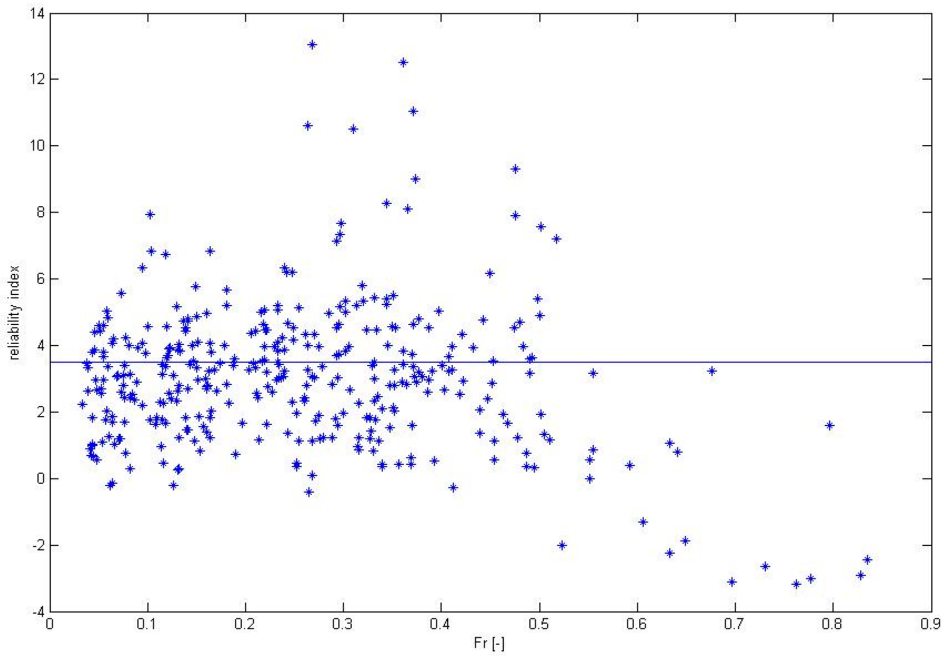




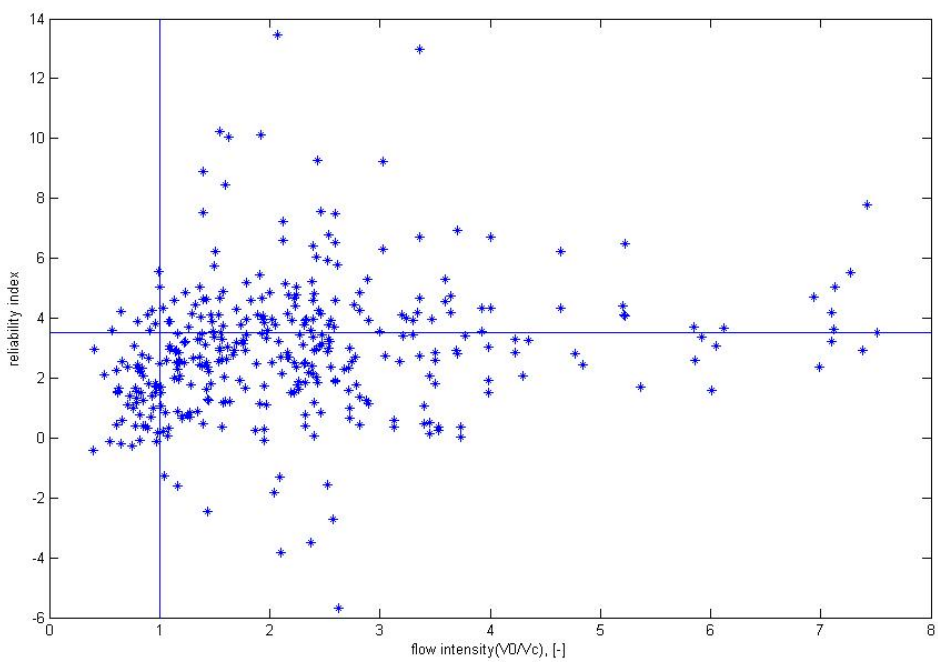
10^2
10^1
10^0
10^{-1}
10^{-2}
reliability index, log 10 scale
10^{-2}
10^{-1}
10^0
10^1
10^2
10^3
discharge per flow width, log 10 scale, [m²/s]



10^2
10^1
10^0
10^{-1}
10^{-2}
reliability index, log 10 scale
10^{-2}
10^{-1}
10^0
10^1
10^2
10^3
discharge per pier, log 10 scale, [m³/s]



reliability index
Fr [-]



reliability index
flow intensity(V0/Vc), [-]

When using a basis-10-logarithmic scale for the reliability index over the **flow velocity** respectively, it reveals that the scattering increases while the flow velocity is increasing, too. After sorting the beta values by increasing flow velocity and dividing into three approximately equally spaced groups of 119 (120), the following means can be determined: 2.49, 2,75, 4.12. The coefficients of variation take the following values: 3.51, 4.58, 9.19. Concluding with increasing flow velocity both, the reliability and the variation of the beta values increase. For faster streams the HEC-equation obtains conservative results. Still, the percentage of reliabilities less than 3.5 are 72.5%, 66.7% and 41.5% sorted by increasing flow velocity.

Usually the **discharge rate** is determined as water volume per time and bridge pier. When calculating it following the first equation above as discharge per meter flow width the observed dependencies clear out more accurate. This can be found when sorting the determined reliability indices by the discharge rate not using the pier width b and another time sorting by the discharge rate per pier which does include the pier width b. I divided each time into three groups which are approximately equally spaced with 120 (118) data sets.

Discharge per Flow Width:

small discharge	mean	1.64
	COV	5.88
	sets >3.5	14.20%
	sets <3.5	85.80%
medium discharge	mean	3.15
	COV	2.49
	sets >3.5	36.7
	sets <3.5	63.3
large discharge	mean	4.02
	COV	3.67
	sets >3.5	55.9
	sets <3.5	44.1

Discharge per Pier:

small discharge	mean	1.81
	COV	6.85
	sets >3.5	18.30%
	sets <3.5	81.70%
medium discharge	mean	2.93
	COV	2.34
	sets >3.5	28.3
	sets <3.5	71.7
large discharge	mean	4.08
	COV	3.17
	sets >3.5	60.2
	sets <3.5	39.8

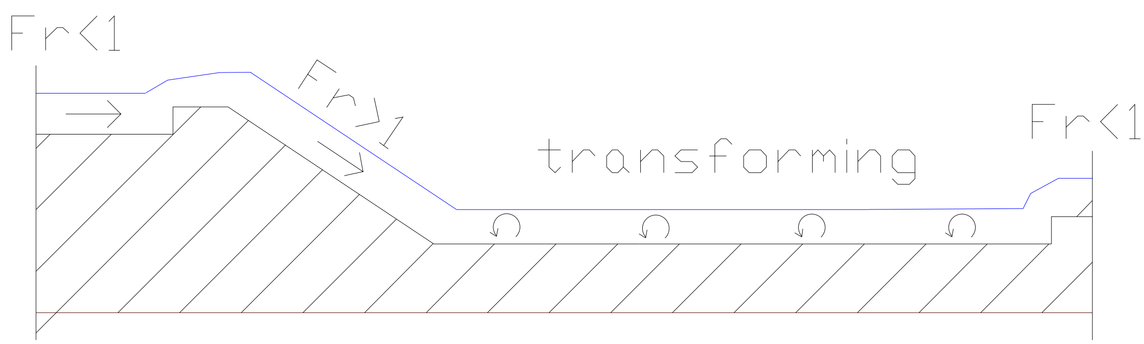
The change in average reliability is more significant for the discharge per flow width. For the COV no certain rule can be observed. Looking at the percentage of sets in each group which exceed a reliability index of 3.5 it can be found that the difference between group one and two (small and medium discharge) is bigger for a discharge per flow width while for the difference between group two and three (medium and large discharge) the opposite can be found. Looking at both plots the behaviour of the general reliability as a function of the discharge rate can be found to be quite similar. With an increasing discharge rate the reliability seems to get close to the desired value of 3.5 than it is for a small discharge rate. This counts no matter in which way the discharge rate is been calculated and no matter whether the pier width is included or not.

Quite an interesting notice can be taken when comparing reliability plotted over the flow velocity and a second time reliability over the **Froude number**. Although the flow velocity can be found in the enumerator of the latter, the behaviour of β with increasing Froude number is actually different to the one from β wich increasing flow velocity.
I divided the data sets into three parts:

1. $0 \le Fr < 0.25$
2. $0.25 \le Fr < 0.5$
3. $0.5 \le Fr < 1.0$

Due to this ranges the observation numbers are not equally spaced but consist 187, 148 and 23 data sets. While in the beginning the reliability indices get bigger with increasing Froude numbers when getting closer to the critical Froude number of 1.0 β starts decreasing extremely. The mean values of β are 2.89, 3.47 and -0.18 for groups 1 to 3 respectively. Unlike this the variation is increasing continuously with COVs of 2.77, 5.57 and 7.84. As one could expect the percentages of data sets which have a reliability index greater than 3.5 behaves analogically to the mean β. Speaking in numbers a percentage of 32.6%, 43.2% and 8.7% can be observed. The significant difference between flow velocity and Froude number and especially its origin is of outstanding interest.

Trying to explain on possible reason I will give some more information on the Froude number:
When looking at a dam Fr would usually be subcritical before and after the dam but supercritical when accelerating while falling down the dam.



Exceeding the critical Froude number Fr=1 is quite uninterrupted but once the flow has a negative acceleration and the Froude number falls below the critical value intense vortices occur which again cause a lot of erosion. For this reason designer set huge rocks or concrete blocks right underneath the dam to make sure the transformation happens where they want it to prevent scour of the stream bed. Additionally some people use concrete for the whole basin next to the dam as it will sustain the erosion.
Knowing this one has to consider an extreme flood event. The discharge will not be constant any more but increase. Also the flow velocity and the depth will increase. But as the flow

depth is in the denominator to the 0.5th power and also the flow velocity will raise more intense the Froude number is going to transform to the supercritical area at one point. After a certain period of time this process will go back analogically to the procedure described for the dam and therefore will cause intense erosion.

