Theory of Aquifer Tests

U.S. GEOLOGICAL SURVEY WATER-SUPPLY PAPER 1536-E

projected net generally will no longer be a system of squares, and the equipotential and stream lines will not intersect at right angles.

For areally nonhomogeneous aquifers—that is, those comprising subareas of homogeneous and isotropic media but of different transmissibility—the flow pattern cannot, according to theory, be represented by a single system of squares. If the flow net were constructed so that each flow path conducted the same quantity of water, one subarea could be represented by a system of squares, but the nets in the other subareas would consist of rectangles in which the ratio of the lengths of the sides would be proportional to the differences in transmissibility. If the flow lines from one subarea enter another subarea at an angle, the flow lines (and equipotential lines) would be refracted according to the tangent law. The graphical construction of a flow net under such conditions is extremely difficult and, with the data that are available for most ground-water problems, is generally impossible. However, Bennett and Meyer (1952, p. 54–58) have shown that by generalizing the flow net for such an area into a system of squares and determining the quantity of flow by making an inventory of pumpage in each of the subareas, the approximate transmissibility of the subareas may be determined. Although such an application of the method departs somewhat from theory, it is likely that for many areas it provides more realistic areal transmissibilities than could be obtained by use of pumping-test methods alone. Whereas pumping tests may provide accurate values of transmissibilities they generally represent only a small "sample" of the aquifer. Flow-net analysis on the other hand may include large parts of the aquifer, and hence provide an integrated and more realistic value of the areal transmissibility. Moreover, by including comparatively large parts of the aquifer, the local irregularities that may appreciably affect some pumping-test analyses generally have an insignificant effect on the overall flow patterns.

The application of flow-net analysis to ground-water problems has not received the attention it deserves; however as the versatility of flow-net analysis becomes more widely known, its use will become more common. Such a method of analysis greatly strengthens the hydrologist's insight into ground-water flow systems; it provides quantitative procedures for analyzing and interpreting contour maps of the water-table and piezometric surfaces.

For other illustrations of flow-net construction, see figures 36 and 38.

THEORY OF IMAGES AND HYDROLOGIC BOUNDARY ANALYSIS

The development of the equilibrium and nonequilibrium formulas discussed in the preceding sections was predicated in part on the assumption of infinite areal extent of the aquifer, although it is recognized that few if any aquifers completely satisfy this assumption. In many instances the existence of boundaries serves to limit the continuity of the aquifer, in one or more directions, to distances ranging from a few hundred feet to as much as tens of miles. Thus when an aquifer is recognized as having finite dimensions, direct analysis of the test data by the equations previously given is often precluded. It is often possible, however, to circumvent the analytical difficulties posed by the aquifer boundary. The method of images, widely used in the theory of heat conduction in solids, provides a convenient tool for the solution of boundary problems in ground-water flow. Imaginary wells or streams, usually referred to as images, can sometimes be used at strategic locations to duplicate hydraulically the effects on the flow regime caused by the known physical boundary. Use of the image thus is equivalent to removing a physical entity and substituting a hydraulic entity. The finite flow system is thereby transformed by substitution into one involving an aquifer of infinite areal extent, in which several real and imaginary wells or streams can be studied by means of the formulas already given. Such substitution often results in simplifying the problem of analysis to one of adding effects of imaginary and real hydraulic systems in an infinite aquifer.

An aquifer boundary formed by an impermeable barrier, such as a tight fault or the impermeable wall of a buried stream valley that cuts off or prevents ground-water flow, is sometimes termed a "negative boundary." Use of this term is discouraged, however, in favor of the more meaningful and descriptive term "impermeable barrier." A line at or along which the water levels in the aquifer are controlled by a surface body of water such as a stream, or by an adjacent segment of aquifer having a comparatively large transmissibility or water-storage capacity, is sometimes termed a "positive boundary." Again, however, use of the term is discouraged in favor of the more precise terms line source or line sink, as may be appropriate.

Although most geologic boundaries do not occur as abrupt discontinuities, it is often possible to treat them as such. When conditions permit this practical idealization, it is convenient for the purpose of analysis to substitute a hypothetical image system for the boundary conditions of the real system.

In this section, where the analysis of pumping-test data is considered, several examples are given of image systems required to duplicate, hydraulically, the boundaries of certain types of areally restricted aquifers. It should be apparent that similar methods can be used to analyze flow to streams or drains through areally limited aquifers.

PERENNIAL STREAM—LINE SOURCE AT CONSTANT HEAD

An idealized section through a discharging well in an aquifer hydraulically controlled by a perennial stream is shown in figure 35*A*. For thin aquifers the effects of vertical-flow components are small at relatively short distances from the stream, and if the stream stage is not lowered by the flow to the real well there is established the boundary condition that there shall be no drawdown along the stream position. Therefore, for most field situations it can be assumed for practical purposes that the stream is fully penetrating and equivalent

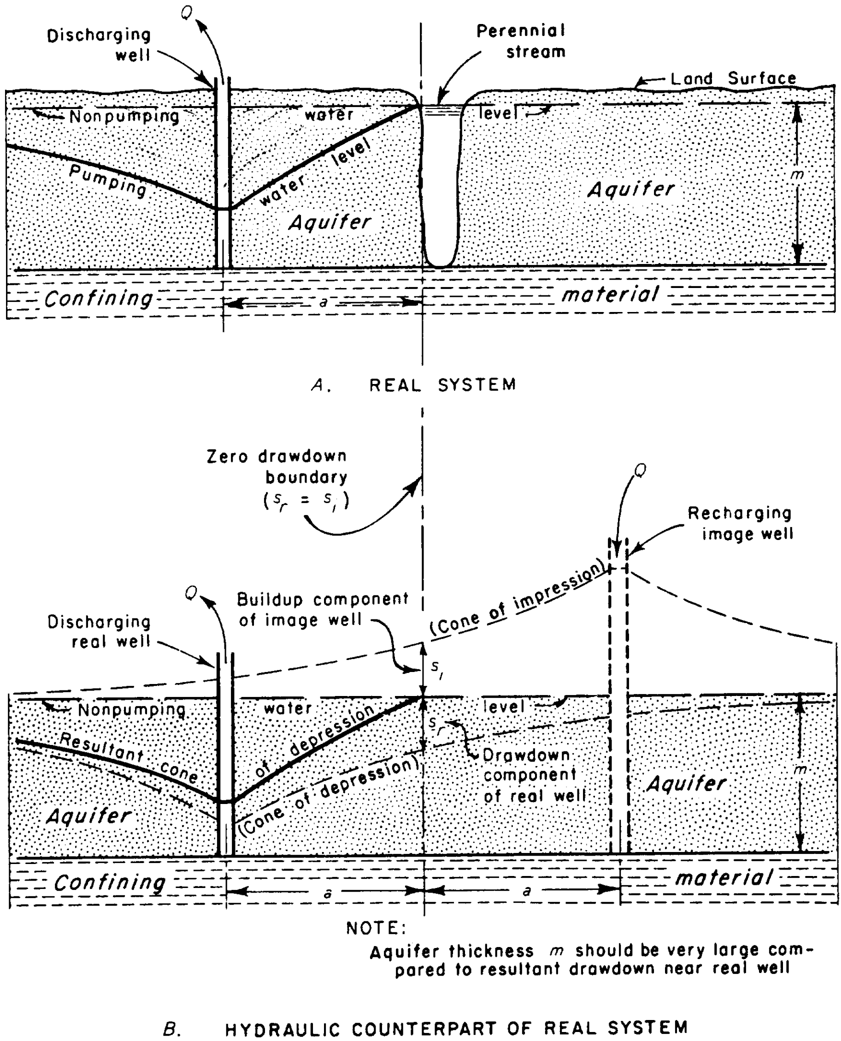

FIGURE 35.—Idealized section views of a discharging well in a semi-infinite aquifer bounded by a perennial stream, and of the equivalent hydraulic system in an infinite aquifer.

