Theory of Aquifer Tests

U.S. GEOLOGICAL SURVEY WATER-SUPPLY PAPER 1536-E

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By J. G. FERRIS, D. B. KNOWLES, R. H. BROWN, *and* R. W. STALLMAN

GROUND-WATER HYDRAULICS

GEOLOGICAL SURVEY WATER-SUPPLY PAPER 1536-E

UNITED STATES GOVERNMENT PRINTING OFFICE, WASHINGTON : 1962

UNITED STATES DEPARTMENT OF THE INTERIOR

MANUEL LUJAN, JR., Secretary

GEOLOGICAL SURVEY

Dallas L. Peck, Director

First Printing 1962
Second printing 1963
Third printng 1989

For sale by the Books and Open-File Reports Section, U.S. Geological Survey,
Federal Center, Box 25425, Denver, CO 80225

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SYMBOLS

[The following symbols are listed in alphabetical order. Each symbol indicates the basic term usually represented, with no attempt to show the many and unavoidable duplicate uses. In the text, various subscripts are used in conjunction with these symbols to denote specific applications of the basic terms. A few of the more important combinations of this type are given; others are defined where they appear in the text.

A	Area of cross section through which flow occurs.
a	Distance from stream or drain to ground-water divide.
e	Base of natural (Napierian) logarithms, numerically equal to 2.7182818.
g	Local acceleration due to gravity.
h	Head of water with respect to some reference datum.
I	Hydraulic gradient.
L	Length (width) of cross section through which flow occurs.
m	Saturated thickness of an aquifer.
m'	Saturated thickness of relatively impermeable bed confining an aquifer.
P	Coefficient of permeability of the material comprising an aquifer.
P'	Coefficient of vertical permeability of the material comprising a relatively impermeable bed that confines an aquifer.
Q	Rate of discharge, or recharge.
r	Radial distance from discharge or recharge well to point of observation.
r_w	Effective radius of discharge or recharge well.
S	Coefficient of storage of an aquifer.
s	Change in head of water, usually expressed as drawdown, or recovery or buildup.
s'	Residual change in head of water, usually reserved for use in conjunction with the term, "drawdown".
T	Coefficient of transmissibility of an aquifer.
t	Elapsed time with respect to an initial reference.
t'	Elapsed time with respect to a second reference.
V	Volume.
W	Rate of accretion or recharge to an aquifer.
w	Spacing of grid lines used to subdivide a region into finite squares.
x	Distance from stream or drain to point of observation.
BE	Barometric efficiency of an aquifer.
TE	Tidal efficiency of an aquifer.
$D(u)_h$	Drain function of u, constant head situation.
$D(u)_q$	Drain function of u, constant discharge situation.
$G(\alpha)$	Well function of α, constant head situation.
$W(u)$	Well function of u, constant discharge situation.
$J_0(x)$	Bessel function of first kind, zero order.
$I_0(x)$	Modified Bessel function of first kind, zero order.
$Y_0(x)$	Bessel function of second kind, zero order.
$K_0(x)$	Modified Bessel function of second kind, zero order.
α	Bulk modulus of compression or vertical compressibility (reciprocal of the bulk modulus of elasticity) of the aquifer skeleton.

β	Bulk modulus of compression, or compressibility of water; approximate value for average ground-water temperature is 3.3×10^{-6} in ^{2}/lb.
γ	Specific weight of a substance.
γ_0	Specific weight of water at a stated reference temperature; numerically equal to 62.4 lb per cubic foot at 4°C or 39°F.
θ	Porosity of an aquifer.
ρ	Density of a substance.

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ABSTRACT

The development of water supplies from wells was placed on a rational basis with Darcy's development of the law governing the movement of fluids through sands and with Dupuit's application of that law to the problem of radial flow toward a pumped well. As field experience increased, confidence in the applicability of quantitative methods was gained and interest in developing solutions for more complex hydrologic problems was stimulated. An important milestone was Theis' development in 1935 of a solution for the nonsteady flow of ground water, which enabled hydrologists for the first time to predict future changes in ground-water levels resulting from pumping or recharging of wells. In the quarter century since, quantitative ground-water hydrology has been enlarging so rapidly as to discourage the preparation of comprehensive textbooks.

This report surveys developments in fluid mechanics that apply to ground-water hydrology. It emphasizes concepts and principles, and the delineation of limits of applicability of mathematical models for analysis of flow systems in the field. It stresses the importance of the geologic variable and its role in governing the flow regimen.

The report discusses the origin, occurrence, and motion of underground water in relation to the development of terminology and analytic expressions for selected flow systems. It describes the underlying assumptions necessary for mathematical treatment of these flow systems, with particular reference to the way in which the assumptions limit the validity of the treatment.

INTRODUCTION

Lectures on ground-water hydraulics by John G. Ferris provide most of the source material for this paper, which was organized by Doyle B. Knowles. Subsequent refinements of concepts and standardization of nomenclature and method of presentation were accomplished by Russell H. Brown and Robert W. Stallman, with the important collaboration of Edwin W. Reed. Appropriate individual authorship is recognized for several sections of the text.

The material presented herewith concerns the theory supporting many hydraulic concepts. Applications of the theory to field problems are to be shown in another report.

AQUIFER TESTS—THE PROBLEM

The basic objective of ground-water studies of the U.S. Geological Survey is to evaluate the occurrence, availability, and quality of ground water. The science of ground-water hydrology is applied toward attaining that goal. Although many ground-water investigations are of a qualitative nature, quantitative studies are necessarily an integral component of the complete evaluation of occurrence and availability. The worth of an aquifer as a fully developed source of water depends largely on two inherent characteristics: its ability to store and its ability to transmit water. Furthermore, quantitative knowledge of these characteristics facilitates measurement of hydrologic entities such as recharge, leakage, and evapotranspiration. It is recognized that these two characteristics, referred to as the coefficients of storage and transmissibility, generally provide the very foundation on which quantitative studies are constructed. Within the science of ground-water hydrology, ground-water hydraulics methods are applied to determine these constants from field data.

Ground-water hydraulics, as now defined by common practice, can be described as the process of combining observed field data on water levels, water-level fluctuations, natural or artificial discharges, etc., with suitable equations or computing methods to find the hydraulic characteristics of the aquifer; it includes the logical extension of these data and computing methods to the prediction of water levels, to the design of well fields, the determination of optimum well yields, and other hydraulic uses—all under stated conditions. The selection of equations or computing procedures to be used for analysis is governed largely by the physical conditions of the aquifer studies, insofar as they establish the hydraulic boundaries of the system. The extraordinary variability in the coefficients of storage and transmissibility, combined with the irregularities in the shape of flow systems encountered in many ground-water studies, precludes uninhibited support of calculated coefficients based on vague or meager data. One quantitative test does not satisfy the demand for a quantitative study of an aquifer. It is merely a guidepost, indicator, or segment of knowledge, which must be supported by additional tests. Often the initially calculated results may require revision on the basis of the discoveries resulting from additional testing as the field investigation proceeds.