The measurements of flow velocity and depth given in the Landers database are collected while no flood event was present. As some data sets are already pretty close to the critical Froude number under normal conditions they tend to get to a supercritical level more likely than those which have lower Froude numbers under normal conditions. Again this could cause a lower reliability index.

The above said proofs the necessity to account for the subcritical or supercritical flow behaviour of a river and does not support the opinion of some authors who claim the Froude number to be of identical explanatory power as the flow velocity. Both are important hydrodynamic parameters and have to be kept under consideration.

The **flow intensity** is an important parameter and equals the ratio of flow velocity to critical flow velocity. The latter is the velocity at which an incipient motion of the average sized soil particle can be observed. Several equations have been developed to determine the critical velocity. I am going to use the one used in the Simplified Chinese Equation:

$$V_c = (\frac{y_0}{D_m})^{0.14} * (17.6 * (\frac{\rho_s - \rho}{\rho}) * D_m + 6.05E(-7) * (\frac{10 + y_0}{D_m{}^{0.72}}))^{0.5}$$

Where:

- y_0 = flow depth,

- D_m = average particle size,
- ρ = density of water,
- ρ_s = density of soil,

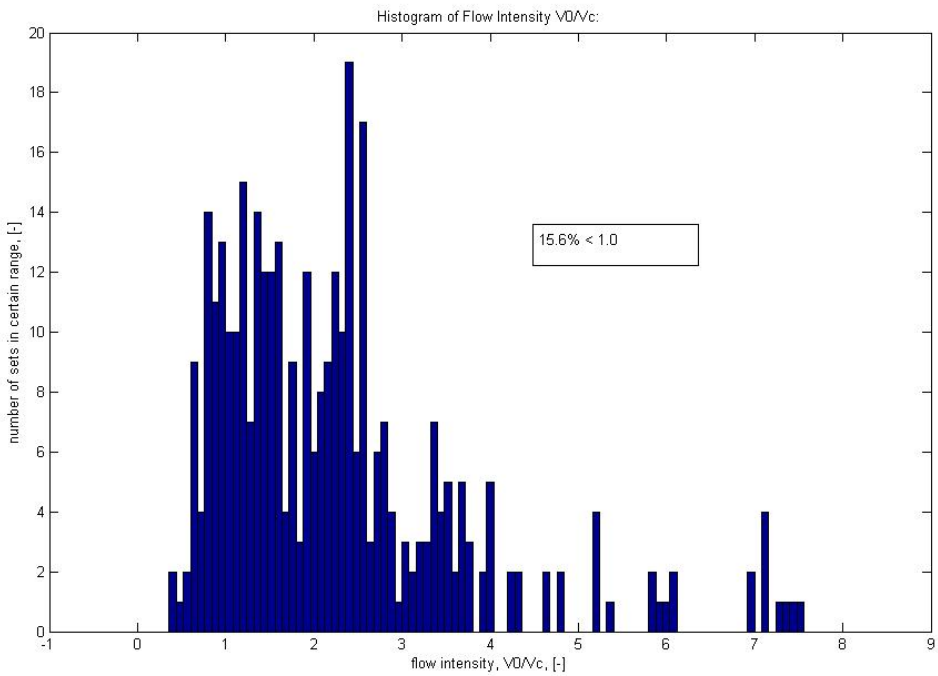
Knowing the flow velocity at which the average sized soil particle starts floating one can calculate the flow intensity defined as the actual flow velocity over the critical flow velocity.

$$Int = \frac{V_0}{V_c}$$

In order to calculate the flow intensity several approaches have been provided especially to account for the difference between V_c in the open river compared to the corresponding V_c in the constricted reach of the pier site. The Simplified Chinese Equation uses another definition to account for the flow intensity which includes the "…approach velocity corresponding to the critical velocity and incipient scour in the accelerated flow region at the pier" V_c'. I am going to use the V_c for the open river conditions to find out about general relations and dependencies.

A general subdivision has to be taken to consider whether clear-water or live-bed conditions are present. The necessity for this subdivision is due to the fact that for clear-water conditions, there is no soil particle "input" into the scour hole. In other words the soil volume taken out of the scour hole equals the volume of the future hole. It is not being filled up afterwards either. Whereas for live-bed conditions soil is being transported out of the scour hole and into the scour hole at the same time. Therefore the volume of the future scour hole equals the volume of soil taken out minus the volume of soil put in the hole. Also, once the flood event is over the hole will be filled up to a certain level. The limit between clear-water and live-bed conditions is symbolized by the vertical straight even in the plot below whereas the horizontal even equals a desired reliability index of 3.5.

The following plot illustrates the distribution of the entirety of calculated flow intensities. Note that a flow intensity equal to 1.0 is the limit state between clear-water and live-bed conditions.



As for most of the sites live-bed conditions were found to be present the explanatory power of the 56 data sets which represent only 15.6% of the whole database is questionable.

Despite this I want to mention the differences in the averages of reliability indices which equals 3.15 for live-bed conditions but only 1.76 for clear-water conditions. This indicates that the HEC Equation is predestined to be used with live-bed conditions although it tries to account for channel bed conditions by the different values of factor K_3.

I break down those β values for live-bed conditions one more time.

- $1 \leq Int < 3.5$

- $3.5 \leq Int$

The average reliability increases while the variation decreases. The averages and COVs are respectively:

Averages:

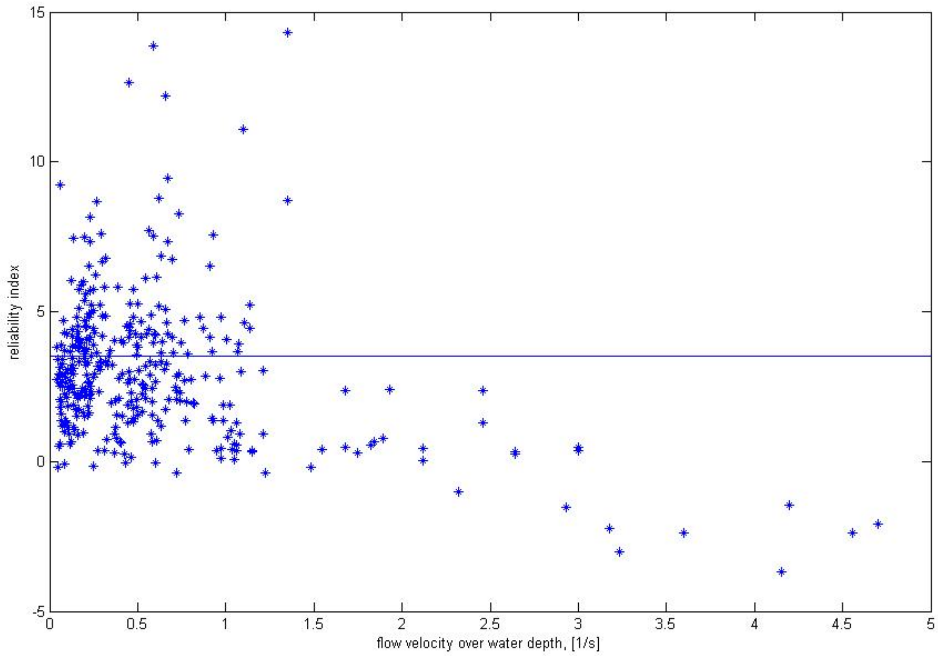
- 3.08
- 3.49

Coefficients of variation:

- 5.71
- 2.89

The result which one can interpret due to these observations is that for a ratio of $\frac{V_0}{V_c}$ >3.5 the average reliability is pretty close to the desired 3.5 and although the COV is still quite high, it is way less the one for lower values of the flow intensity. Still, one has to keep in mind that for the observed 50 sets where the flow intensity exceeded 3.5 still 20% of sets had an reliability index βless than 2.0 which meaning is a probability of failure of approximately 1:80.

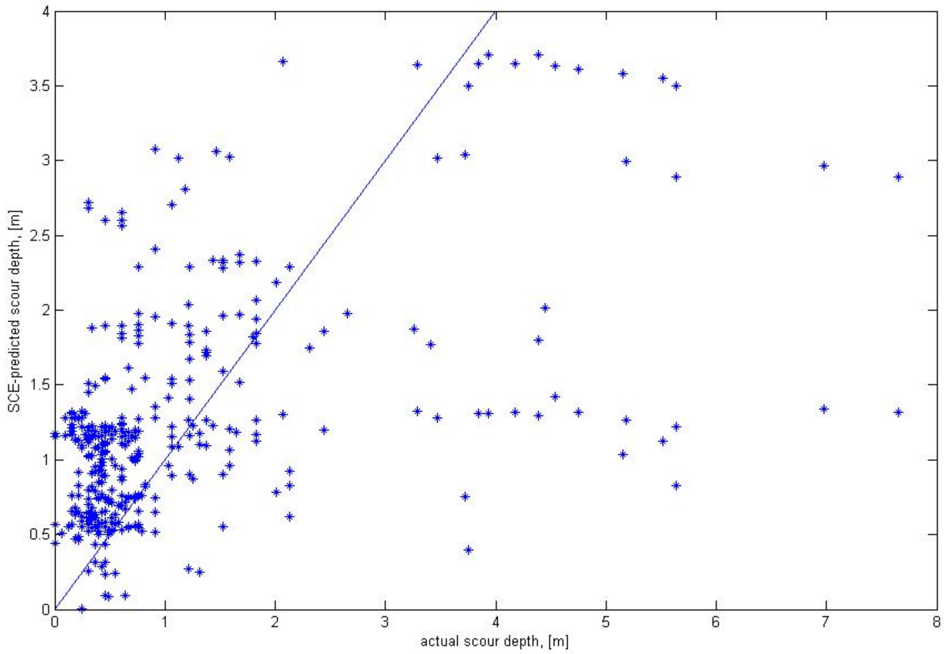
As the flow depth inherent in the Froude number's denominator has the power of 0.5, its weight in the resulting value is reduced. When taking the ratio of **flow velocity over water depth**, this reduction is taken away and the trend is more obvious as can be seen in the plot below. Although the vast majority of measurements provide a ratio of less or equal 1.0, a trend can be found: As the above mentioned ratio increases, the average reliability index decreases. Although only 26 sets which represent 7% of the whole database have a $\frac{V_0}{y_0}$ ratio greater 1.5 one can observe the reliability indices to decrease abruptly for this range. For a ratio less than 1.5 the average β equals 3.2 with a COV of 4.05 whereas for those sets with a ratio greater than 1.5 the average β beta is even negative and equals -0.51 with a COV of 3.81.