to a line source at constant head. An image system that satisfies the foregoing boundary condition, as shown in figure 35*B*, allows a solution of the real problem through use, in this example, of the Theis nonequilibrium formula. Note in figure 35*B* that an imaginary recharging well has been placed at the same distance as the real well from the line source but on the opposite side. Both wells are situated on a common line perpendicular to the line source. The imaginary recharge well operates simultaneously with the real well and returns water to the aquifer at the same rate that it is withdrawn by the real well. It can be seen that this image well produces a buildup of head everywhere along the position of the line source that is equal to and cancels the drawdown caused by the real well which satisfies the boundary condition of the problem. The resultant drawdown at any point on the cone of depression in the real region is the algebraic sum of the drawdown caused by the real well and the buildup produced by its image. The resultant profile of the cone of depression, shown in figure 19*B*, is flatter on the landward side of the well and steeper on the riverward side, as compared with the shape it would have if no boundary were present. Figure 36 is a generalized plan view of a flow net for the situation given in figure 35*A*. The distribution of stream lines and potential lines about the real discharging well and its recharging image, in an infinite aquifer, is shown. If the image region is omitted, the figure represents the stream lines and potential lines as they might be observed in the vicinity of a discharging well obtaining water from a river by induced infiltration.

IMPERMEABLE BARRIER

An idealized section through a discharging well in an aquifer bounded on one side by an impermeable barrier is shown in figure 37*A*. It is assumed that the irregularly sloping boundary can, for practical purposes, be replaced by a vertical boundary, occupying the position shown by the vertical dashed line, without sensibly changing the nature of the problem. The hydraulic condition imposed by the veritcal boundary is that there can be no ground-water flow across it, for the impermeable material cannot contribute water to the pumped well. The image system that satisfies this condition and permits a solution of the real problem by the Theis equation is shown in figure 37*B*. An imaginary discharging well has been placed at the same distance as the real well from the boundary but on the opposite side, and both wells are on a common line perpendicular to the boundary. At the boundary the drawdown produced by the image well is equal to the drawdown caused by the real well. Evidently, therefore, the drawdown cones for the real and the image wells will be symmetrical and will produce a ground-water divide at every point along the boundary line. Because there can be no flow

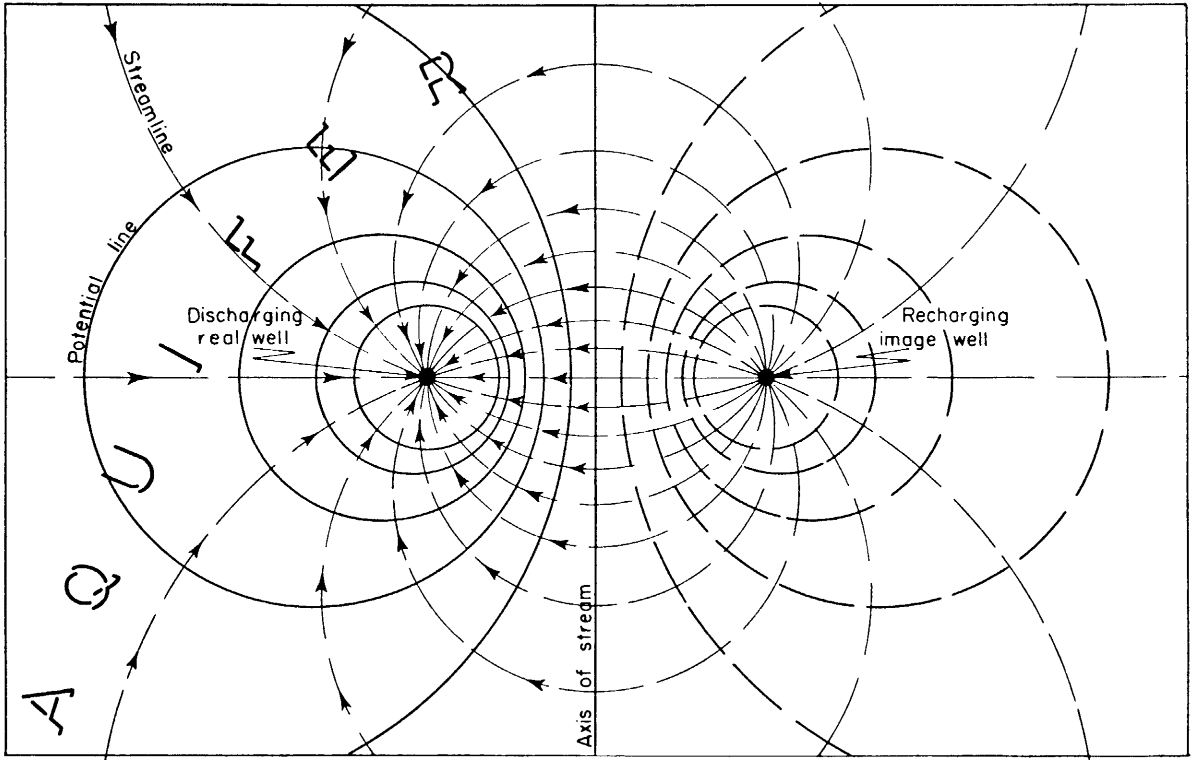

FIGURE 36.—Generalized flow net showing stream lines and potential lines in the vicinity of a discharging well dependent upon induced infiltration from a nearby stream.

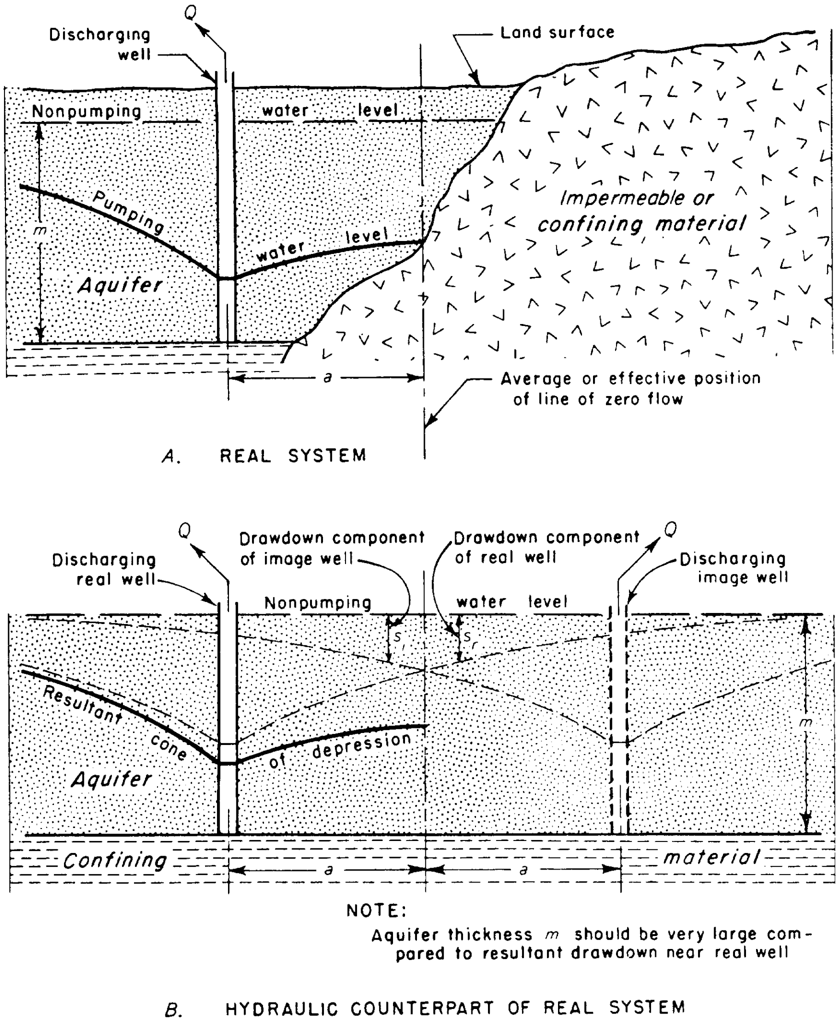

FIGURE 37.—Idealized section views of a discharging well in a semi-infinite aquifer bounded by an impermeable formation, and of the equivalent hydraulic system in an infinite aquifer.