Obviously the results from ground-water hydraulics must be completely in accord with the geologic characteristics of the aquifer or of the area under investigation. Circumstance frequently demands that tests be conducted without prior knowledge of the geology in the vicinity of the test site. To varying degrees, lack of knowledge of the geology in most cases reduces the reliability of the test results to a semiquantitative category until more adequate support is found.

The principal method of ground-water hydraulics analysis is the application of equations derived for particular boundary conditions. The number of equations available has grown rapidly and steadily during the past few years. These are described in a wide assortment of publications, some of which are not conveniently available to many engaged in studies of ground-water hydraulics. The essence of each of many concepts of hydraulics is presented and briefly discussed, but frequent recourse should be made to the more exhaustive treatment given in the cited reference.

Where the definition of a hydraulics or ground-water term is considered necessary, it is stated where the term first appears, and the symbol and units in which the term is ordinarily expressed are given. Much of the terminology is assembled under "Symbols," following the table of contents.

DARCY'S LAW

Hagen (1839) and Poiseuille (1846) were the first to study the law of flow of water through capillary tubes. They found that the rate of flow is proportional to the hydraulic gradient. Later Darcy (1856) verified this observation and demonstrated its applicability to the laminar (viscous, streamline) flow of water through porous material while he was investigating the flow of water through horizontal filter beds discharging at atmospheric pressure. He observed that, at low rates of flow, the velocity varied directly with the loss of head per unit length of sand column through which the flow occurred and expressed this law as

$$v=\frac{Ph}{l}$$

in which v is velocity of the water through a column of permeable material, h is the difference in head at the ends of the column, l is the length of the column, and P is a constant that depends on the character of the material, especially the size and arrangement of the grains.

The velocity component in laminar flow is proportional to the first power of the hydraulic gradient. It can be seen, therefore, that Darcy's law is valid only for laminar flow. The flow is probably turbulent or in a transitional stage from laminar to turbulent flow near the screens of many large-capacity wells. Jacob (1950) agrees with Meinzer and Fishel (1934) that because water behaves as a viscous fluid at extremely low hydraulic gradients, it will obey Darcy's law at gradients much smaller than can be measured in the laboratory. He points out, however, that Darcy's law may not be valid for the flow of water in sands that are not completely saturated, or in extremely fine grained materials.

COEFFICIENTS OF PERMEABILITY AND TRANSMISSIBILITY

The *coefficient of permeability*, *P*, of material comprising a formation is a measure of the material's capacity to transmit water. The coefficient of permeability was expressed by Meinzer (Stearns, N.D., 1928) as the rate of flow of water in gallons per day through a cross-sectional area of 1 square foot under a hydraulic gradient of 1 foot per foot at a temperature of 60°F. In figure 17, then, it would be the flow of water through opening A, which is 1 foot square. In field practice the ajdustment to the standard temperature of 60°F. is commonly ignored and permeability is then understood to be a field coefficient at the prevailing water temperature. Theis (1935) introduced the term *coefficient of transmissibility*, *T*, which is expressed

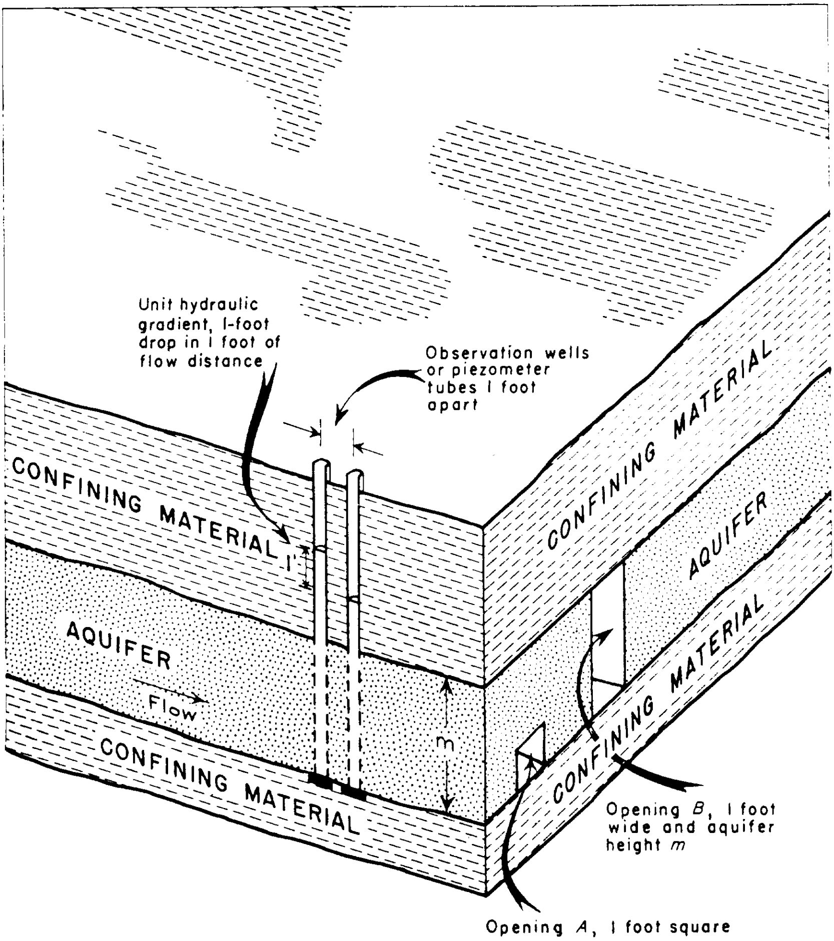


FIGURE 17.—Diagram for coefficients of permeability and transmissibility.

as the rate of flow of water, at the prevailing water temperature, in gallons per day, through a vertical strip of the aquifer 1 foot wide extending the full saturated height of the aquifer under a hydraulic gradient of 100 percent. In figure 17 it would be the flow through opening B, which has a width of 1 foot and a height equal to the thickness, m, of the aquifer. A hydraulic gradient of 100 percent means a 1-foot drop in head in 1 foot of flow distance as shown schematically by the pair of observation wells in figure 17. It is seldom necessary to adjust the coefficient of transmissibility to an equivalent value for the standard temperature of 60°F., because the temperature range (and, hence, range in viscosity) in most aquifers is not large. The relation between the coefficient of transmissibility and the field coefficient of permeability, as they apply to flow in an aquifer, can be seen in figure 17.

A useful form of Darcy's law, which is often applied in studies of ground-water hydraulics problems, is given by the expression

$$Q_d = PIA$$

in which Q_d is the discharge, in gallons per day; P is the coefficient of permeability, in gallons per day per square foot; I is the hydraulic gradient, in feet per foot; and A is the cross-sectional area, in square feet, through which the discharge occurs. For most ground-water problems, this expression can be more conveniently written as

$$Q_d = TIL$$

in which Q_d and I are defined as above, T is the coefficient of transmissibility in gallons per day per foot, and L is the width, in feet, of the cross section through which the discharge occurs. In many field problems it may be more practical to express I in feet per mile and L in miles. The units for T and Q_d will remain as already stated. The coefficient of transmissibility may be determined by means of field observations of the effects of wells or surface-water systems on ground-water levels. It is then possible to determine the field coefficient of permeability from the formula $P = T/m$. Physically, however, P has limited significance under these conditions. It merely represents the overall average permeability of an ideal aquifer that behaves hydraulically like the aquifer tested.