Simplified Chinese Equation Analysis

When attempting to interpret the scour depths predicted by the Simplified Chinese Equation one has to know that the purpose of this equation is not to be used directly as a design equation but to predict the expected scour depth pretty accurate. In other words the result obtained by the SC-Equation is not supposed to be conservative.

Plotting the SCE-predicted scour depth over the actual measured scour depth delivers some facts of interest. Note that the straight even represents an optimal prediction where predicted scour depth equals measured scour depth.



Like the HEC-equation, the uncertainties increase for an actual scour depth greater than 3.5m (11.5ft). Unlike the HEC-equation underestimations can already be observed for actual scour depth less than 3.5m, too. As already mentioned this is due to the fact that the Simplified Chinese Equation lacks of a safety factor as the HEC-equation has. The SCE tends to overestimates though.

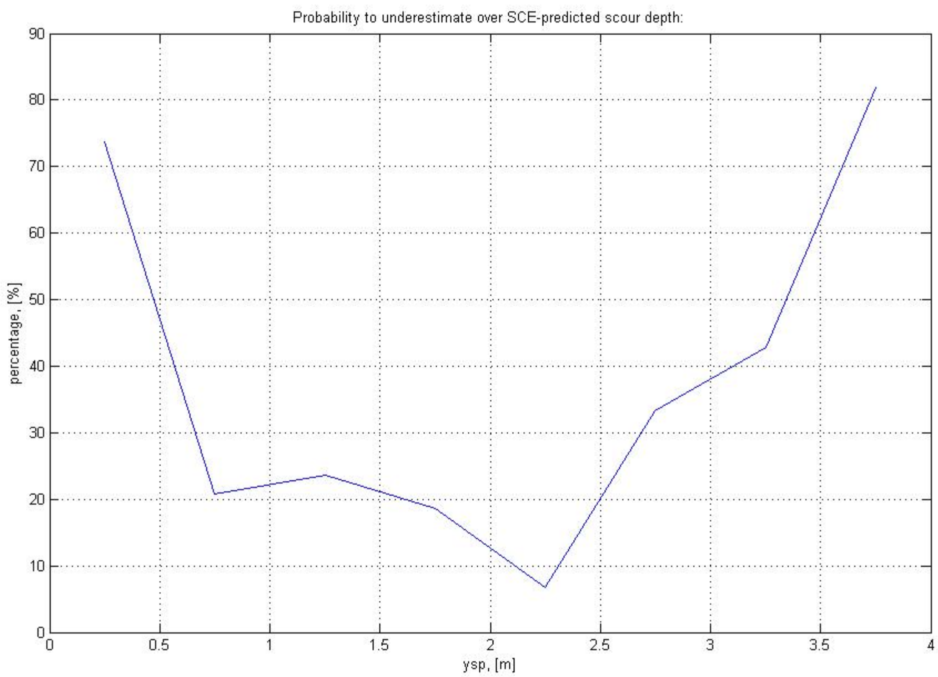
Two ranges of predicted scour depth can be found for which the SCE varies quite a lot compared to the actual scour depth:

$$\approx 1.3m$$

$$\approx 3.6m$$

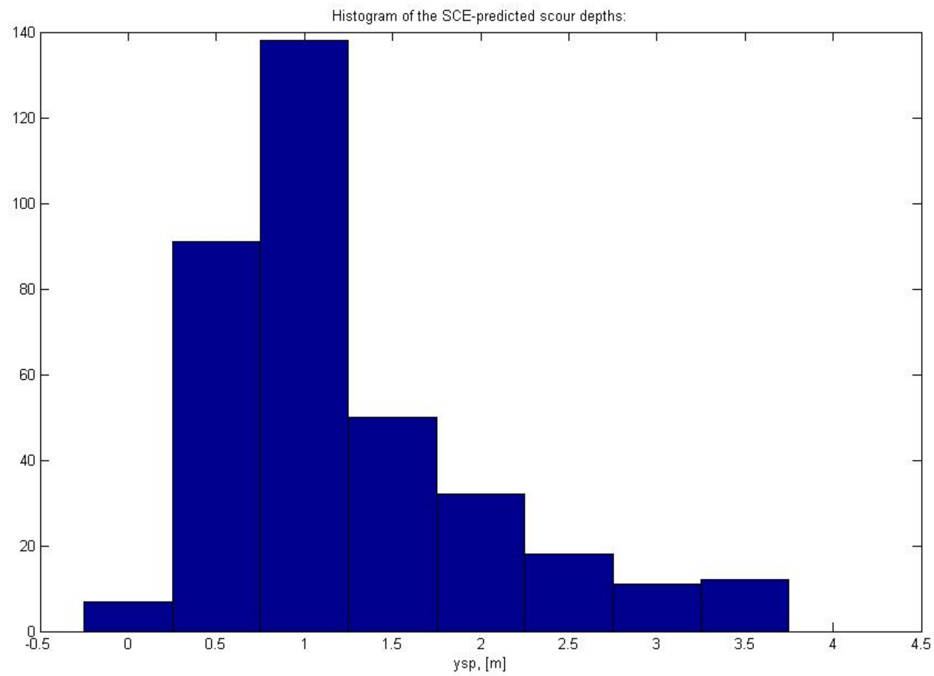
When comparing the SCE results to the measured scour depth one finds underestimations of 10 to 15 times which is quite enormous.

These similarities with the results of the regression analysis are quite interesting as one could presume those to be of the same origin. I will focus on that later on and try to find reasonable explanations. Assuming that the used database it statistically significant one can approach a prediction to find out about the probability of underestimation.



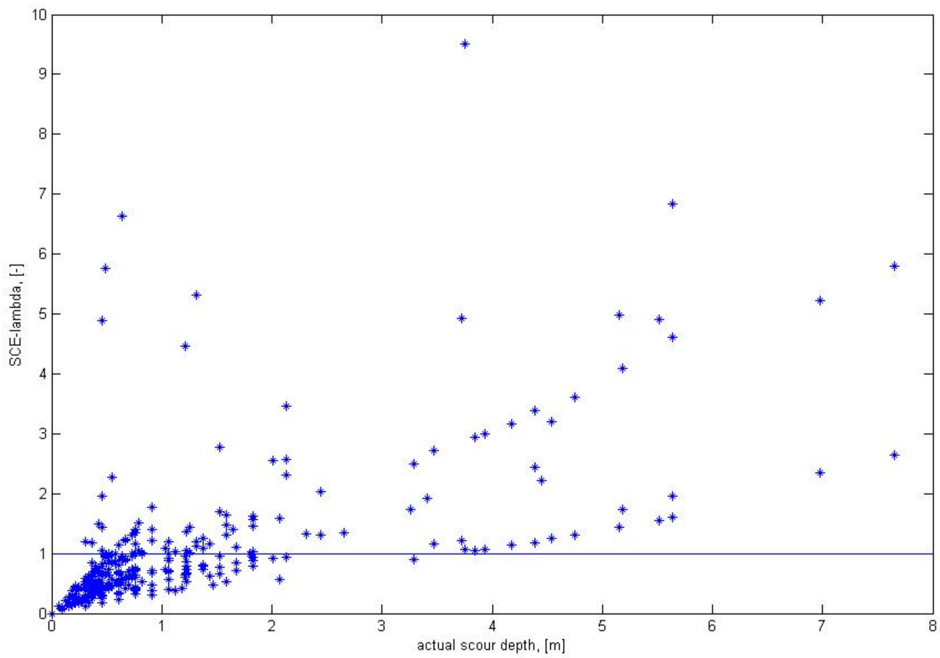
As can be seen with increasing predicted scour depth the probability to underestimate first increases until y_{sp} equals approximately 2.25m (7.4ft) after which it goes up again. The minimum probability is 6.7% while the greatest slightly exceeds 80%.

To find out how useful this plot actually is one needs to know about the distribution of the SCE-predicted depths. I will plot the histogram in steps of 0.5m.



For predictions less 0.5m (1.64ft) and greater than 2m (6.5ft) the number of sets in this range falls below 20 thus the statistically significance can be questioned out of those limits. Within these limits the found percentages might be a rough support to interpret the predicted scour depths and its accuracy.

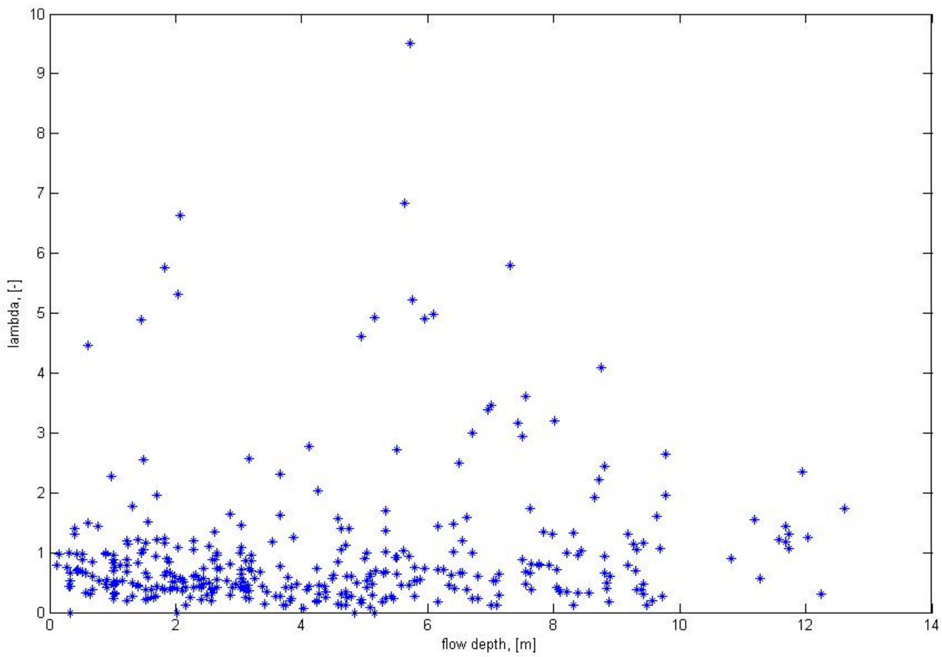
Another way to judge the over- and underestimation is to plot lambda over the measured scour depths. Remember that $\lambda = \frac{s}{y_{sp}}$ where s equals the actual scour depth and y_{sp} is the SCE-predicted depth. The even shows the location of the optimal prediction meaning that the predicted scour depth equals the actual scour depth.