across a divide, the image system satisfies the boundary condition of the real problem and analysis is simplified to consideration of two discharging wells in an infinite aquifer. The resultant drawdown at any point on the cone of depression in the real region is the algebraic sum of the drawdowns produced at that point by the real well and its image. The resultant profile of the cone of depression, shown in figure 37*B*, is flatter on the side of the well toward the boundary and steeper on the opposite side away from the boundary than it would be if no boundary were present. Figure 38 is a general-

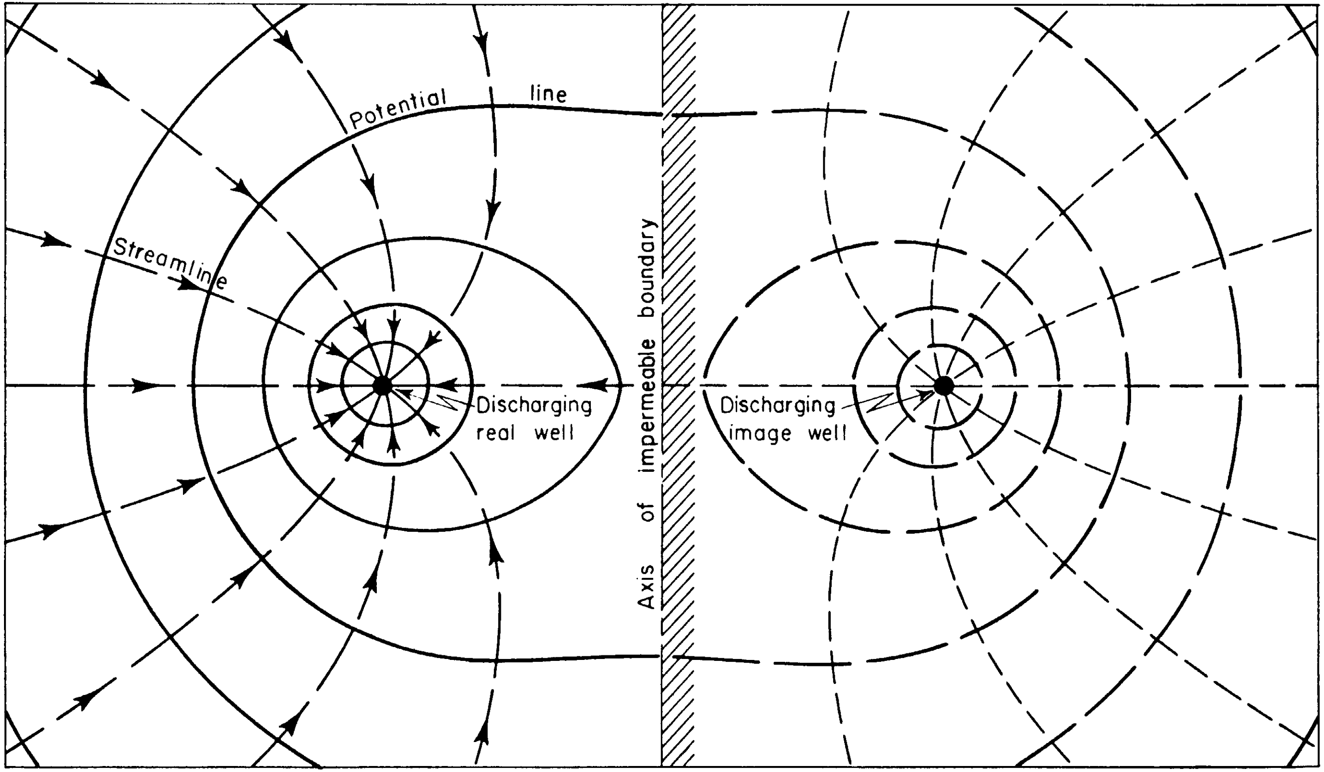

FIGURE 38.—Generalized flow net showing stream lines and potential lines in the vicinity of a discharging well near an impermeable boundary.

ized plan view of a flow net for the situation given in figure 37*A*. The distribution of stream lines and potential lines about the real discharging well and its discharging image, in an infinite aquifer, is shown. If the image region is omitted, the diagram represents the flow net as it might be observed in the vicinity of a discharging well located near an impermeable boundary.

TWO IMPERMEABLE BARRIERS INTERSECTING AT RIGHT ANGLES

The image-well system for a discharging well in an aquifer bounded on two sides by impermeable barriers that intersect at right angles is shown in figure 39. Although the drawdown effects of the primary image wells, I_1 and I_2, combine in the desired manner with the effect

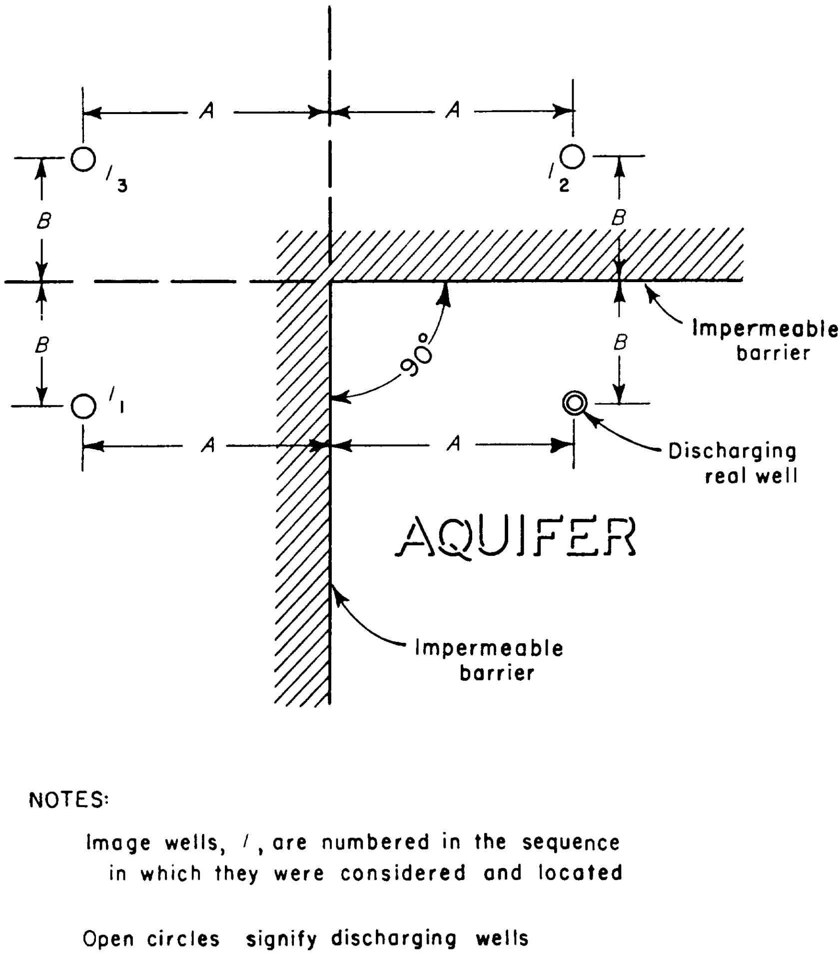

FIGURE 39.—Plan of image-well system for a discharging well in an aquifer bounded by two impermeable barriers intersecting at right angles.

of the real well at their respective boundaries, each image well produces an unbalanced drawdown at the extension (reflection) of the other boundary. These unbalanced drawdowns at the boundaries produce a hydraulic gradient, with consequent flow across the extension of each boundary, and therefore do not completely satisfy the requirement of no flow across the boundaries of the real system. It is necessary, therefore, to use a secondary image well, I_3, which balances the residual effects of the two primary image wells at the two extensions of the boundaries. The image system is then hydraulically in complete accord with the physical boundary conditions. The problem thereby has been simplified to consideration of four discharging wells in an infinite aquifer.