In general, laboratory measurements of permeability should be applied with extreme caution. The packing arrangement of a poorly sorted sediment is a critical factor in determining the permeability, and large variations in permeability may be introduced by repacking a disturbed sample. Furthermore, a laboratory measurement of permeability on one sample is representative of only a minute part of the water-bearing formation. Obviously, therefore, if quantitative data are to be developed by laboratory methods, it is desirable to collect samples of the water-bearing material at close intervals of depth and at as many locations within the aquifer as is feasible.

COEFFICIENT OF STORAGE

The *coefficient of storage*, *S*, of an aquifer is defined as the volume of water it releases from or takes into storage per unit surface area of the aquifer per unit change in the component of head normal to that surface.

A simple way of visualizing this concept is to imagine an artesian aquifer which is elastic and uniform in thickness, and which is assumed, for convenience, to be horizontal. If the head of water in that aquifer is decreased, there will be released from storage some finite volume of water that is proportional to the change in head. Because the aquifer is horizontal, the full observed head change is evidently effective perpendicular to the aquifer surface. Imagine further a representative prism extending vertically from the top to the bottom of this aquifer, and extending laterally so that its cross-sectional area is coextensive with the aquifer-surface area over which the head change occurs. The volume of water released from storage in that prism, divided by the product of the prism's cross-sectional area and the change in head, results in a dimensionless number which is the coefficient of storage. If this example were revised slightly, it could be used to demonstrate the same concept of coefficient of storage for a horizontal water-table aquifer or for a situation in which the head of water in the aquifer is increased.

As with almost any concise definition of a basic concept it is necessary to develop its full significance, its limitations, and its practical use and application through elaborative discussion. The coefficient of storage is no exception in this respect, and the following discussion will serve to bring out a few ideas that are important in applying the concept to artesian and water-table aquifers in horizontal or inclined attitudes.

Observe that the statement of the storage-coefficient concept first focuses attention on the volume of water that the aquifer releases from or takes into storage. Identification and measurement of this volume poses no particular problem, but it should be recognized that it is measured outside the aquifer under the natural local conditions of temperature and atmospheric pressure; it is not the volume that the same amount of water would occupy if viewed in place in the aquifer.

Although the example used to depict the concept of the storage coefficient was arbitrarily developed around a horizontally disposed artesian aquifer, the concept applies equally well to water-table aquifers and is not compromised by the attitude of the aquifer. This flexibility of application relies importantly, however, on relating the storage-coefficient concept to the surface area of the aquifer and to the component of head change that is normal to that surface. In turn this relation presupposes that the particular aquifer prism involved in the movement of water into or out of storage is that prism whose length equals the saturated thickness of the aquifer, measured normal to the aquifer surface, and whose cross-sectional area equals the area of the aquifer surface over which the head change occurs. Furthermore, water moves into or out of storage in this prism in direct proportion only to that part of the head change that acts to compress or distend the length of the prism. In other words, the component of the head change to be considered in the release or storage of water is that which acts normal to the aquifer surface. The mathematical models devised for analyzing ground-water flow usually require uniform thickness of aquifer. However, the storage coefficient concept, as defined here, applies equally well to aquifers that thicken or thin substantially, if the "surface area" is measured in the plane that divides the aquifer into upper and lower halves that are symmetrical with respect to flow. The imaginary prism would then be taken perpendicular to this mean plane of flow.

THE ARTESIAN CASE

Consider an artesian aquifer, in any given attitude, in which the head of water is changed, but which remains saturated before, during, and after the change. It is assumed that the beds of impermeable material confining the aquifer are fluid in the sense that they have no inherent ability to absorb or dissipate changes in forces external to or within the aquifer. Inasmuch as no dewatering or filling of the aquifer is involved, the water released from or taken into storage can be attributed only to the compressibility of the aquifer material and of the water. By definition the term "*head* of water" and any changes therein connote measurements in a vertical direction with reference to some datum. In a practical field problem the change in head very likely would be observed as a change in water-level *elevation* in a well. The change in head is an indication of the change in pressure in the aquifer prism, and the total change in force tending to compress the prism is equal to the product of the change in pressure multiplied by the end area of the prism. Obviously this change in force is not affected by the inclination of the aquifer, inasmuch as a confined pressure system is involved and the component of force due to pressure always acts normal to the confining surface. Thus any conventional method of observing head change will correctly identify the change in pressure normal to the aquifer surface and may be considered as a component of head acting normal to that surface.

Examine figure 18*A*, which depicts, in schematic fashion, a horizontal artesian aquifer. Shown within the aquifer is a prism of unit cross-sectional area and of height, m, equal to the aquifer thickness. If the piezometric surface is lowered a unit distance, x, as shown, a certain amount of water will be released from the aquifer prism. This occurs in response to a slight expansion of the water itself and a slight decrease in porosity due to distortion of the grains of material composing the aquifer skeleton.

Summary statement.—For an artesian aquifer, regardless of its attitude, the water released from or taken into storage, in response to a change in head, is attributed solely to compressibility of the aquifer material and of the water. The volume of water (measured outside the aquifer) thus released or stored, divided by the product of the head change and the area of aquifer surface over which it is effective, correctly determines the storage coefficient of the aquifer. Although rigid limits cannot be established, the storage coefficients of artesian aquifers may range from about 0.00001 to 0.001.

THE WATER-TABLE CASE

Application of the storage coefficient concept to water-table aquifers is more complex, though reasoning similar to that developed in the preceding paragraphs can be applied to the saturated zone of an inclined water-table aquifer. Consider a water-table aquifer, in any given attitude, in which the head of water is changed. Obviously there will now be dewatering or refilling of the aquifer, inasmuch as it is an open gravity system with no confinement of its upper surface. Thus the volume of water released from or taken into storage must now be attributed not only to the compressibility of the aquifer material and of the water, in the saturated zone of the aquifer, but also to gravity drainage or refilling in the zone through which the water table moves. The volume of water involved in the gravity drainage or refilling, divided by the volume of the zone through which the water table moves, is the specific yield. Except in aquifers of low porosity the volume of water involved in gravity drainage or refilling will ordinarily be so many hundreds or thousands of times greater than the volume attributable to compressibility that for practical purposes it can be said that the coefficient of storage equals the specific yield. The conventional method of measuring change in head by observing change in water-level *elevation* in a well evidently identifies the *vertical* change in position of the water table. In other words, head change equals vertical movement of the water table. It can be seen that the volume of the zone through which the water table moves is equal to the area of aquifer surface over which the head change occurs, multiplied by the head change, multiplied by the cosine