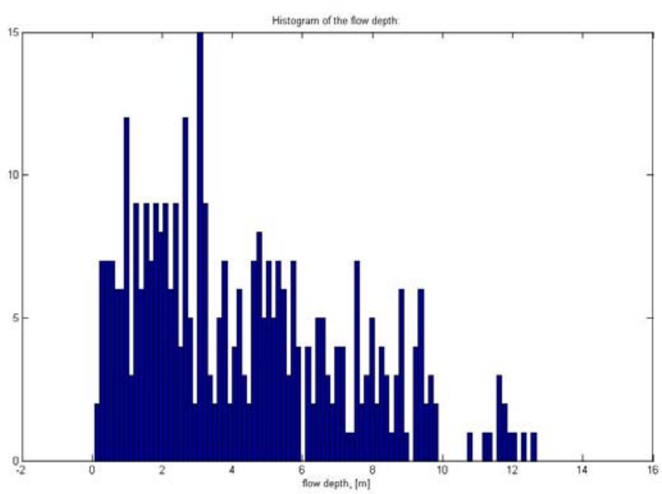
The scattering appears somewhat similar to a shaft of light starting in the axes' origin. In other words with increasing actual scour depth both the mean lambda as well as the COV of the lambdas increase. This could be expected as the actual scour depth s happens to be in the enumerator of l and for this reason an increasing s causes an increasing l, too. The accession in scattering can be explained due to the fact that with an increasing s a deflection of a certain amount between s and the SCE-predicted scour depth y_{sp} causes a larger difference for the l-value.

For actual scour depths less than 1m the mean l equals 0.76 with a coefficient of variation of 593%. Ranging from 1m to 3m of measured scour depth the mean increases to 1.19 varying by a COV of only 70%. The average lambda for all sets with an actual scour depth exceeding 3m equals 2.85, the COV goes up to a value of 360%.

Although some dependencies can be found the **flow depth** directly upstream of the bridge pier seems to have a rather small influence on l. While the average lambdas take values between 0 and 1.5 an increasing ratio of actual over predicted scour depth can be obtained for flow depths between the limits of approximately 4m (13.1ft) and 10m (32.8ft).

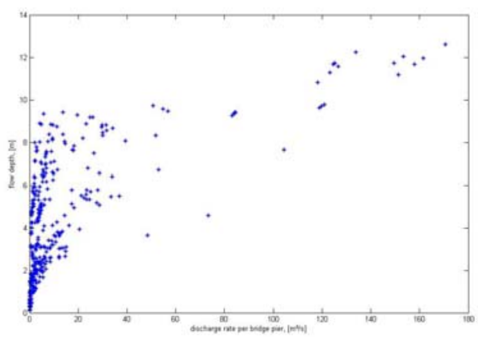


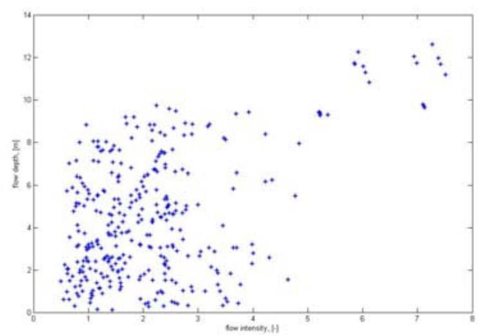
The majority of data sets happen to show a flow depth between 0m and 4m but there are still quite a few sets with y_0 increasing 4m. See the histogram of the flow depth on the right side.



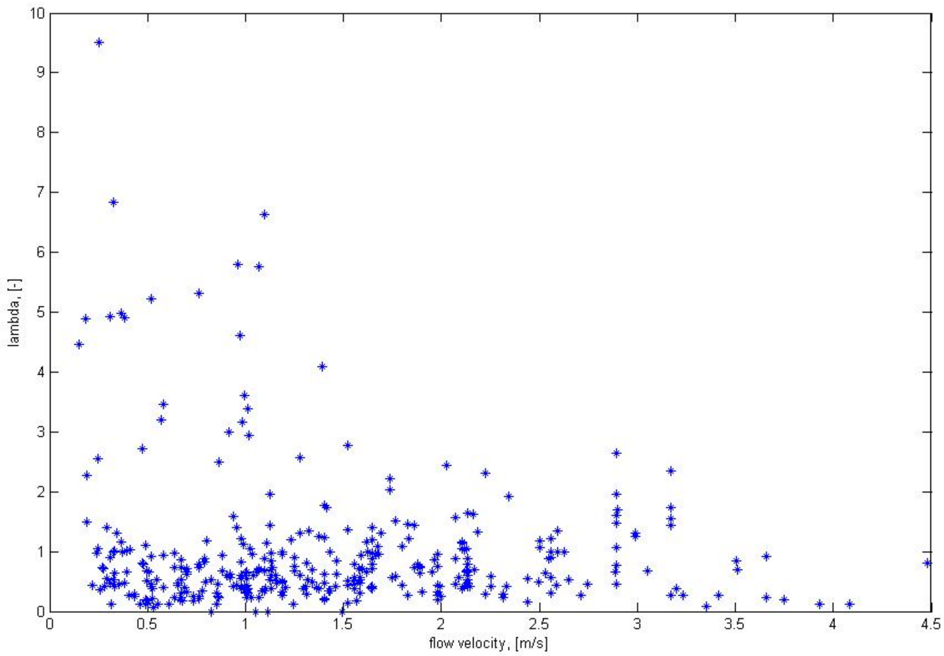
The reason for the increase of lambda values for a range of flow depth of 4m to 10m could be that starting in this range both, the discharge rate per bridge pier as well as the flow intensity start to increase. The SCE seems to be more accurate once the incipient increase of flow intensity is completed.

The plots below show the flow depth over the discharge rate per bridge pier (left) and the flow intensity (right) respectively.





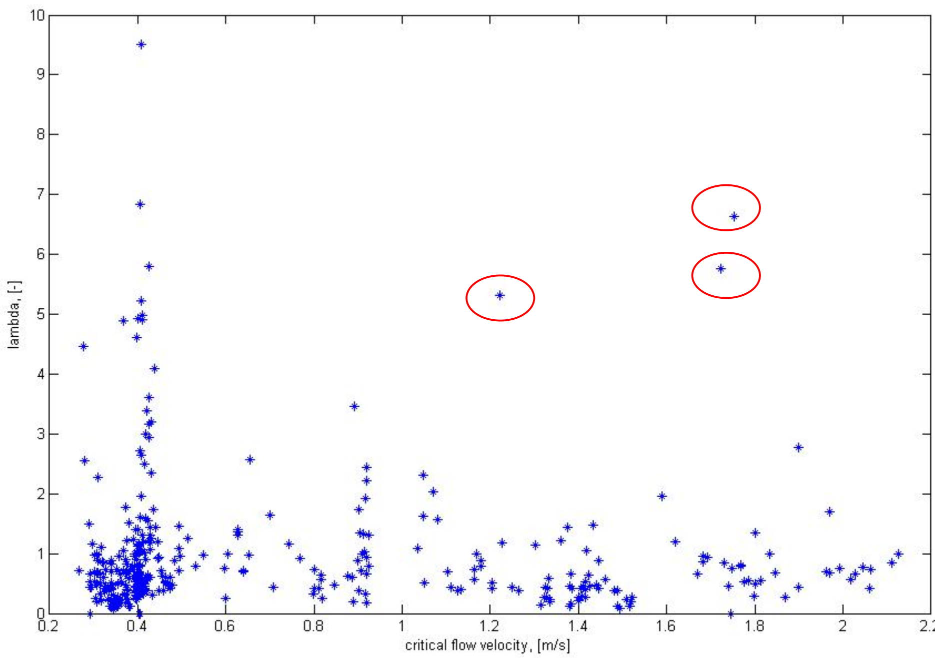
Unlike the flow depth the lambda values occur to be much more linked to the **flow velocity** V_0. For $V_0 < 1.5\frac{m}{s}$ the mean lambda equals 1.24 with a huge COV of 815%. Those represent about 60% of all data sets. The lambdas for flow velocities that exceed $1.5\frac{m}{s}$ the average lambda equals 0.84 while the COV cuts down to 30%.



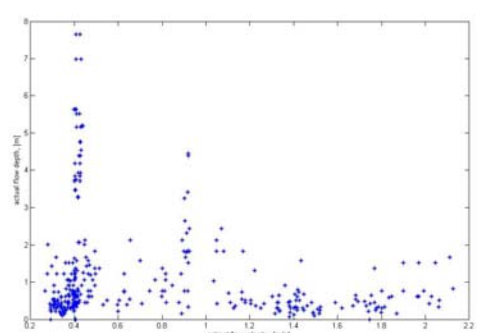
The plot above illustrates that the scour causing potential of slow flow velocities is being underestimated. A conclusion related to that is that apparently the SCE assumes a distinct dependency between scour depth and flow velocity which does not exist to that extent. Although the flow velocity influences the predicted scour depth in different ways in the HEC and SCE equation the same underestimation can be found using the HEC equation. For the HEC equation the flow velocity appears in the enumerator of the Froude number which itself is taken to the 0.43th power. Looking at the SCE-equation the flow velocity is inherent in the flow intensity term:

$$\frac{V_0 - V_c'}{V_c - V_c'}$$

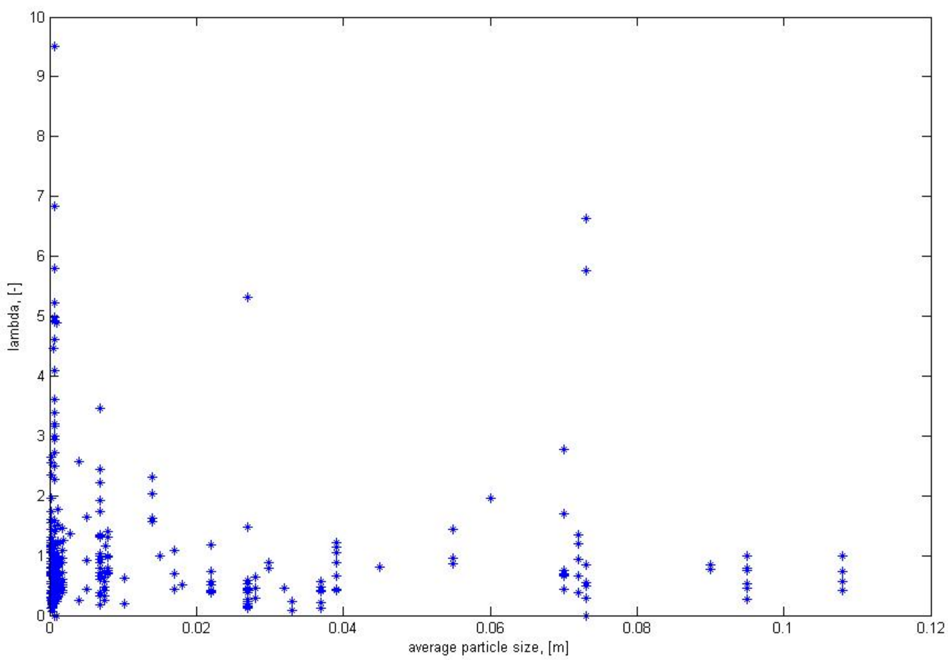
Neglecting the three marked outliers with increasing **critical flow velocity** V_c lambda decreases continuously.

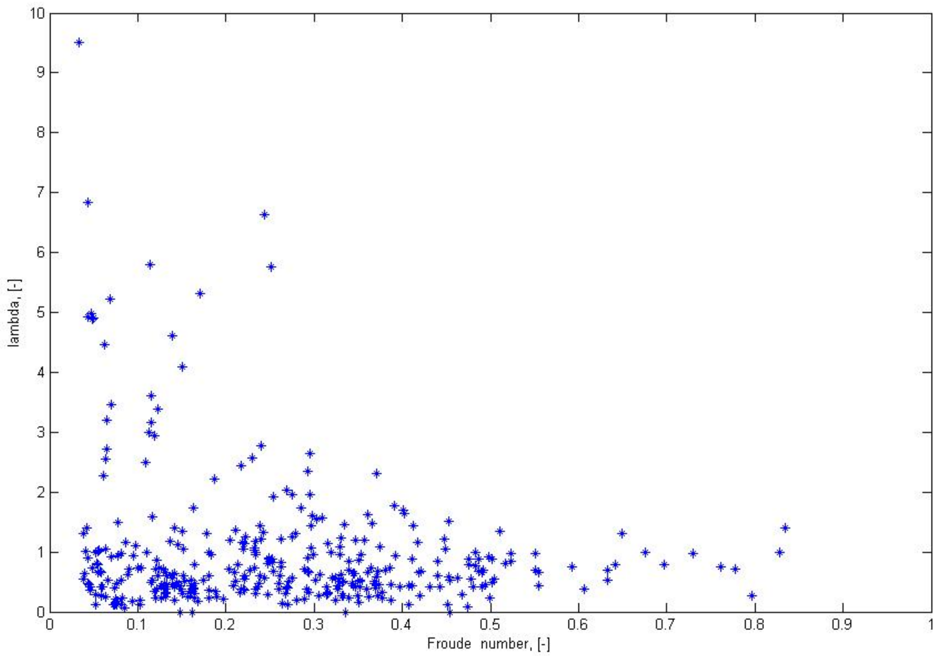


Comparing the plot above to the one on the right which shows actual scour depth over critical flow velocity reveals that there is a pronounced dependency between critical flow velocity and scour depth which the SCE does not account for to a pertinent extent yet.



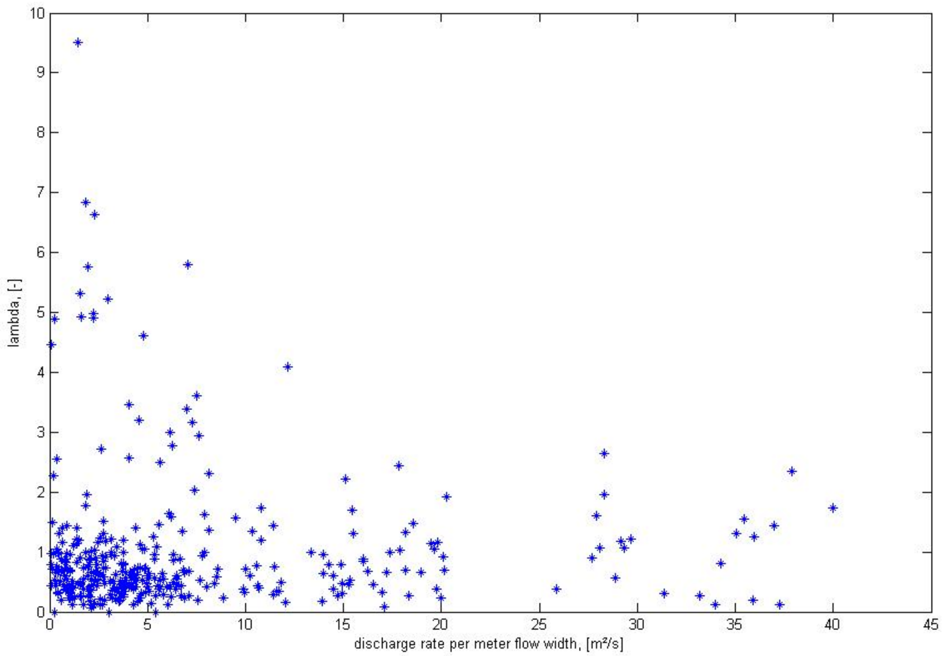
The vast majority (75%) of average particle sizes happens to be smaller than 0.02m (0.8inch) which leads to a limited statistical significance of the remaining data sets. Lambdas seem to take greater values for small D_m and for those around 0.075m (3inches). As mentioned before the explanatory power of this observation is questionable.





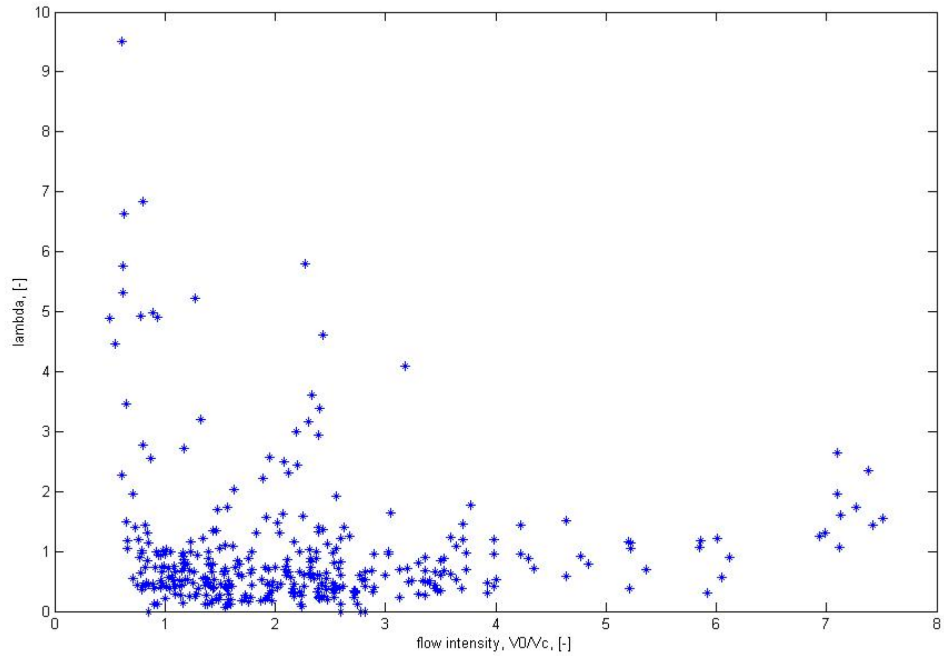
The plot above shows l over the **Froude number** Fr. As could be expected the plot appears somewhat similar to l over V_0 as the latter happens to be in the enumerator of the Froude number. Unlike the plot showing the flow velocity on the abscissa, the graphic including the Froude number shows lambda values close to 1.0 for Fr>0.5. Note that for fast flow velocities l appears somewhat asymptotically to 0.

Besides this difference it can be found that for increasing flow velocities as well as for increasing Froude numbers the lambda values decrease and generally speaking the SCE results become more conservative.



To find out about the links between lambda and the **discharge rate** Q I will use the discharge rate per meter flow width. It equals flow velocity times flow depth. The flow depth does have a less distinct influence on l thus the plot shown above is quite similar to lambda over flow velocity. The maximum scour occurs when the discharge is maximal while this happens during flood events. Assuming that the measurements on which the data is based were taken under ordinary conditions the calculated Q would not be the discharge rates which caused the measured scour depths. Again for live-bed scour this assumes that the scour hole is not filled up again yet.