IMPERMEABLE BARRIER AND PERENNIAL STREAM INTERSECTING AT RIGHT ANGLES

The image-well system for a discharging well in an aquifer bounded on two sides by an impermeable barrier and a perennial stream which intersect at right angles is shown in figure 40. The perennial stream of figure 40 might also represent a canal, drain, lake, sea, or any other line source of recharge sufficient to maintain a constant head at this boundary. As before, the drawdown effects of the primary images, I_1 and I_2, combine in the desired manner with the effects of the real well at their respective boundaries. However, discharging image well I_1 produces a drawdown at the extension of the line source, which is a no-drawdown boundary, and recharging image well I_2 causes flow across the extension of the impermeable barrier, which is a no-flow boundary. By placing a secondary recharging image well, I_3, at the appropriate distance from the extension of each boundary, the system is balanced so that no flow occurs across the impermeable barrier and no drawdown occurs at the perennial stream. Thus again the problem has been simplified to consideration of an infinite aquifer in which there operate simultaneously two discharging and two recharging wells.

The simplest way to analyze any multiple-boundary problem is to consider each boundary separately and determine how best to meet the condition of no flow or no drawdown, as the case may be, at that boundary. After the positions of the primary image wells have been established, the boundary positions should be reexamined to see if the net drawdown effects of the primary image wells satisfy all stipulated conditions of no flow or no drawdown. For each primary image causing an unbalance at a boundary position, or extension thereof, it is necessary to place a secondary image well at the same distance from the boundary but on the opposite side, both wells occupying a common line perpendicular to the boundary. When the combined drawdown (or buildup) effects of all image wells are found to produce the desired effect at this boundary the same procedure is executed with

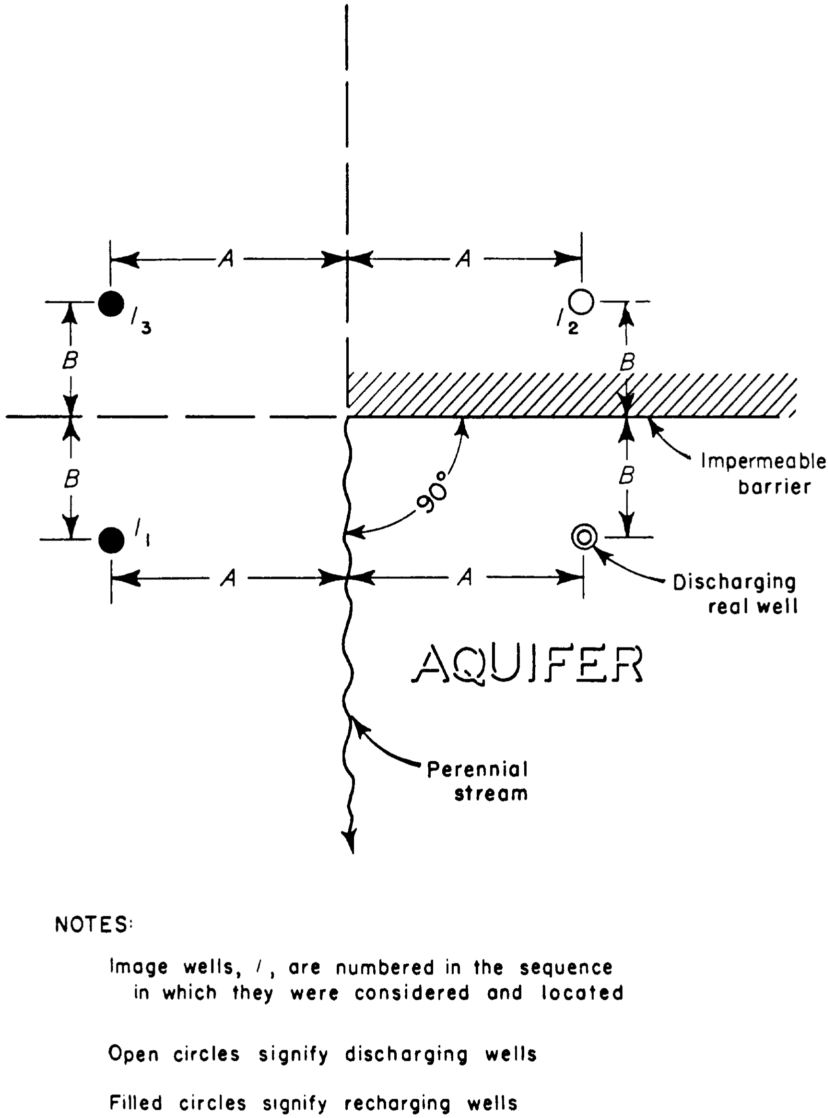

FIGURE 40.—Plan of image-well system for a discharging well in an aquifer bounded by an impermeable barrier intersected at right angles by a perennial stream.

respect to the second boundary. Thus, the inspection and balancing process is repeated around the system until everything is in balance and all boundary conditions are satisfied, or until the effects of additional image wells are negligible compared to the total effect.

TWO IMPERMEABLE BARRIERS INTERSECTING AT AN ANGLE OF 45°

Although it is intended here to consider the particular image-well system required for analyzing flow to a well in a 45-degree wedge-shaped aquifer, it is appropriate first to comment briefly on some general aspects of image-well systems in wedge-shaped aquifers. By analogy with similar heat-flow situations it is possible to analyze the flow to a well in a wedge-shaped aquifer, and equivalent image systems can be constructed regardless of the wedge angle involved. However, closed image systems that are the simplest to construct and analyze occur when the wedge angle, θ, of the aquifer equals (or can be approximated as equal to) one of certain aliquot parts of 360°. These particular values of θ may be specified as follows (after Walton, 1953, p. 17), keeping in mind that it is required to analyze flow to a single pumped well situated anywhere in the aquifer wedge: If the aquifer wedge boundaries are of like character, θ must be an aliquot part of 180°. If the boundaries are not of like character, θ must be an aliquot part of 90°.

Other simple solutions not covered by the above rule appear possible when θ is an odd aliquot part of 360°, the pumped well is on the bisector of the wedge angle, and the boundaries are similar and impermeable. For any of the foregoing special situations it can be shown, with the aid of geometry, that the number of image wells, n, required in analyzing the flow toward the single real pumping well is given by the relation

$$n=\frac{360°}{\theta}-1. \tag{79}$$

It can also be shown that the locus of all image-well locations, for a given aquifer-wedge problem, is a circle whose center is at the wedge apex and whose radius equals the distance from the apex to the real discharging well (see figure 45).