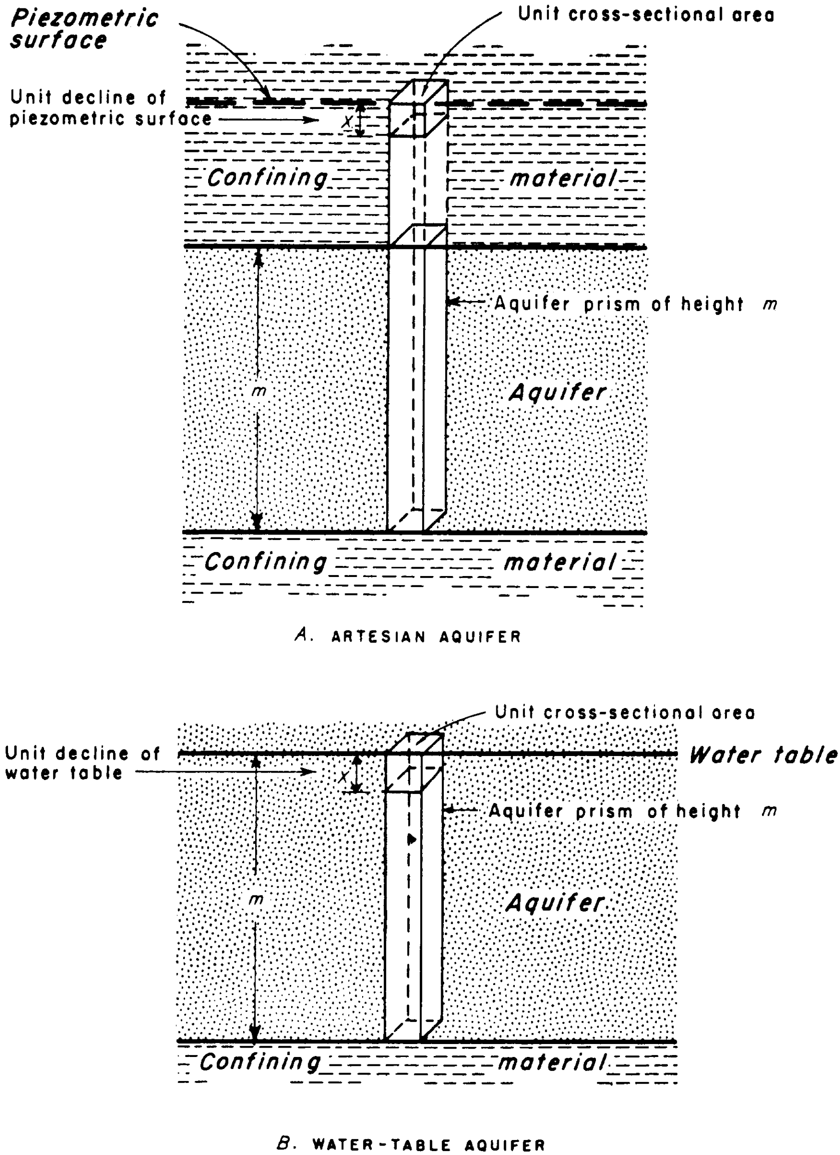


FIGURE 18.—Diagrams for coefficient of storage.

of the angle of inclination of the water table. The product of the last two factors is the component of head change acting normal to the aquifer surface. The importance of interpreting correctly the phrase "component of head change" which appears in the definition of the storage coefficient cannot be overemphasized.

Examine figure 18*B*, which depicts, in schematic fashion, a horizontal water-table aquifer. Again a unit prism of the aquifer is shown, and it is assumed that the water table is lowered a unit distance, x. Usually the water that is thereby released represents, for practical purposes, the gravity drainage from the x portion of the aquifer prism. Theoretically, however, a slight amount of water comes from the portion of the prism that remains saturated, in accord with the principles discussed for the artesian case.

Summary statement.—For a water-table aquifer, regardless of its attitude, the water released from or taken into storage, in response to a change in head, is attributed partly to gravity drainage or refilling of the zone through which the water table moves, and partly to compressibility of the water and aquifer material in the saturated zone. The volume of water thus released or stored, divided by the product of the area of aquifer surface over which the head change occurs and the component of head change normal to that surface, correctly determines the storage coefficient of the aquifer. Usually the volume of water attributable to compressibility is a negligible proportion of the total volume of water released or stored and can be ignored. The storage coefficient then is sensibly equal to the specific yield. The storage coefficients of water-table aquifers range from about 0.05 to 0.30.

ELASTICITY OF ARTESIAN AQUIFERS

It has long been recognized that artesian aquifers have volume elasticity. D. G. Thompson, though not the first to publish on the subject, apparently was among the first in the Geological Survey to recognize this phenomenon. In studying the relation between the decline in artesian head and the withdrawals of water from the Dakota sandstone in North Dakota, Meinzer (Meinzer and Hard, 1925) came to the conclusion that the water was derived locally from storage. He found that the withdrawals could not be accounted for by the compressibility of the water alone, but might be accounted for by the compressibility of the aquifer.

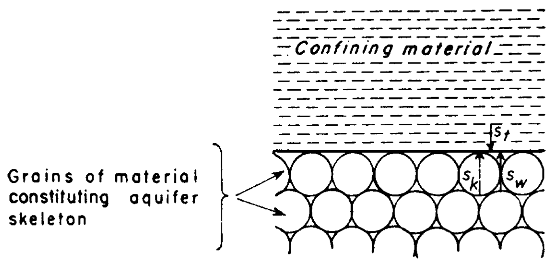
INTERNAL FORCES

The diagram in figure 19*A* shows the forces acting at the interface between an artesian aquifer and the confining material. These forces may be expressed algebraically as

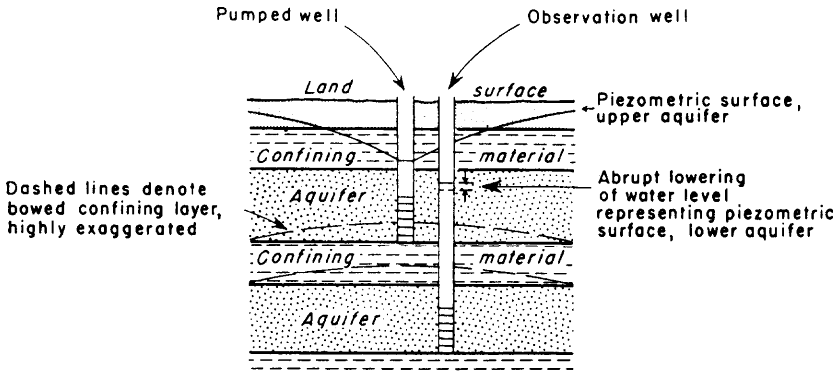
$$s_t = s_w + s_k$$

where s_t is the total load exerted on a unit area of the aquifer, s_w is that part of the total load borne by the confined water, and s_k is that part borne by the structural skeleton of the aquifer. Assume that the total load (s_t) exerted on the aquifer is constant. If s_w is reduced, as a result of pumping, the load borne by the skeleton of the aquifer increases and there is slight distortion of the component grains of material. At the same time, the water expands to the extent permitted by its elasticity. Distortion of the grains of the aquifer skeleton means that they will encroach somewhat on pore space formerly occupied by water.