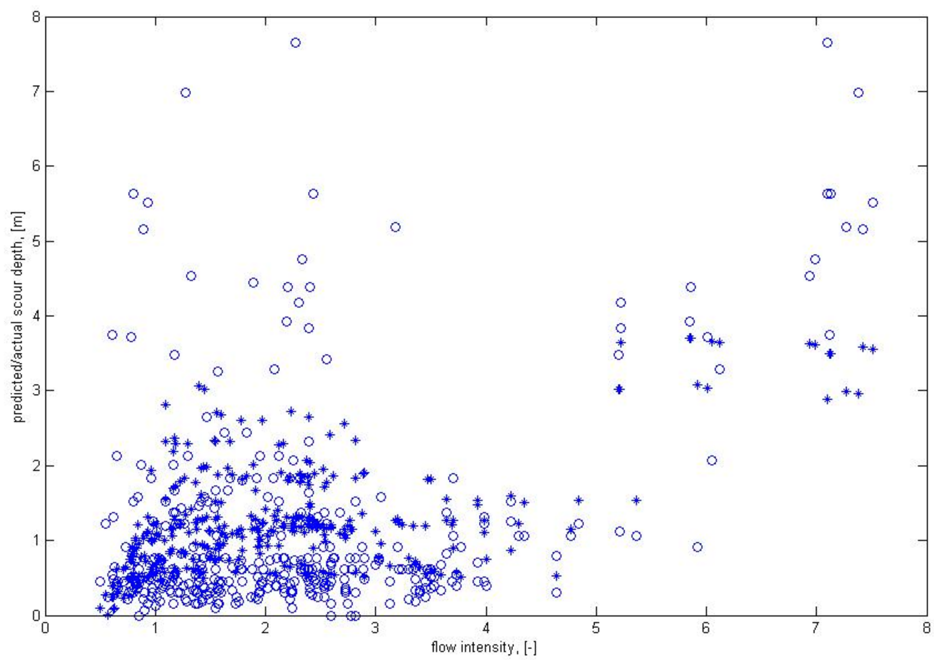
Knowing that the **flow intensity** equals $\frac{V_0}{V_c}$ the plot below can be interpreted. It is quite similar to both plots: l over V_0 and l over V_c. This indicates that the influence of V_0 is more intense than the one from V_c.



For a ratio of V_0 to V_c greater than 3.5 the lambda values level off where nearly all values are less than 2.0. Dividing the plot above into three ranges delivers a trend regarding to average values and coefficients of variation:

- Int<2.0: mean: 1.24
  COV: 913%
- $2.0 \leq$ Int<4.0 mean: 0.85
  COV: 77%
- $4.0 \leq$ Int mean: 1.14
  COV: 30%

To figure out whether the flow intensity is being accounted for in a pertinent way one shall look at the plot below which shows predicted (*) and actual (o) scour depth over flow intensity.

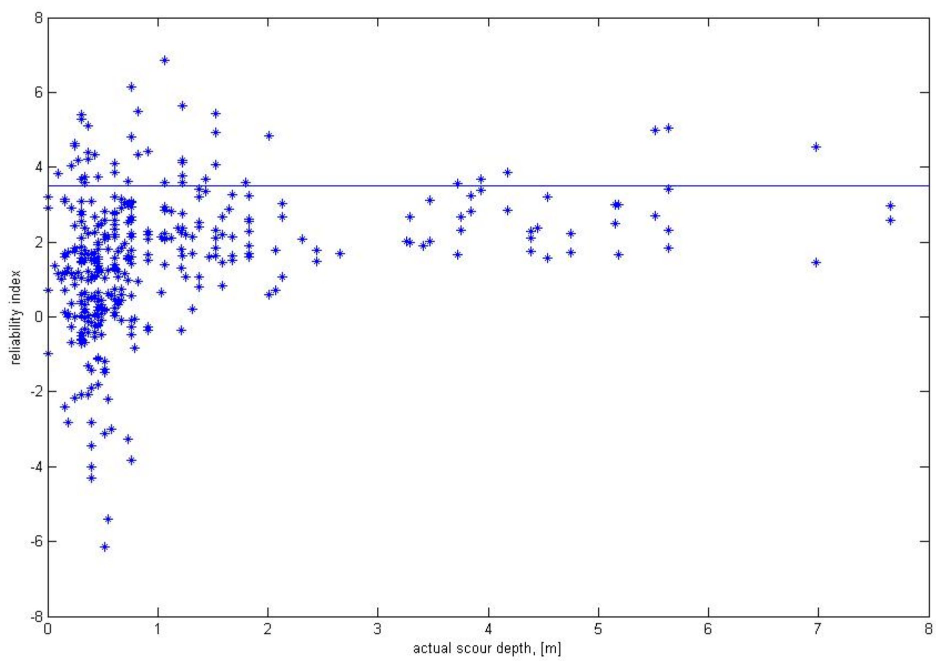


The trend appears to be quite similar: Most of the values are in a flow intensity range less than 3.5 (86%) and a scour depth less than 2.5m (92% of y_{sp}, 89% of s). For flow intensities greater than 5 the scour depth increases for both, the predicted as well as the actual scour depth.

Although the mentioned similarities can be observed there are still recognizable differences. 32 values (representing 9% of the whole database) of the actual scour depth with a flow intensity less than 3.5 have greater actual scour depth than the maximal predicted scour depth. The latter equals 3.71m while the maximal actual scour depth equals 7.65m. The same trend can be observed for flow intensities ≥ 3.5.

Besides these huge differences which can be found, for a small range of 3.5<Int<5 the predicted and actual scour depths do not vary that much. Looking at the predicted (actual) scour depth a mean of 0.97 (0.81) with a COV of 15% (18%) can be observed. Those values represent 8.1% of the whole amount of available data sets.

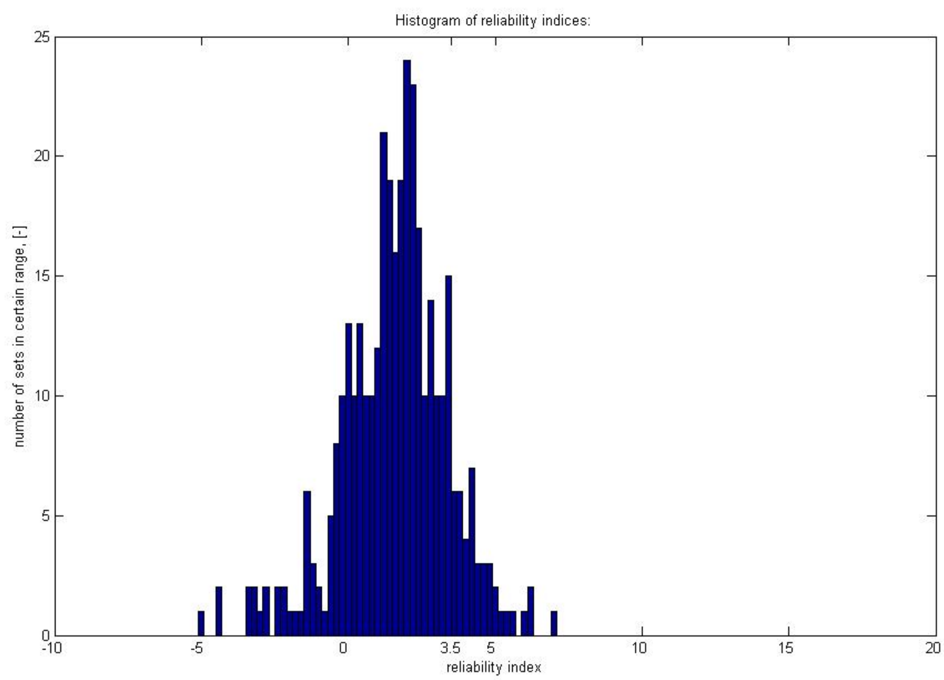
The reliability index is plotted over the actual scour depth while the straight even illustrates the location of the desired reliability index of 3.5.



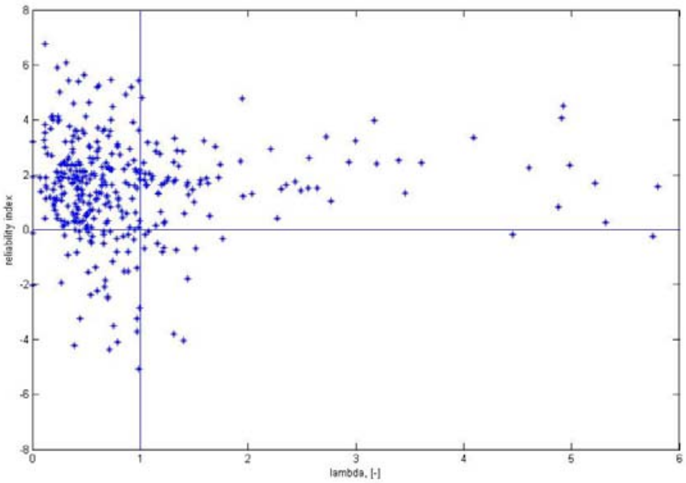
Running the reliability analysis 1,000 times the mean reliability index β equals 1.53 and produces a coefficient of variation of 337%. Taking a look at the target reliability index of 3.5 an average of 37.9 data sets (10.6%) have a β actually greater than that. In other words 10.6% of the SCE-predicted scour depths satisfy the claim to have a probability of failure less than $\approx$1:1,050.

The shown plot itself looks similar to the graphic which shows β over actual scour depth for the HEC-equation.

The distribution of reliability indices can be visualized with the help of the plot below.

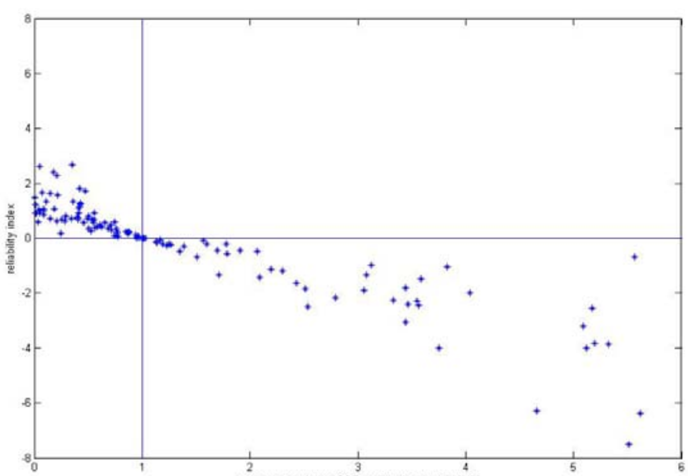


One simple way to check the results is two plot β over **lambda** which reveals the same deficiencies as could be found analyzing the HEC-equation. The regression equation seems to include some uncertainties.