The image-well system for a discharging well in a wedge-shaped aquifer bounded by two impermeable barriers intersecting at an angle of 45° is shown in figure 41. The real discharging well is reflected across each of the two boundaries which results in location of the two primary image wells I_1 and I_2 as shown. Considering boundary 1 only, the effects of the real well and image well I_1, are seen to combine so that, as desired, no flow occurs across that boundary. However, image well I_2 will produce flow across boundary 1 unless image well I_3 is added at the location shown. The system now satisfies the condition of no flow across boundary 1. Repeating this examination process for boundary 2 only, it is seen that the effects of the real well and image well I_2 combine, as desired, to produce no flow across boundary 2. However, image wells I_1 and I_3 will produce flow across this boundary unless image wells I_4 and I_5 are added as shown. The image system now satisfies the condition of no flow across boundary 2. Reexamining, it is seen that image wells I_4 and I_5 will produce flow across boundary 1 unless image wells I_6 and I_7 are added as shown. A final appraisal of the effects at boundary 2, shows that the entire system of image wells, plus the real well, satisfies the requirement of no flow across the boundary. Thus the flow field caused by a discharging well in this wedge-shaped aquifer can be simulated by a total of eight discharging wells in an infinite aquifer. The seven image wells have replaced the two barriers. The drawdown at any point between the two barriers can then be computed by adding the

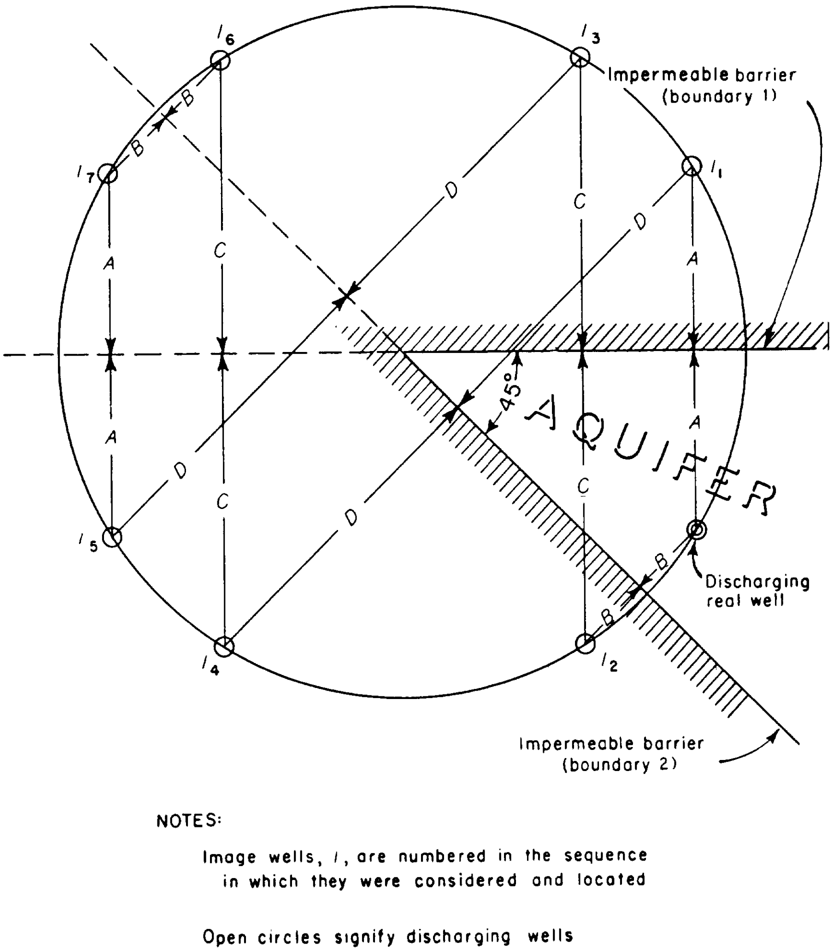

FIGURE 41.—Plan of image-well system for a discharging well in an aquifer bounded by two impermeable barriers intersecting at an angle of 45°.

effects produced at that point by the real well and the seven image wells. Each image well begins discharging at the same rate and at the same time as the real well.

IMPERMEABLE BARRIER PARALLEL TO A PERENNIAL STREAM

Shown in figure 42 is the image-well system for a discharging well in an aquifer bounded by an impermeable barrier and cut by a fully penetrating perennial stream parallel to the barrier. A recharging image well, I_1, and a discharging image well, I_2, are placed as shown to satisfy respectively the conditions that no drawdown can occur along the line source, and no flow can occur across the impermeable barrier. Although these two primary image wells produce, in conjunction with the real well, the desired effects at their respective boundaries, each image well produces a residual effect at the opposite boundary which conflicts with the stipulated boundary conditions. It is therefore necessary to add a secondary set of image wells, I_3 and I_4, as shown, to produce effects that will combine properly with the residual effects of the primary images. Each image well in the secondary set will again produce residual effects at the opposite boundary, and similarly with each successively added image pair there will be residual effects at the boundaries. It should be evident, however, that as more pairs of image wells are added the effects of adding a new pair have lesser influence on the cumulative effect at each boundary. In other words it is only necessary to add pairs of image wells until the residual effects associated with addition of the next pair can be considered to have negligible influence on the cumulative effect at each boundary. It is seen in figure 42 that there is a repeating pattern in the locations of the image wells. Therefore, after the positions of the first images have been determined, it is possible to locate by inspection as many more as are needed for the practical solution of the problem. Once the required number of image pairs has been determined, the aquifer boundaries can be ignored and the problem analyzed like any other multiple-well problem in an infinite aquifer.

If the two parallel boundaries are of like character—that is, if the perennial stream in figure 42 were replaced by an impermeable barrier or if the impermeable barrier were replaced by a perennial stream—the positions of the image wells would not be changed. In the first case, however, all the images would be discharging wells, and in the second case the image system would be an alternating series of recharging and discharging wells.

TWO PARALLEL IMPERMEABLE BARRIERS INTERSECTED AT RIGHT ANGLES BY A THIRD IMPERMEABLE BARRIER

The image-well system for a discharging well in this type of areally restricted aquifer is shown in figure 43. The positions of the images are determined as before by adding imaginary discharging wells so

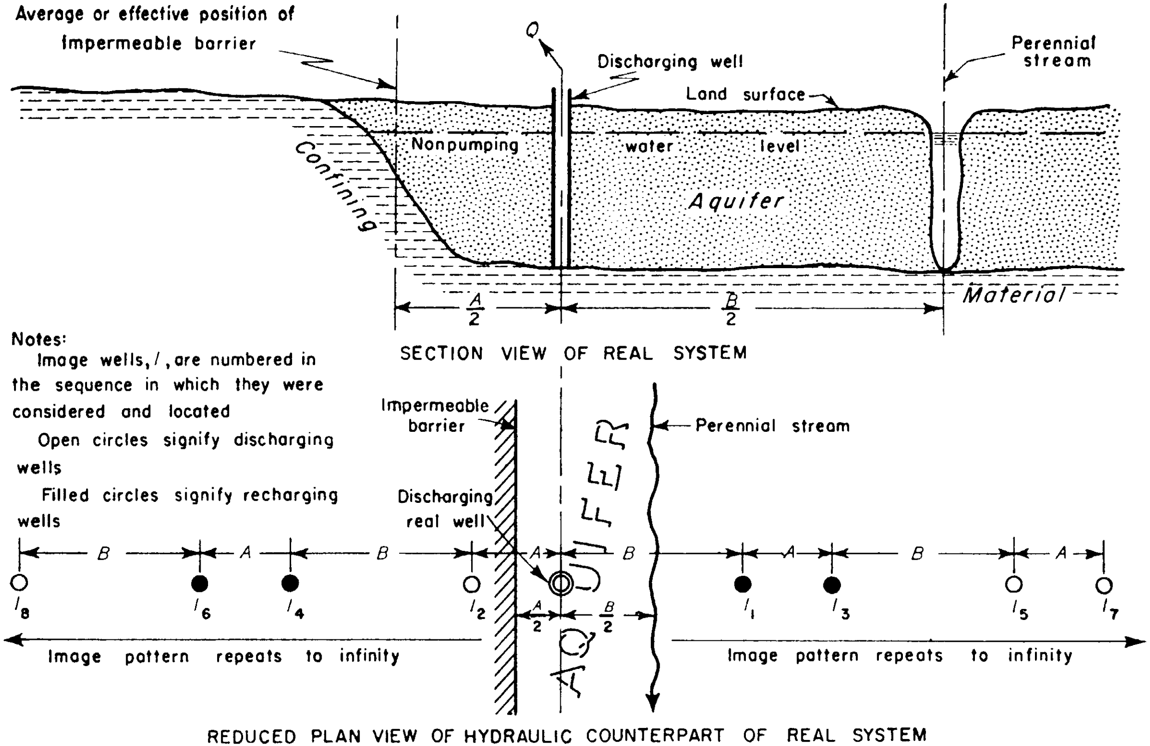

FIGURE 42.—Image-well system for a discharging well in an aquifer bounded by an impermeable barrier parallel to a perennial stream.