Conversely, if s_w is increased, as in response to cessation of pumping, the piezometric head builds up again gradually approaching its original value, and the water itself undergoes slight contraction. With an increase in s_w there is an accompanying decrease in s_k and the grains of material in the aquifer skeleton return to their former



A MICROSCOPIC VIEW OF FORCES ACTING AT INTERFACE BETWEEN ARTESIAN AQUIFER AND CONFINING MATERIAL



B. DIAGRAM SHOWING THE EFFECT ON A DEEP ARTESIAN AQUIFER OF PUMPING FROM A SHALLOW ARTESIAN AQUIFER

FIGURE 19.—Diagrams for elastic phenomena in artesian aquifers.

shape. This releases pore space that can now be reoccupied by water moving into the part of the formation that was influenced by the compression.

TRANSMISSION OF FORCES BETWEEN AQUIFERS

It has been observed in some places that a well pumping from an aquifer affects the water level in a nearby well that is screened in a deeper or shallower artesian aquifer. Consider the case shown in figure 19*B*. The well screened in the upper aquifer, for convenience depicted as artesian, is pumped and the water level in the well screened in the lower aquifer abruptly declines when pumping begins. As pumping continues, the water level of the lower aquifer ceases to decline and gradually recovers its initial position. Although not proved, a logical explanation of this phenomenon may be as follows: When the well in the upper aquifer begins pumping, there is a lowering of the pressure head in the vicinity of the pumped well. The decrease in pressure head unbalances the external forces that were acting on the upper and lower surfaces of the confining layer separating the two aquifers. In seeking a new static balance the confining layer will be bowed upward slightly thereby creating additional water storage space in the lower aquifer. The abrupt lowering of water level in the observation well represents the response to the newly created storage space and the reduction in the forces s_k and s_w in the lower aquifer. The subsequent water-level recovery, approaching the initial position, represents filling of the new storage space and return to the original pressure head as water in the lower aquifer moves in from more remote regions.

An interesting phenomenon that has been observed a few times, but for which no completely satisfactory explanation has yet been given, is that where pumping a well in one artesian aquifer causes a rise in the water level in a nearby well producing from a different artesian aquifer. (See Barksdale, Sundstrom, and Brunstein, 1936; and Andreasen and Brookhart, 1952.)

EFFECTS OF CHANGES IN LOADING

EXCAVATIONS

"Blowthroughs" may occur if deep excavations are made in the confining materials overlying an artesian aquifer that has a high artesian head. This is shown diagramatically in figure 20. An excavation lowers the total load on part of the aquifer, which means that the forces s_w and s_k (see fig. 19*A*), are now the dominant forces acting on the remaining layer of confining material, separating the bottom of the excavation from the top of the aquifer. Thus this layer will be bowed upward and if it is incompetent to contain the

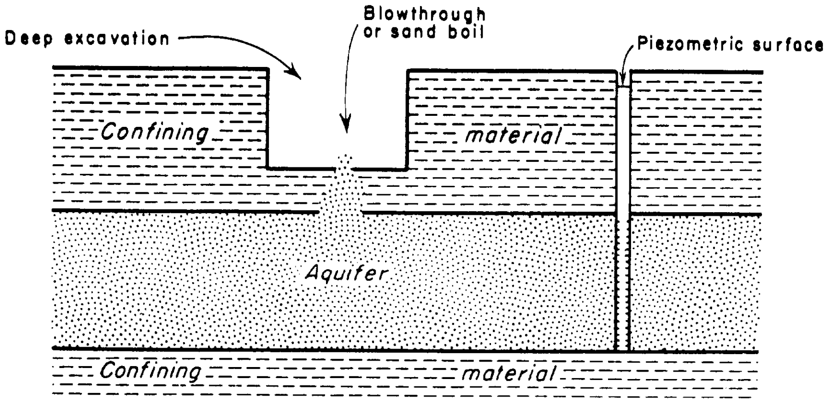


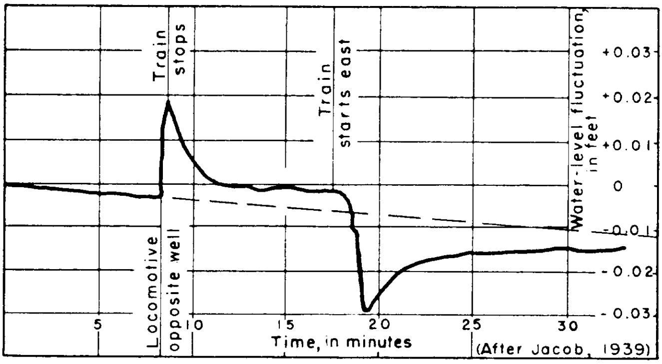
FIGURE 20.—Excavating in material overlying artesian aquifers,

bowing forces, it will rupture in the form of "blowthroughs" or "sand boils." It is the practice in Holland (Krul and Liefrinck, 1946), where this situation is commonly encountered, to install relief wells to lower the artesian head until an excavation is refilled.

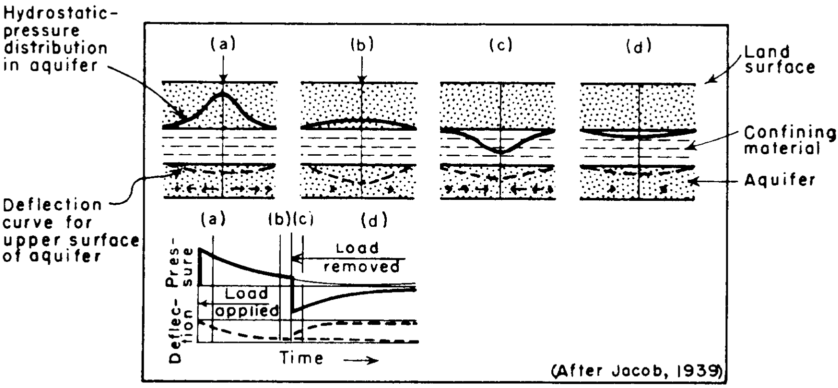
MOVING RAILROAD TRAINS

It is a frequent observation that a passing railroad train affects the water levels in nearby artesian wells. This is another demonstration of the elasticity of artesian aquifers. The fluctuation of water level in a well on Long Island, N.Y., produced by a passing railroad train is shown in figure 21*A*. As the train approaches the well, an additional load is placed on the aquifer. This load tends to compress the aquifer, causing a rapid rise in water level that reaches a maximum when, or shortly after, the locomotive is opposite the well. As the aquifer becomes adjusted to the new loading, the water level declines toward its initial position. When the entire train has passed the well, the aquifer expands and the water level in the well declines rapidly and reaches a minimum shortly after the train has left the well. The water level then recovers toward its initial position as the aquifer again becomes adjusted to this new condition of loading. The time required for this cycle of events is commonly a few minutes. Thus the fluctuations in water levels caused by the passing of a train usually appear as vertical lines on water-stage recorder charts because the time scale ordinarily used is too small to record the fluctuations in any greater detail.