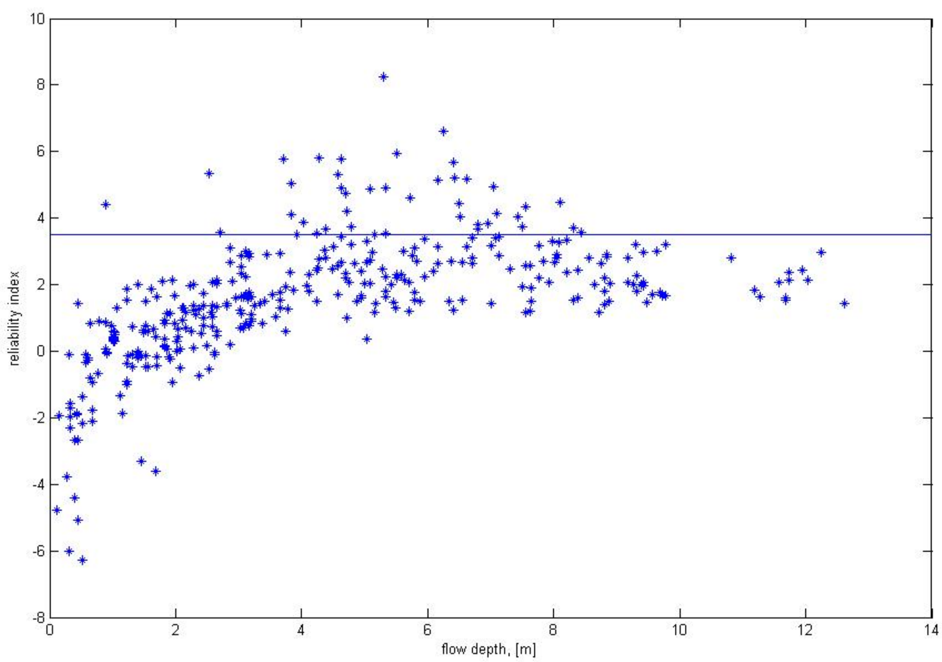
For l>1.0 the reliability indices should be all in the negative range which most of them are not.

Again replacing lambda by the quotient of regression result over SCE-predicted scour depth obtains a plot that appears more logically.



Although this new plot appears somehow theoretically correct it proofs that the regression results and the actual scour depths still have discrepancies. If they would be identical the plots of beta over lambda and beta over the quotient of regression result and SCE-prediction should be the same. To find a regression equation which fits accurately is hard and I will focus on that later on. Theoretically the other way to get rid of the uncertainties described above would be to use the actual measured scour depth for the determination of the expected scour depth μ_Q as well instead of using a regression equation. The reason why this is not recommendable at all is because one should treat μ_Q as a random variable to account for its nature as a random distributed variable. Therefore if one does not want to accept an uncertainty in the result the only way is to find another improved regression equation. Nevertheless I will use this one first regression equation for all of the data sets for the further calculations and come up with an improvement equation later on. Note that one will have to accept a certain amount of inaccuracy inherent in the regression equation no matter which one is used. The less these uncertainties are the better the results will be and the more realistic the calculated reliabilities are.

For shallow rivers a limited reliability can be found. With very small β values going down to -6 the SCE-results become more reliable with increasing **river depth** where the increase of beta is somewhat linear until a flow depth of approximately 4.0m (13.1ft) is reached. From that point on the reliability indices level off but still quite a huge variation can be found for $4.0m \leq y_0 \leq 8m$.



Dividing the flow depth in three ranges the following data can be found:

- $y_0 < 4.0m$: mean β 0.54

  COV of β 316%

- $4.0m \leq y_0 < 8.0m$: mean β 2.94

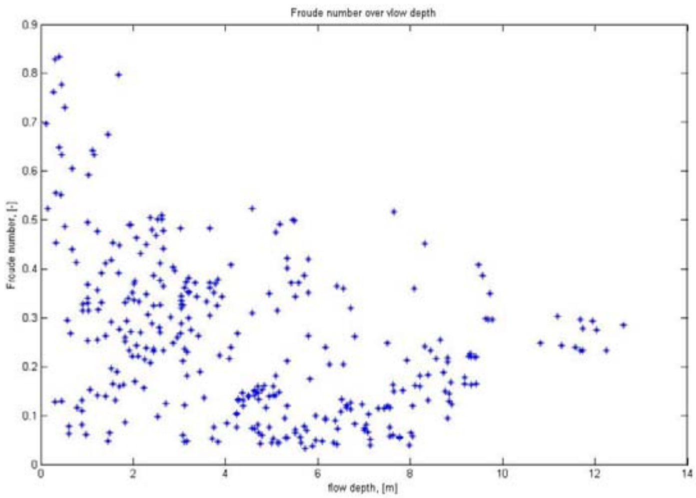
  COV of β 177%

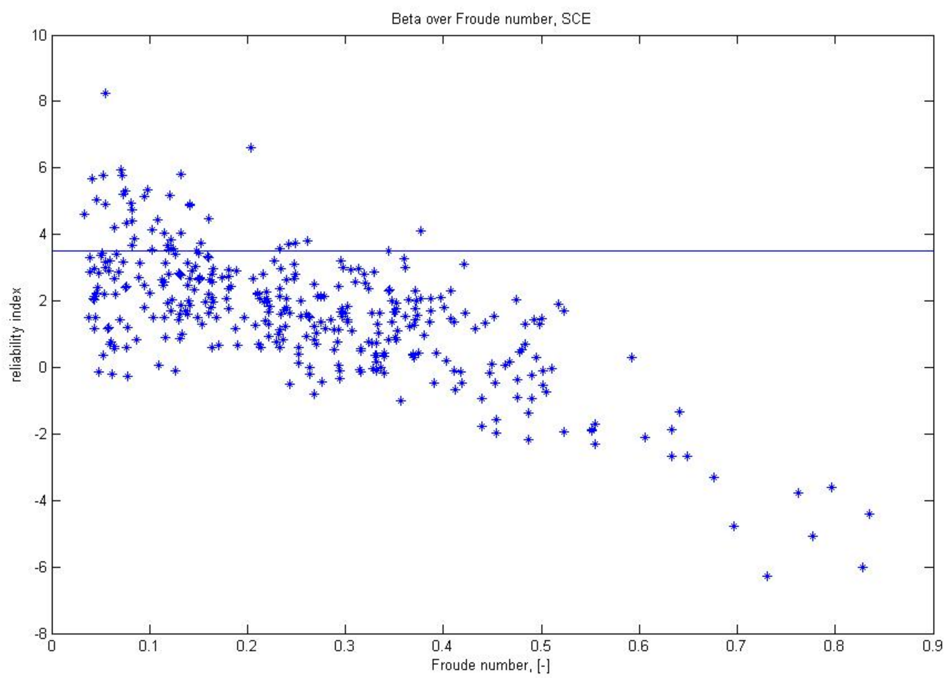
- $y_0 \geq 8.0m$: mean β 2.32

  COV of β 50%

As the flow depth is part of the Froude number looking at the plot above which reveals a distinct dependency between the reliability index β and the flow depth y_0 one expects a relation between the Froude number and the reliability against failure due to scour as well.

The following plot shows the **Froude number** over the river depth to proof and illustrate the relation between Fr and y_0 while the plot below that shows β over Fr. As usual the even symbolizes the location of a desired β=3.5.

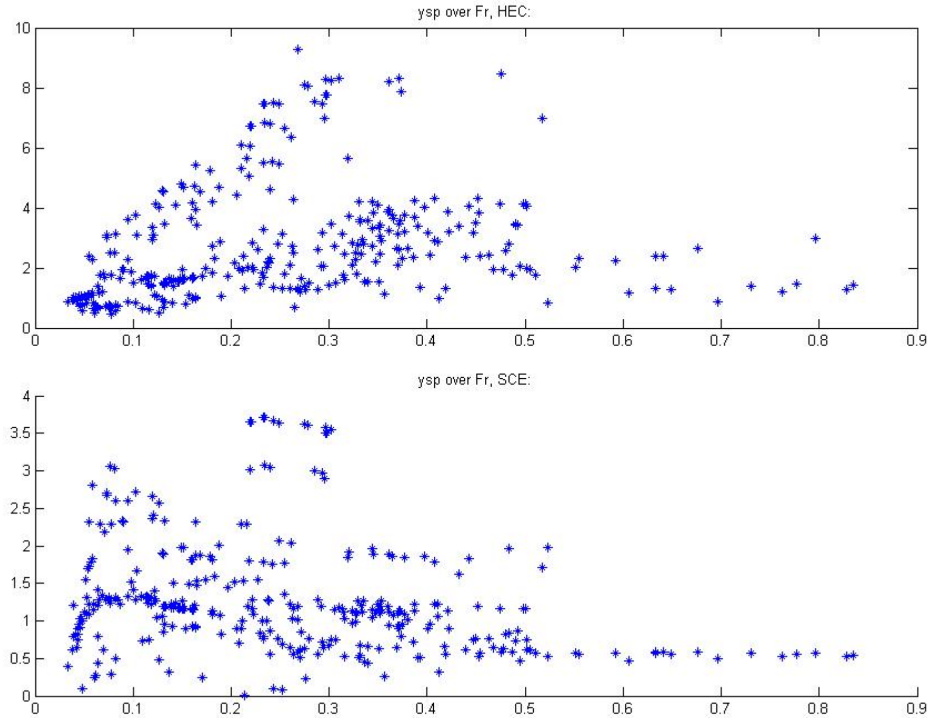




Looking at the lower plot shown above one can find an important difference between the results obtained by the HEC equation and those from the SCE equation. While one could find rather less clear dependencies for the HEC results the ones that can be find at this point are more helpful. While for small Froude numbers the variation of reliability indices still is quite huge the COV becomes less with increasing Froude number. As can be seen the average β clearly decreases once the Froude number gets closer to 1.0. Although less obvious the latter trend could also be found for the HEC equation and the explanation given at the comments in the HEC chapter of this thesis are very likely to be the reason for the decrease found in the plot above, too: For a river with Fr close to a critical value of 1.0 it is more likely to exceed it when a flood event occurs. Once the flood is gone and the Froude number drops below 1.0, the occurring scour is immense. For further details look at the chapter about the HEC equation and its comments on that topic.