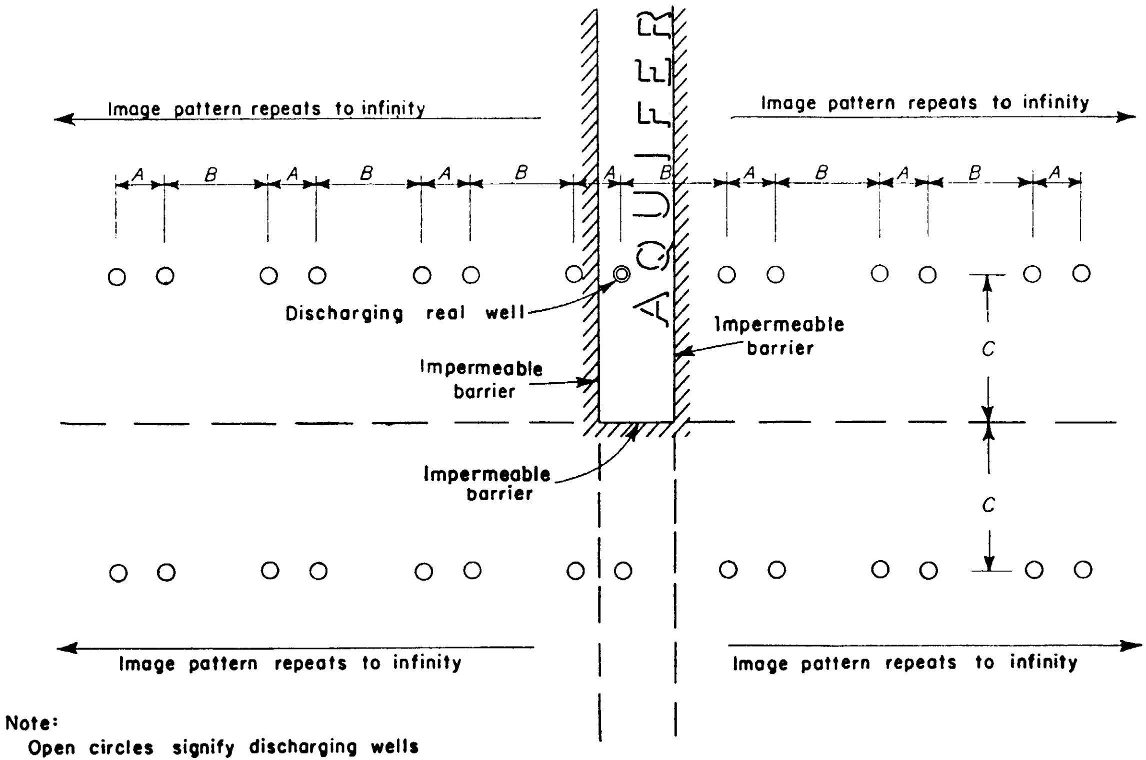

FIGURE 43.—Plan of image-well system for a discharging well in an aquifer bounded by two parallel impermeable barriers intersected at right angles by a third impermeable barrier.

that, in combination with the real discharging well, the condition of no ground-water flow across any of the three boundaries is established. As shown in the figure, two parallel lines of discharging image wells are required, separated by twice the distance between the real well and the barrier that intersects the two parallel barriers. Theoretically the two lines of image wells extend to infinity in both directions from the real well. The practical analysis of a problem of this kind, however, requires the addition of only enough images so that the effect of adding the next image, in any of the directions involved, has a negligible influence on the cumulative effect at each of the boundaries. It is seen from figure 43 that there is a repeating pattern in the positions of the image wells, so that the locations of only the first few images are required to determine the locations of as many succeeding image wells as are needed. For the case of two parallel impermeable barriers intersected at right angles by a perennial stream, the image system would be the same as shown by figure 43 except that all images on the line reflected across the stream would be recharging wells.

RECTANGULAR AQUIFER BOUNDED BY TWO INTERSECTING IMPERMEABLE BARRIERS PARALLELING PERENNIAL STREAMS

The image-well system for a discharging well in such an aquifer is shown by figure 44. The positions of the images are determined in the manner previously described. It is seen from figure 44 that there is again a repeating pattern that extends to infinity in all directions from the real well. Thus only the first few images need be located to determine the positions of as many succeeding images as are required in the practical solution of the problem. If the four boundaries in figure 44 were all impermeable barriers, all images would be discharging wells; and if the four boundaries were all perennial streams, the image system would be alternating series of recharging and discharging wells.

APPLICABILITY OF IMAGE THEORY INVOLVING INFINITE SYSTEMS OF IMAGE WELLS

Referring to the three problems discussed in the three preceding sections, it will be observed that in each situation the aquifer involved is limited in areal extent by two or more boundaries. Furthermore, the arrangement of the boundaries is such that at least two are parallel to each other, which means that analysis by the image theory requires use of an image-well system extending to infinity.

It has been stated, in discussing the practical aspects of using an infinite image-well system, that the individual effects of image wells need be added only out to the point where the effect associated with the addition of the next more distant well (or wells, depending on the symmetry of the array) can be considered to have negligible influence on the cumulative effect. Although this criterion ostensibly provides

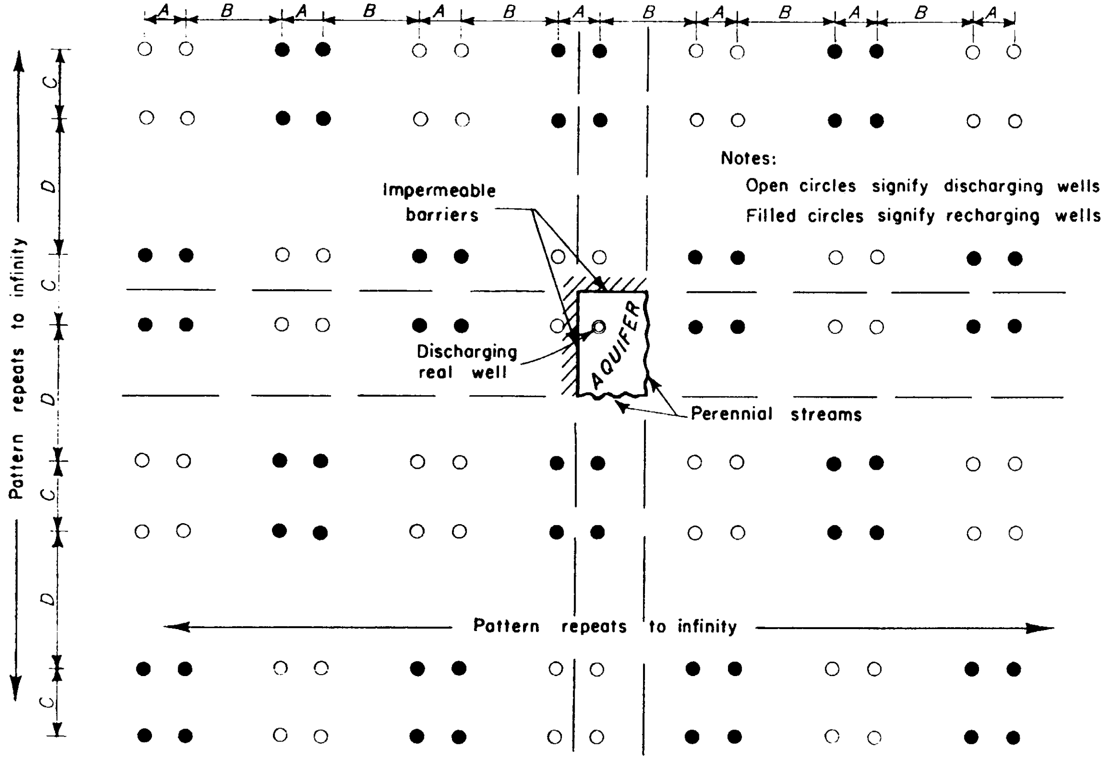

FIGURE 44.—Plan of image-well system for a discharging well in a rectangular aquifer enclosed by 2 intersecting impermeable barriers and 2 intersecting perennial streams.

a reliable and practical means of terminating what would otherwise be an endless analytical process, closer scrutiny appears warranted. There is no reason to state categorically that this practical approach to a solution should never be tried. Undoubtedly there will occur situations wherein sensible results can be obtained. On the other hand it seems prudent to observe that if the process of algebraically summing the individual effects of an infinite system of image wells is terminated anywhere short of infinity, there is no simple way of determining what proportion of the infinite summation is represented by the partial summation. Although addition of the next image well (or wells) might have a negligible influence on the sum of all image-well effects considered out to that point, there is no simple way of deciding whether the same may be said of the total influence represented by adding the effects of say the next 10 or 20 or 100 more distant image wells. Thus it would appear wise to keep in mind the possible limitations of any solution involving the use of an infinite system of image wells.