The diagram in figure 21*B* shows, schematically, the effect of an instantaneously applied load on the pressure distribution within an elastic artesian aquifer and on the compression and subsequent ex-



A. WATER-LEVEL FLUCTUATION IN WELL S-201, LONG ISLAND, N.Y., PRODUCED BY EAST-BOUND FREIGHT TRAIN, MARCH 21, 1938



B. SCHEMATIC DIAGRAM SHOWING EFFECT OF INSTANTANEOUS APPLICATION AND SUBSEQUENT REMOVAL OF A SINGLE CONCENTRATED LOAD AT THE LAND SURFACE ON THE PRESSURE DISTRIBUTION AND ON THE COMPRESSION AND SUBSEQUENT EXPANSION OF AN ELASTIC ARTESIAN AQUIFER

FIGURE 21.—Effects of changes in loading on artesian aquifer.

pansion of the aquifer after removal of the load. In discussing figure 21*B* Jacob (1939) says:

The upper diagrams a, b, c, and d show the distribution of pressure and the deflection of the upper surface of the aquifer at the respective times indicated on the time-pressure and time-distribution curves. The hydrostatic pressure in the aquifer is plotted as a full line, the upper limit of the confining layer arbitrarily being adopted as a base. The deflection curve for the upper surface of the aquifer is plotted as a dashed line. (The lower surface of the aquifer is assumed fixed.) These quantities are, of course, grossly exaggerated and are obviously plotted to quite different scales. The length of the arrows indicates the relative magnitude of the velocity of flow at various distances from the load. The lower diagram . . . shows, by the heavy full line, the change in pressure produced by the load, and, by the heavy dashed line, the deflection of the upper surface of the aquifer, plotted against time.

CHANGES IN ATMOSPHERIC PRESSURE

It has often been observed that water levels in wells tapping artesian aquifers respond to changes in atmospheric pressure. An increase in the atmospheric pressure causes the water level to decline, and a decrease in atmospheric pressure causes the water level to rise. The diagrams shown in figure 22 will aid in explaining why this phenomenon is observed in artesian wells and why it ordinarily is not observed in water-table wells.

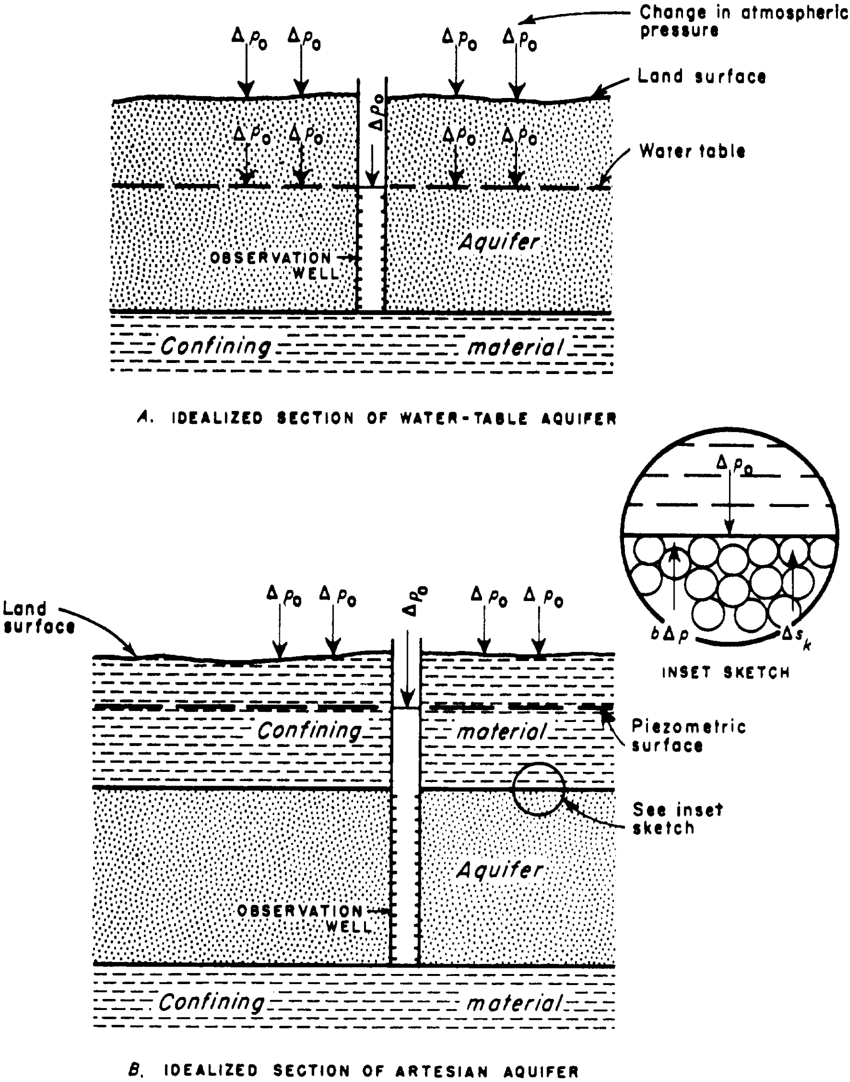


FIGURE 22.—Effect of atmospheric pressure loading on aquifers.

Referring to diagram *A*, figure 22, the force Δp_0, representing the change in atmospheric pressure, is exerted on the free water surface in the well. The same force Δp_0 is also exerted simultaneously on the water table because there is direct communication between the atmosphere and the water table through the unsaturated pore space of the soil. Thus the system of forces remains in balance and there is no appreciable change in water level in the well with changes in atmospheric pressure. Some water-table wells exhibit barometric fluctuations if the soil is frozen or saturated with water. But either of these conditions is, in effect, only a special case of the artesian condition.

Referring to diagram *B*, figure 22, the force Δp_0, which again represents the change in atmospheric pressure, acts on the free water surface in the well and also on the layer of material confining the artesian aquifer. Jacob (1940) in discussing this situation, reasons that barometric fluctuations in a well are an index of the elasticity of the aquifer. In other words the confining layer, viewed as a unit, has no beam strength or resistance to deflection sufficient to withstand or contain any sensible part of an applied load. Thus in effect any changes in the atmospheric pressure loading on a confining layer are transmitted through it undiminished in magnitude. The forces acting at a point at the interface between the aquifer and the confining layer may then be drawn as shown in the inset sketch. Observe that the change in atmospheric pressure, Δp_0, is now accommodated by a change in stress in the skeleton of the aquifer, Δs_k, plus a change in the water pressure in the aquifer, Δp, applied over the percentage b of the interface, where the water is in direct contact with the confining layer. It is evident, therefore, that in an artesian situation there will be a pressure differential between an observation well where the water is directly subject to the full change in atmospheric pressure, and a point out in the aquifer where the water is required to accept only part of the change in atmospheric pressure. Thus barometric fluctuations will be observed in the well. Some wells near the outcrop of an artesian aquifer or near a discontinuity in the confining layer will show little or no response to atmospheric pressure changes.

Although not associated with the elasticity of artesian aquifers, it is interesting to note that the phenomena of blowing and sucking wells, which exhibit a pronounced updraft or downdraft of air at the well mouth, may also be related to changes in atmospheric pressure. In areas where such wells have been noted, a bed of fine-grained, relatively impermeable material usually lies some distance above the water table, thereby effectively confining, in the intervening unsaturated pore space, a body of air that can communicate with the atmosphere only through wells.

