Report as of FY2006 for 2004AZ68G: "Forward and Inverse Transient Analytic Element Models of Groundwater Flow"

Publications

- Conference Proceedings:
  - Kuhlman, Kristopher L. and Shlomo P. Neuman. Recent advances in Laplace transform analytic element method (LT-AEM) theory and application to transient groundwater flow, In Computation Methods in Water Resources, Volume XVI, 2006.
  - Kuhlman, K.L. and S.P. Neuman, Recent advances in Laplace-transform analytic element method (LT-AEM) theory and application to transient groundwater flow, Proc. XVIth Intern. Conf. Computational Methods in Water Resources XVI (CMWR XV), June 2005, Copenhagen, Denemark, 2006.

Report Follows

Progress Report

Mr. Kris Kuhlman, a doctoral student in the department of Hydrology and Water Resources, has been working as Research Associate on this project. The work described below has been accomplished by him under the supervision of the PI in consultation with our USGS co-PI, Dr. Paul A. Hsieh.

The project focuses on developing and implementing a novel transient extension to the analytic element method (AEM), by posing the transient problem in Laplace space. A general numerical Laplace transform inversion method is used to compute the approximate solution in the time domain to a high degree of accuracy. The resultant Laplace transform AEM (LT-AEM) provides accurate solutions to transient groundwater flow problems which are far more general than most transient analytic solutions (e.g., flow to a well) and far less complex than fully gridded flow simulation codes such as MODFLOW.

To date, Kris derived and implemented active and passive elliptical elements in addition to circular and point LT-AEM elements. Finite line segments are implemented as degenerate ellipses (the line connecting the two foci of the ellipse), allowing much greater flexibility and accuracy than other attempts to implement analytic expressions for transient line sources.

To implement ellipses and line segments in Laplace space, a special algorithm was developed to compute radial and angular Mathieu functions of a complex parameter. The algorithm is general and works in almost all cases of interest.

Kris implemented an improved direct solution method that alleviates common convergence problems arising from the previously used fixed-point iterative scheme. In the case of complex geometries the direct solution method is much faster than the iterative scheme, which is now used primarily as an option to obtain added accuracy.

The LT-AEM is now implemented using modern platform-independent Fortran-2003 programming (utilizing modules, intrinsic vector / matrix operations and overloaded procedures). The program is being made modular to admit a variety of element types and numerical Laplace transform inversion algorithms.

Current work is focusing on adding intersecting elements (e.g., the union of two overlapping circles) and approximate rectangular elements to the list of available elements in LT-AEM. An important role of rectangular and polygonal elements would be to allow coupling our LT-AEM program with grid-based codes to allow (a) an easy treatment of far-field conditions in these codes without explicitly extending the solution mesh and (b) the treatment of domain within which the flow equations are nonlinear. The coupling can be iterative or, in the case of gridded models operating in the Laplace domain, directly (e.g., by incorporating the LT-AEM into the stiffness matrix of a finite element model).

In a separate program Kris is planning to implement some simple three-dimensional LT-AEM elements for which he has already developed the requisite mathematics.

Kris has completed his graduate coursework and plans to complete his doctoral comprehensive exams this Spring Semester 2007.

In June 2006 Kris presented a paper based on his research at the XVI-th International Conference

on Computational Methods in Water Resources (CMWR XVI) in Copenhagen, Denmark. The paper is available on the conference permanent website under the title

Kuhlman, K.L. and S.P. Neuman, Recent advances in Laplace-transform analytic element method (LT-AEM) theory and application to transient groundwater flow, Proc. XVIth Intern. Conf. *Computational Methods in Water Resources XVI* (CMWR XV), June 2005, Copenhagen, Denemark, 2006.

In March 2007 Kris presented a talk based on his research at the annual Dia del Agua workshop of the University of Arizona Department of Hydrology and Water Resources. He received the coveted Earl Montgomery Prize for best presentation at the workshop.

Kris' work will be the focus of an invited paper in the Journal of Engineering Mathematics that we plan to prepare over the summer.