COROLLARY EQUATIONS FOR APPLICATION OF IMAGE THEORY

The nature and location of hydrologic boundaries of water-bearing formations in some cases can be determined from the analysis of pumping-test data. Considering the discussion in the preceding sections, it should be evident that in an aquifer whose extent is limited by one or more boundaries, a plot of drawdown or recovery data will depart from the form that would be expected if the aquifer were of infinite extent. Thus, in a problem involving a discharging well in a semi-infinite aquifer bounded by an impermeable barrier, some part of a time–drawdown plot may be steepened by the boundary effects. Conversely, if the boundary involved in the same type of problem were a perennial stream, a part of the time–drawdown plot may be flattened because of the boundary effects.

Imagine a pumping test made in an aquifer whose extent is limited by one or more boundaries. During the early part of the test, the drawdown data for observation wells close to the pumped well will reflect principally the pumping effects. As the test continues, however, there will very likely come a time for each observation well when the measured drawdowns reflect the net effect of the pumped well and any boundaries that are present. At distant observation wells boundary effects may arrive almost simultaneously with the effect of the real discharging well. Thus determination of the aquifer coefficients of transmissibility and storage should be based on the early drawdown data, as observed in a well near the pumped well, before the boundary effects complicate the analysis. Superposition and matching of a plot of these early data (s versus r^2/t) on the Theis type curve permits drawing in the type-curve trace. Extension or extrapolation of this trace beyond the early data indicates the trend the drawdowns would have taken if the pumping had occurred in an infinite aquifer. The departure, s_i, of the later observed data from this type-curve trace represents effects of the boundaries on the drawdown. The subscript i refers to the image-well system substituted as the hydraulic equivalent of the boundaries. Usually it is convenient to note values of s_i at a number of points along the data curve and to replot these departures versus values of r_r^2/t on the same graph sheet that was used in determining the coefficient of storage and transmissibility from the early data. The subscript r refers to the real discharging well. The latter part of the replotted departure data may again deviate from the type-curve trace if the cone of depression has intercepted a second boundary. As before, the departures can be replotted against corresponding values of r_r^2/t to form a second departure curve. This process should be repeated until the last departure curve shows no deviation from the type curve. The observed data array will then have been separated into its component parts which can be used to compute the distances between the observation wells and the image wells.

Inasmuch as the aquifer is assumed to be homogeneous (that is, the coefficients of transmissibility and storage are constant throughout the aquifer) it follows from equation 8 that

$$\frac{1.87S}{T}=\frac{u_r}{r_r^2/t}=\frac{u_i}{r_i^2/t}, \qquad (80)$$

where the subscripts r and i have the significance previously given. If on the plots of early drawdown data and first-departure curve a pair of points is selected so that the drawdown component caused by the real well, s_r, and the drawdown component caused by the image well, s_i, are equal, it follows that $u_r=u_i$. On the plots of observed early drawdowns and first departures just described, s_r and s_i obviously occur at different elapsed times, which can be labelled t_r and t_i respectively. Equation 80 can therefore be rewritten as follows:

$$\frac{r_i^2}{t_i}=\frac{r_r^2}{t_r}, \qquad (81)$$

or

$$r_i=r_r\sqrt{\frac{t_i}{t_r}}. \qquad (82)$$

Equation 81, known as the "law of times" in the physics of heat conduction, shows that at a given observation well location the times of occurrence of equal drawdown components vary directly and only as the squares of the distances from the observation well to the pumped well and to its image.

Referring to the data plots mentioned earlier in this section, note, for the pair of points selected, that values of s_r and r_r^2/t_r will be read from the early drawdown data while values of s_i and r_r^2/t_i will be read from the first departure curve. Equation 82 can be made more useful, therefore, if it is rewritten in the form

$$r_i=r_r\sqrt{\frac{r_r^2/t_r}{r_r^2/t_i}}. \tag{83}$$

Equation 83 now affords a ready means of computing the distance from an observation well to an image well. Similar analysis may be made of each departure curve constructed from the original drawdown data.

Stallman (1952) has described a convenient method for computing r_i when the observed drawdown in the aquifer represents the algebraic sum of the drawdown effects from one real well and one image well. If equation 6 is used to provide expressions for s_r and s_i, and $W(u)$ is substituted as a symbolic form of the exponential integral, it is seen that the drawdown at the observation well is

$$s=s_r\pm s_i=\frac{114.6Q}{T}\,[W(u)_r\pm W(u)_i]. \tag{84}$$

From equation 80,

$$\frac{r_i}{r_r}=\sqrt{\frac{u_i}{u_r}},$$

or

$$r_i=r_r\sqrt{\frac{u_i}{u_r}}. \tag{85}$$

From equations 84 and 85, it can be seen that r_i and the sum of the $W(u)$ terms in equation 84 can be expressed in terms of r_r and the ratio u_i/u_r. Thus for any given values of $\sqrt{u_i/u_r}=K$, a type curve can be constructed by plotting assumed values of u_r against corresponding computed values of the bracketed portion of equation 84 [which may be written in abbreviated form as $\sum_{r,i} W(u)$]. The data plot, s versus t, will match this constructed type of curve if the observation well is located so that the ratio r_i/r_r equals the given value of K. However, if a family of type curves is drawn for a number of given values of K, the observed data plot, s versus t, for any observation well, can be compared with the set of type curves. Once the best matching curve is found, any convenient matchpoint is selected and the coordinate values, s, t, u_r, $\sum_{r,i} W(u)$, and K are noted. These values, substituted in equations 80, 84, and 85 provide the means for computing T, S, and r_i.

Stallman's set of curves is the familiar type-curve u versus $W(u)$, used in conjunction with the Theis formula, with a series of appendage curves (two for each value of K) asymptotic to it. The trend of the appendage curve for a recharging image well is below, and for a discharging image above, the Theis curve. Appendage curves could have been constructed by assuming values of u_i instead of u_r. In this event, however, the matching process would not be as direct inasmuch as the parent type curve, instead of occupying a single position, would shift along the u axis with each pair of appendage (K) curves.

The appendage curves, computed by Stallman, are for ideal image wells—those which are pumped or recharged at the same rate as the real well. The hydrogeologic structure which gives rise to the hypothetical image is not always ideal; therefore the hypothetical images are not always ideal. For this case the method of plotting departures may yield an erroneous and misleading analysis. On the other hand, the deviations from ideality can be seen immediately if the observed data plot s versus t is matched to Stallman's set of type curves. Furthermore, for nonideal images, the most accurate selection of K is made by utilizing the portion of the appendage curve that is nearest the parent or Theis type curve.