The barometric efficiency of an aquifer may be expressed as

$$BE=\frac{s_w}{s_b}$$

where s_w is the net change in water level observed in a well tapping the aquifer and s_b is the corresponding net change in atmospheric pressure, both expressed in feet of water. It is frequently convenient to determine the barometric efficiency by plotting the water-level changes as ordinates and the corresponding changes in atmospheric pressure as abscissas on rectangular coordinate paper. The slope of the straight line drawn through the plotted points is the barometric efficiency.

TIDAL FLUCTUATIONS

OCEAN, LAKE, OR STREAM TIDES

Water levels in wells near the ocean or near some lakes or streams exhibit semidiurnal fluctuations in response to tidal fluctuations. In wells tapping water-table aquifers, the water-level response to tidal fluctuations is due to actual movement of water in the aquifer. However, in wells tapping artesian aquifers that are effectively separated from the body of surface water by an extensive confining layer, the response is due to the changing load on the aquifer, transmitted through the confining layer with the changing tide. Thus with the rise of the tide the load on the aquifer is increased, which means that in the aquifer there will be compensating increases of the water pressure and of the stress in the skeleton. Accordingly, the water-level rise in the well is but a reflection of the increased pressure head in the aquifer caused by the tidal loading.

An artesian well that responds to tidal fluctuations should also respond to changes in atmospheric pressure, because the same mechanism in the aquifer produces both types of response.

The tidal efficiency of an aquifer may be expressed as

$$TE=\frac{s_w}{s_t}$$

where s_w is the range of water-level fluctuation, in feet, in a well tapping the aquifer, and s_t is the range of the tide, in feet, corrected for density when necessary. There is a direct relation between the tidal efficiency or the barometric efficiency and the coefficient of storage. This relation will be discussed in a later section of this report.

Jacob (1950, p. 331–332) has derived expressions relating the tidal and barometric efficiency and the elasticity of an artesian aquifer. The two pertinent equations are

$$TE=\frac{\alpha/\theta\beta}{1+\alpha/\theta\beta}$$

and

$$BE=\frac{1}{1+\alpha/\theta\beta}$$

where α is the bulk modulus of compression of the solid skeleton of the aquifer, β is the bulk modulus of compression of water (reciprocal of the bulk modulus of elasticity), and θ is the porosity of the aquifer. If these two equations are added it is evident that the sum of the barometric efficiency and the tidal efficiency equals unity, that is,

$$BE+TE=1$$

EARTH TIDES

It has been observed that earth tides, which are caused by the forces exerted on the earth's surface by the sun and the moon, may produce water-level fluctuations in artesian wells. Water-level fluctuations due to earth tides were apparently first observed by Klonne (1880) in a flooded coal mine at Dux, Bohemia. Such fluctuations in wells were first observed by Young (1913) near Cradock, South Africa. After the water levels have been adjusted for changes in atmospheric pressure the "high" water levels have been observed near moonrise and moonset and the "low" water levels near the upper and lower culminations of the moon.

For a well near Carlsbad, New Mexico, and for a well at Iowa City, Iowa, Robinson (1939) showed that the water-level fluctuations, after adjustment for changes in atmospheric pressure, are coincident with the earth tides. The low water levels showed a tendency to precede the culmination of the moon, suggesting that the tide in the well precedes the culmination of the moon.

According to Theis (1939), the possible effects of tidal forces acting either directly upon the water in the aquifer or upon the aquifer itself by varying the weight of the overburden could not account for the observed water-level fluctuations. The explanation for these water-level fluctuations is probably the distortion of the earth's crust. In this regard Theis (1939) states:

> As the crust of the earth in any given area rises and falls with the deformation of the earth caused by the tidal forces the crust is most probably alternately

expanded and compressed laterally—expanded when the earth bulges up and compressed when it subsides. Water in an artesian aquifer making up part of the crust shares in this deformation. In localities distant from points of outflow of the water, it is in effect confined without possibility of outflow within the period of tidal fluctuations. The slight hydraulic gradient imposed by the tidal distortion is too small to cause effective release of pressure. Hence the aquifer is essentially sealed with respect to its included fluid. With the expansion of the aquifer incident to the tidal bulge the hydrostatic pressure falls and with its compression incident to tidal depression the hydrostatic pressure rises.

EARTHQUAKES

Fluctuations in water levels due to earthquakes have been observed in many wells equipped with water-stage recorders. Veatch (1906, p. 70) was apparently one of the first hydrologists in this country to recognize that some water-level fluctuations might be in response to earthquake disturbances. Subsequent investigators who have published papers on the subject include H. T. Stearns (1928), Piper (1933), Leggett and Taylor (1935), LaRocque (1941), Parker and Stringfield (1950), and Vorhis (1953). An earthquake may be defined as a vibration or oscillation of the earth's crust caused by a transient disturbance of the elastic or gravitational equilibrium of the rocks at or beneath the land surface. Earthquakes are classified as shallow or deep depending on the vertical position, relative to the land surface, of the source of the disturbance. Shock waves, propagated by an earthquake, travel through the earth and along the earth's surface. Because the earth is an elastic body it is first compressed by the shock waves and subsequently it expands after the shock wave is dissipated. Where an aquifer is included in the segment of the earth affected by the shock waves of an earthquake there will first be an abrupt increase in water pressure as the water assumes part of the imposed compressive stress, followed by an abrupt decrease in water pressure as the imposed stress is removed. In attempting to adjust to the pressure changes, the water level in an artesian well first rises and then falls. The amounts of the rise and fall of the water level, with respect to the initial position, are approximately the same. Cases have been recorded, however, where the water level did not return to its initial position (Brown, 1948, p. 193–195). This is presumably due to permanent rearrangement of the grains of material composing the aquifer. Fluctuations of greater magnitude have been observed in wells in limestone aquifers than in wells in granular material.

The following table gives the types of shock waves caused by earthquakes, the approximate average velocities at which they travel and the path they take.

TABLE 1.—*Approximate average velocities and paths taken by different types of shock waves caused by earthquakes*

[After Byerly, P., 1933, p. 155]

Type of shock wave	Approximate average velocity				Path
	(km per sec)	(ft per sec)	(mi per min)	(mi per hr)	
Deep-seated	5.5	18,000±	205±	12,030±	Chord
Surface	3.2	10,500±	120±	7,200±	Arc

COEFFICIENT OF STORAGE AND ITS RELATION TO ELASTICITY

The coefficient of storage is a function of the elasticity of an artesian aquifer. Jacob (1950) has expressed the relation as

$$S=\gamma_0\theta m\left(\beta+\frac{\alpha}{\theta}\right)$$

where in this instance γ_0 is the specific weight of the water at a stated reference temperature, and θ, m, β, and α are as defined earlier in this report. This formula assumes no leakage from or into contiguous beds.