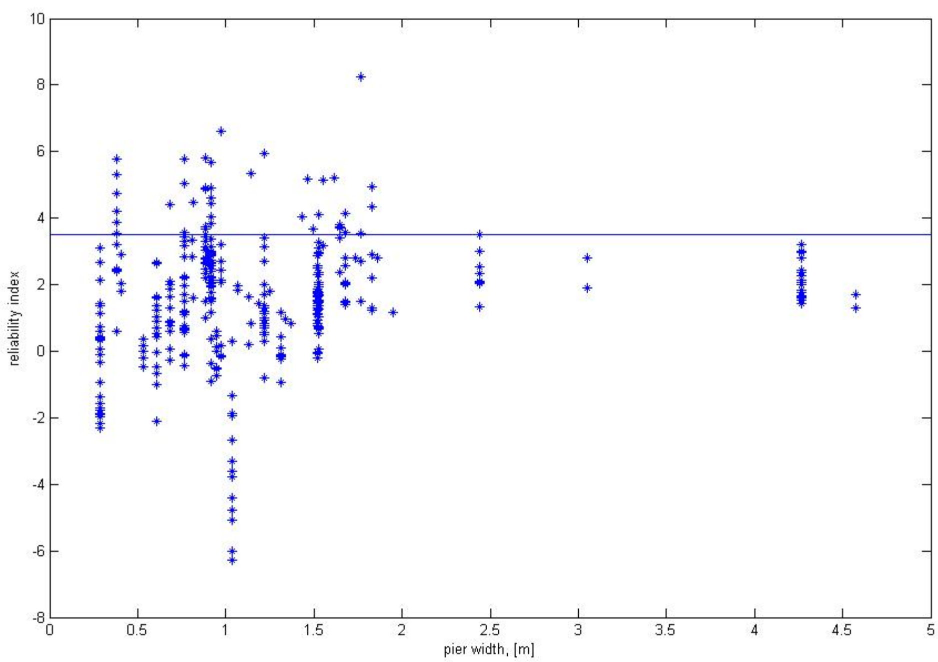
The difference in dependency between reliability indices and Froude numbers when comparing HEC and SCE predictions is due to the fact that the Froude number is already used to predict scour depth in the HEC but not in the SCE equation. Therefore it is quite useful to improve the SCE predictions using Fr while the help it provides for the HEC results is not that reasonable. The Froude number has already been used for the HEC equation and for this reason its predictions do not depend on Fr anymore.

The following plots illustrate the above said. Those show predicted scour depth over Froude number for the HEC (upper plot) and the SCE (lower plot) equation. Although linear dependencies can only be found in certain ranges of Froude numbers it is quite obvious that the HEC predictions depend more on the Froude number than those provided by the SCE equation.

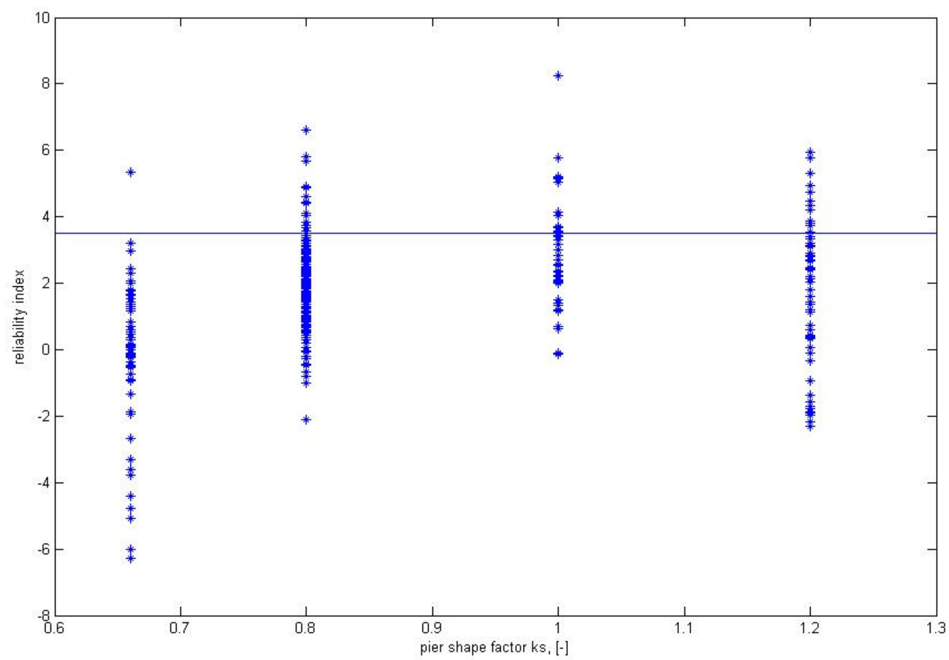


Generally speaking parameters that are already inherent in the SC-equation have a less distinct dependency on the reliability indices β than those which has not been accounted for yet. This statement assumes a pertinent input of the certain parameter in the prediction equation. The above said is true not matter which scour depth prediction equation is used, the equation out of the Hydraulic Engineering Circular or the Simplified Chinese Equation.

Looking at the relations between beta and the **pier width** b shows that for small pier widths the variation of the produced reliability indices is huge while it becomes smaller with increasing b. Note that for a b equal to 1.04m (3.4ft) β drops down to values less reliable than -6. To find out a reason for that I will take wave lengths, wave periods, orbital velocities of the water particle, etc. into consideration later on.



Close to the pier width b is the **correction factor** k_s which accounts for the pier shape. Although it would be inaccurate to get safety factors out of the plot of reliability index over k_s a trend can be found. For small values equal to 0.66 (sharp nosed piers) of the pier shape factor an average β of -0.1 can be found. For k_s=0.8 (round nosed piers) the mean beta equals 1.9, for k_s=1.0 it is 2.8 and for k_s=1.2 the average beta equals 1.7. Therefore one can find the general rule that the greater k_s is, the greater the average reliability index will be. The observations regarding to this safety factor have to be taken carefully though as the values it takes are man-made and not measurements.



The flow intensity can be determined as the flow velocity over the critical velocity:

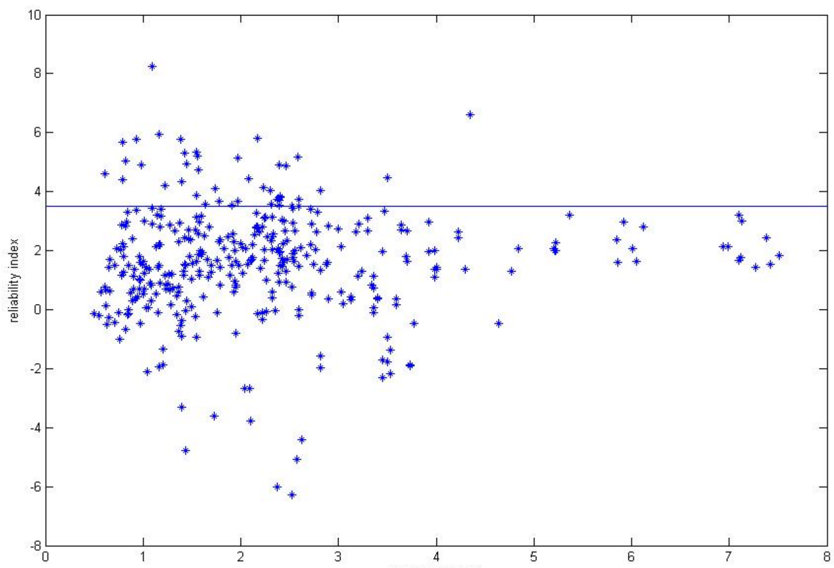
$$Int = \frac{V_0}{V_c}$$

For usual values of Int the reliability indices scatter a lot whereas for greater values of Int the variation is quite small. Unfortunately the number of sets in this range is quite small, too, thus the statistical significance is questionable.

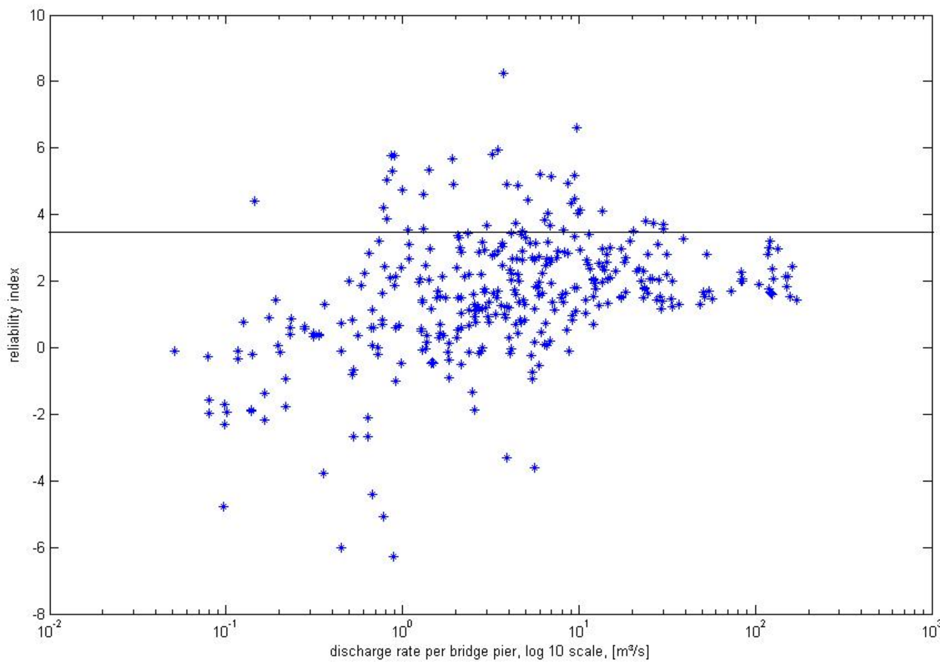
When subdividing the database into two parts the mean betas and pertinent COVs are respectively:

• $Int < 4.0$	mean β	=	1.55
	COV	=	374%
• $4.0 \le Int$	mean β	=	2.08
	COV	=	142%

Note that the first range represents 91.4% of all sets while the second one represents only 8.6%.



Most of the **discharge rates per bridge pier** that could be found take values less or equal to 30[m³/s] (1,060 ft³/s). Plotting the abscissa in a logarithmic scale to the basis 10 while keeping the ordinate in linear scale points out that all the negative reliability indices occur for a discharge rate Q less than 10 m³/s. For greater Q the noted β start levelling off while still having quite a huge variation. The data sets which actually exceed a reliability of 3.5 are located in the medium range of flow intensities in a range between approximately 0.6 m³/s to about 30 m³/s.



Comparison of the two scour prediction models and analysis of the effect of changing land use

This work is in progress.