If little is known of the possible location of a local hydraulic boundary a minimum of three observation wells is required to fix the position of an image well, which in turn permits location of the boundary. After the distances from the individual observation wells to the image well have been computed, arcs are scribed with their centers at the observation wells and their radii equal to the respective computed distances to the image well. The intersection of the arcs at a common point fixes the location of the image well, and the strike of the boundary is represented by the perpendicular bisector of a line connecting the pumped well and the image well.

Another graphical method for locating a hydraulic boundary in the vicinity of a discharging well was devised by E. A. Moulder (1951, written communication, p. 61). The geometry is shown in figure 45. A circle is scribed whose center is at a nearby observation well, O, and whose radius, r_i, is equal to the computed distance from the observation well to the image well. The image well lies somewhere on this circle, say, at point I. Lines are drawn from the selected point I to the observation well and to the real discharging well, P. If point I is the image-well location and if A is the midpoint of the line IP, then point A lies on the boundary. It can be proved by geometry, that the locus of all points A determined in this manner is a circle, of radius BA or $r_i/2$, with its center, B, located midway between the discharing well and the observation well. Moulder's method is particularly useful in aquifer-test situations where data from only one or two observation wells are available for locating a boundary position. If the approximate position of a suspected boundary is known before a pumping test begins, it is desirable to locate most of the observation wells along a line parallel with the boundary and passing through the pumped well. If feasible the range of distances from the observation wells to the pumped well should be distributed logarithmically to assure well-defined arc intersections in the graphics of locating a point on that boundary. At least one observation well

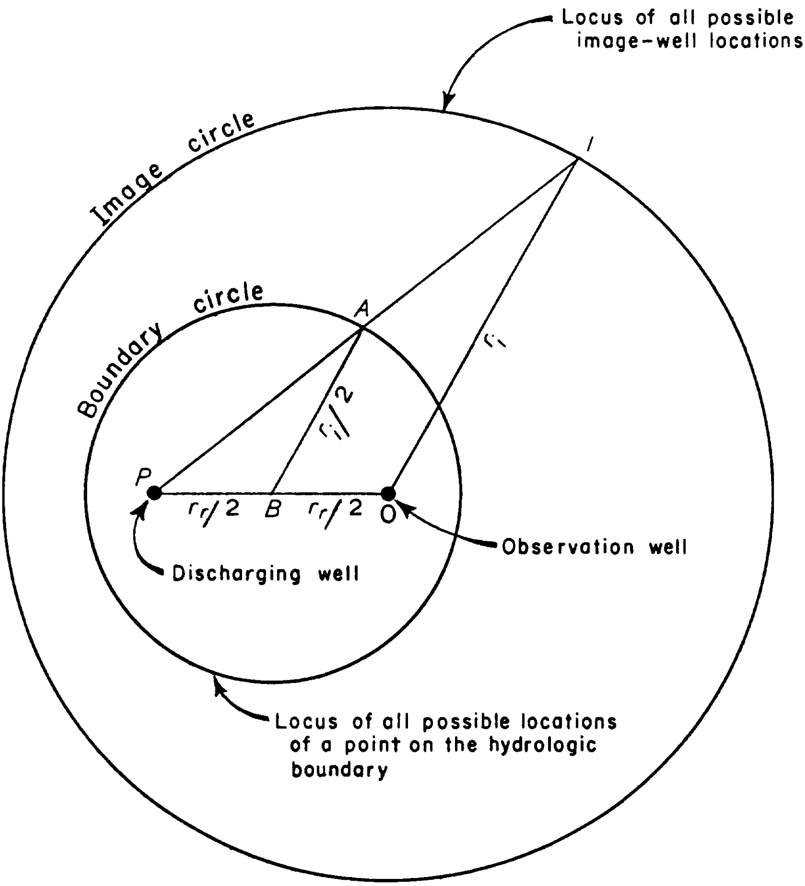

FIGURE 45.—Geometry for locating a point on a hydrologic boundary, with reference to the locations of a discharging well and a nearby observation well.

should be located close enough to the pumped well so that the early drawdown data, unaffected by the boundary, can be used in computing the aquifer coefficients of storage and transmissibility.

APPLICABILITY OF ANALYTICAL EQUATIONS

The assumptions used in developing the equations presented in this report include the stipulation that the aquifer is homogeneous and isotropic. Even though most naturally deposited sediments do not satisfy this condition, the equations may still be applied and the results qualified according to the extent of nonhomogeneity. It should be realized that homogeneity is a relative term with respect to time and space. As an illustration, consider an aquifer composed of two types of material—a fine sand and a very coarse sand. Assume that these materials occur individually in deposits having the shape of cubes one-eighth of a mile on a side, and that alternate rows of cubes (squares in plan view) are offset a distance equal to one-half the length of one side of the cube (that is, one-sixteenth of a mile). Let the fine and coarse sand occur in alternate cubes along the continuous rows, and assume that water occurs, in the aquifer thus created, under water-table conditions. Strictly speaking, this aquifer, of infinite extent, would now be described as nonhomogeneous. However, the areal extent of the portion of the aquifer sampled in a test would be significant in judging this element of the aquifer's description. For example, if a discharging well test is conducted in the center of one of the squares and if the test is terminated before the area of influence reaches the perimeter of the square, the test results probably would be considered excellent and the aquifer described as homogeneous. The results would in no way differ from the results to be expected if a similar test were made on an infinite "homogeneous" aquifer, composed of material identical to that occurring in the limited area here tested.

As another example, again consider an aquifer test using a discharging well in the center of one of the squares of the hypothetical aquifer. The nearest of several observation wells is at a radius of 5 miles from the pumped well, and the test is run until the area of influence is described by a circle 10 miles in radius. Coefficients of transmissibility computed from data collected at all the observation wells should be in close agreement (although not equal to the values obtained from the previously described test), and again the hypothetical aquifer, even on the larger scale represented in this sample, would be adjudged homogeneous. This judgment relies upon the reasoning that, for the distances involved, the slightly meandering path of water, as it moves toward the well, may be described statistically as conforming to the concept of radial flow. For any case in which nonhomogeneity is so distributed that the flow field statistically fits the geometry of the mathematical model, the mathematical solution will provide a sound analysis. Conversely, when the flow field or a portion thereof is significantly distorted in the area of observation, the assumption of homogeneity is incorrect. Thus, for the hypothetical aquifer considered in the two preceding examples, the distorted condition is seen to exist if the area of influence of the discharging well were to extend to a radius of about ¼ to 1 mile—that is, a little beyond the limits of one cube of the aquifer material.

Often the field situation is encountered where a zone of relatively impermeable material such as a clay lens, of limited thickness and extent, occurs in an aquifer. It should be evident from the foregoing examples and discussion, however, that the presence of this clay lens in the flow field will have less influence on aquifer test results when the effects of the test encompass an area of large radius than when the area affected is of small radius.

An important criterion, therefore, regarding the applicability of the equations discussed in this report, is the amount the flow field is distorted, as compared with the flow field that would have been observed in an ideal aquifer.

It should be understood that the numerical results obtained by substituting aquifer-test data in an appropriate mathematical model indicate the transmissibility and storage coefficients for an ideal aquifer. The hydrologist must judge how closely the real aquifer resembles this particular ideal. It is usually recognized, for example, that in short pumping tests under water-table conditions the water does not drain from the smaller openings in the unwatered portion of the aquifer in a manner even approximating the instantaneous release assumed in devising the mathematical model. Similarly, in testing artesian aquifers it is recognized that the aquifer skeleton does not adjust instantaneously to the change in head, that considerable water is often contributed by intercalated clay beds, and furthermore that water leaks through the confining beds, which in the mathematical model have been assumed to be impermeable. However, these recognized departures from the ideal do not constitute grounds for abandoning, or rarely using, available analytical equations. Such departures simply add emphasis to the admonition that mere substitution of aquifer-test data in an equation will not of itself assure anyone of establishing the correct hydraulic properties for that aquifer. The mechanics of applying any of the analytical equations in this report must be accomplished with sound professional judgment, followed by critical evaluation and testing of the results.

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