Digressing momentarily, the specific weight, γ, of a fluid at a stated reference temperature is defined as its density, ρ, multiplied by the local acceleration due to gravity, g. Stated another way, it is the weight per unit volume that takes into account the magnitude of the local gravitational force. The manner in which specific weight is related to such more commonly used properties as mass, weight, and density can be developed in the following fashion. First it should be recognized that in the English engineering system the unit of mass is termed a "slug". It is the mass in which an acceleration of 1 ft per sec per sec. is produced by a force of 1 lb. Thus 1 slug of mass is approximately equal to 32.2 lb of mass. The mass, M, in slugs, of any substance is determined from the relation

$$M=\frac{W}{g}$$

where W is the weight of the substance in pounds. Inasmuch as weight is dependent on the local gravitational force it obviously varies with location. Thus, the fraction W/g takes into account in both numerator and denominator the local force of gravity, which shows that mass is an absolute property that does not change with location. Density is defined as mass per unit volume, V. That is:

$$\text{Density}=\frac{M}{V}$$

Thus density also is an absolute property that does not change with location. (Slight changes occur with change in temperature but ordinarily, within the range of ground-water temperatures encountered, no adjustment for this factor is necessary.) If the original relation given for determining mass, M, is solved for the weight, W, and if both sides of the rewritten equation are divided by the volume, V, there follows:

$$\left[\frac{W}{V}\right]=\left[\frac{M}{V}\right]g$$

The brackets on the left side of the equation are now seen to include the fraction equivalent to specific weight and the brackets on the right side include the fraction equivalent to density. That is:

$$\gamma=\rho g$$

Returning to the discussion at hand, it is important to point out that considerable study remains to be done regarding the elastic behavior of artesian reservoirs. It would be worthwhile, for example, to compute the values of the bulk modulus of compression (α) of the solid skeleton of the various artesian reservoirs where coefficients of storage have been determined from various types of aquifer tests. R. R. Bennett (oral communication) has supplied the following data for the principal aquifer in the Baltimore, Md., area:

$$S=0.0002$$

$$\theta=0.30$$

$$\text{m}=100 \text{ feet}=1200 \text{ inches}$$

Recognizing that for water

$$\gamma_0=62.4 \text{ lbs per cu ft}=\frac{62.4}{1728} \text{ (or 0.0361 lb per cu in)}$$

and

$$\beta=\frac{1}{300{,}000}=3.3\times10^{-6} \text{ (sq in per lb)}$$

appropriate substitution of all known quantities is made in Jacob's equation for the storage coefficient and the value of α is computed. Thus

$$0.0002=(0.0361)(0.30)(1200)\left[0.0000033+\frac{\alpha}{0.30}\right]$$

or

$$\alpha=0.00000363 \text{ sq in per lb or } 3.63\times10^{-6} \text{ sq in per lb}$$

Conversely, it is possible to determine the coefficient of storage if α is known. Jacob (1941) determined from pumping tests that the modulus of compression (α) of the Lloyd sand member of the Raritan formation on Long Island, N.Y., was 2.1×10^{-6} in per lb. In a location where this sand has a porosity (θ) of 0.30 and is confined to form an aquifer having a thickness (m) of 50 feet the coefficient of storage may be computed as

$$S=(0.0361)(0.30)(50\times 12)\left[(3.3\times 10^{-6})+\frac{2.1\times 10^{-6}}{0.30}\right]$$

$$S=6.7\times 10^{-5}$$

The coefficient of storage is related to barometric and tidal efficiency. As stated previously, Jacob (1940) showed mathematically that the sum of the barometric efficiency and tidal efficiency must equal unity (BE+TE=1). He showed further than when $b=1$ the storage coefficient is related to the barometric efficiency as follows,

$$S=(\gamma_0\theta m\beta)\left[\frac{1}{\mathrm{BE}}\right]$$

using the same terminology as before and assuming no leakage from or into contiguous beds. By observing tidal fluctuations in a well, screened in the Lloyd sand member of the Raritan formation on Long Island, Jacob (1940) could then determine tidal efficiency and compute the coefficient of storage. His computations for well Q–288, near Rockaway Park, where the tidal efficiency was determined as 42 percent, are as follows:

$$\frac{1}{\mathrm{BE}}=\frac{1}{1-(\mathrm{TE})}=\frac{1}{1-0.42}=1.72.$$

Ascribing a thickness of 200 feet and a porosity of 0.35 to the Lloyd sand member in the vicinity of Rockaway Park, Long Island, and substituting in the foregoing equation for storage coefficient, there results—

$$S=(0.0361)(0.35)(200\times 12)\left[\frac{1}{300{,}000}\right](1.72),$$

$$S=1.7\times 10^{-4}.$$

This value for the coefficient of storage is comparable to the values determined from pumping tests.

Jacob (1941) observed that the compressibility of the Lloyd sand member, as computed from a discharging-well test, was about 2½ times that computed from tidal fluctuations and reasoned that this disparity might be due, in part, to the range of stress involved. He states—

> * * * during the pumping test of 1940 the head in the Lloyd sand declined to a new low over a considerable area in the vicinity of the pumped wells, and consequently the stress in the skeleton of the aquifer reached a new high. It is to be expected that the modulus of elasticity would be smaller for the new, higher range of stress than for the old range over which the stress had fluctuated many times.

AQUIFER TESTS—BASIC THEORY

WELL METHODS—POINT SINK OR POINT SOURCE

CONSTANT DISCHARGE OR RECHARGE WITHOUT VERTICAL LEAKAGE

EQUILIBRIUM FORMULA

Wenzel (1942, p. 79–82) showed that the equilibrium formulas used by Slichter (1899), Turneaure and Russell (1901), Israelson (1950), and Wyckoff, Botset, and Muskat (1932) are essentially modified forms of a method developed by Thiem (1906), as are the formulas developed by Dupuit (1848) and Forchheimer (1901). Thiem apparently was the first to use the equilibrium formula for determining permeability and it is frequently associated with his name. The formula was developed by Thiem from Darcy's law and provides a means for determining aquifer transmissibility if the rate of discharge of a pumped well and the drawdown in each of two observation wells at different known distances from the pumped well are known. The Thiem formula, in nondimensional form, can be written as

$$T=\frac{Q\log_e(r_2/r_1)}{2\pi(s_1-s_2)}, \tag{1}$$

where the subscript e in the log term indicates the natural logarithm. In the usual Geological Survey units (see p. 73), and using common logarithms, equation 1 becomes

$$T=\frac{527.7Q\log_{10}(r_2/r_1)}{s_1-s_2}, \tag{2}$$

where

T=coefficient of transmissibility, in gallons per day per foot,
Q=rate of discharge of the pumped well, in gallons per minute,
r_1 and r_2=distances from the pumped well to the first and second observation wells, in feet, and
s_1 and s_2=drawdowns in the first and second observation wells, in feet.

The derivation of the formula is based on the following assumptions: (*a*) the aquifer is homogeneous, isotropic, and of infinite areal extent; (*b*) the discharging well penetrates and receives water from the entire thickness of the aquifer; (*c*) the coefficient of transmissibility is constant at all times and at all places; (*d*) pumping has continued at a