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Chapter A2

DOCUMENTATION OF A COMPUTER PROGRAM TO SIMULATE AQUIFER-SYSTEM COMPACTION USING THE MODULAR FINITE-DIFFERENCE GROUND-WATER FLOW MODEL

By S.A. Leake and D.E. Prudic

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Chapter A2

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PREFACE

The series of manuals on techniques describes procedures for planning and executing specialized work in water-resources investigations. The material is grouped under major subject headings called books and further subdivided into sections and chapters. Section A of Book 6 is ground-water modeling.

This report presents a computer program for simulating aquifer-system compaction resulting from ground-water storage changes in compressible beds. The program documented in this report is designed for incorporation into the modular finite-difference ground-water flow model developed by the U.S. Geological Survey. The modular model is documented in Techniques of Water-Resources Investigations Book 6, Chapter A1.

The performance of this computer program has been tested in models of both hypothetical and actual ground-water flow systems. Future applications or advancement in knowledge, equipment, or techniques may require that this program be updated. Users may inquire about the availability of updates to this computer program (Interbed-Storage Package) by writing to:

U.S. Geological Survey
375 South Euclid
Tucson, Arizona 85719

Copies of the computer program and test data sets on magnetic tape or diskette are available at cost of processing from:

U.S. Geological Survey
National Water Information System
437 National Center
Reston, VA 22092
Telephone: (703) 648-5695

TECHNIQUES OF WATER-RESOURCES INVESTIGATIONS OF THE UNITED STATES GEOLOGICAL SURVEY

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FIGURES

TABLE

METRIC CONVERSION FACTORS

For readers who prefer to use inch-pound units, conversion factors for the metric (International System) units in this report are listed below:

Multiply metric unit	*By*	*To obtain inch-pound unit*
meter (m)	3.281	foot (ft)
square meter (m^2)	10.76	square foot (ft^2)
cubic meter (m^3)	35.31	cubic foot (ft^3)
per meter (m^{-1})	0.3048	per foot (ft^{-1})
square meter per day (m^2/d)	10.76	square foot per day (ft^2/d)
meter per year (m/yr)	3.281	foot per year (ft/yr)

DOCUMENTATION OF A COMPUTER PROGRAM TO SIMULATE AQUIFER-SYSTEM COMPACTION USING THE MODULAR FINITE-DIFFERENCE GROUND-WATER FLOW MODEL

By S.A. Leake and D.E. Prudic

Abstract

Removal of ground water by pumping from aquifers may result in compaction of compressible fine-grained beds that are within or adjacent to the aquifers. Compaction of the sediments and resulting land subsidence may be permanent if the head declines result in vertical stresses beyond the previous maximum stress. The process of permanent compaction is not routinely included in simulations of ground-water flow. To simulate storage changes from both elastic and inelastic compaction, a computer program was written for use with the U.S. Geological Survey modular finite-difference ground-water flow model. The new program, the Interbed-Storage Package, is designed to be incorporated into this model.

In the Interbed-Storage Package, elastic compaction or expansion is assumed to be proportional to change in head. The constant of proportionality is the product of the skeletal component of elastic specific storage and the thickness of the sediments. Similarly, inelastic compaction is assumed to be proportional to decline in head. The constant of proportionality is the product of the skeletal component of inelastic specific storage and the thickness of the sediments. Storage changes are incorporated into the ground-water flow model by adding an additional term to the right-hand side of the flow equation. Within a model time step, the package appropriately apportions storage changes between elastic and inelastic components on the basis of the relation of simulated head to the previous minimum (preconsolidation) head.

Two tests were performed to verify that the package works correctly. The first test compared model-calculated storage and compaction changes to hand-calculated values for a three-dimensional simulation. Model and hand-calculated values were essentially equal. The second test was performed to compare the results of the Interbed-Storage Package with results of the one-dimensional Helm compaction model. This test problem simulated compaction in doubly draining confining beds stressed by head changes in adjacent aquifers. The Interbed-Storage Package and the Helm model computed essentially equal values of compaction.

Documentation of the Interbed-Storage Package includes data input instructions, flow charts, narratives, and listings for each of the five modules included in the package. The documentation also includes an appendix describing input instructions and a listing of a computer program for time-variant specified-head boundaries. That package was developed to reduce the amount of data input and output associated with one of the Interbed-Storage Package test problems.

Introduction

Ground water is removed from storage when water is pumped from aquifers. Jacob (1940) postulated that stored water in confined aquifers is derived from the expansion of water, the compression of the aquifer material, and compression of the clayey beds that are adjacent to and within the aquifer. He further concluded that the principal source of stored water probably is from the compression of the clayey beds. Compression or compaction of the sediments is elastic if the lowering of fluid (pore) pressures does not result in permanent rearrangement of the skeletal structure of the sediments and if water removed from storage can be replaced when fluid pressures increase. However, if the fluid pressures decrease beyond the interval where the sediments compact elastically, additional water is released from the clayey beds as the skeletal structure is rearranged and permanently compacted. This process is referred to as permanent or inelastic compaction. Water removed from storage by permanent compaction cannot be returned after pumping decreases or ceases. Thus, water released during permanent compaction can be considered a one-time source that cannot be replaced.

Concentrated pumping of large quantities of ground water (mainly from unconsolidated deposits) in many areas has resulted in the permanent compaction of fine-grained sediments and a consequent lowering of land surface (referred to as land subsidence). Land subsidence caused by pumping of ground water has been identified in seven states in the Nation (Poland, 1984, p. 6).

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The process of permanent compaction is not routinely incorporated in ground-water flow simulations. Because of the need to simulate permanent compaction, particularly in areas of concentrated pumping in thick unconsolidated deposits, a subprogram was written to be incorporated into a computer program that simulates three-dimensional ground-water flow (McDonald and Harbaugh, 1988). This computer program was designed to allow for additional capabilities to be easily incorporated.

Purpose and Scope

This report documents a method for the simulation of changes in ground-water storage caused by compressible interbeds in an aquifer system. A package consisting of five subroutines or modules has been incorporated into the computer program of McDonald and Harbaugh (1988). The package is the Interbed-Storage Package. In addition to accounting for changes in storage, the package also calculates net compaction or elastic expansion of interbeds in individual model layers and sums those values to calculate a change in the land surface. The documentation includes a description of how the model computes permanent compaction as well as recoverable compaction of sediments. The documentation also includes data-input instructions and, for each module, includes a narrative, a flow chart, a program listing, and a description of variables. Also included are data-input instructions and a program listing for an additional package, the Time-Variant Specified-Head Package, which was used in a simulation to test the Interbed-Storage Package.

Interbeds

The term "interbed" is used in this report to denote a poorly permeable bed within a relatively permeable aquifer (fig. 1). Such interbeds are assumed to be (1) of significantly lower hydraulic conductivity than the surrounding sediments considered to be aquifer material yet porous and permeable enough to accept or release water in response to head changes in adjacent aquifer material, (2) of insufficient lateral extent to be considered a confining bed (or confining unit) that separates adjacent aquifers, and (3) of relatively small thickness in comparison to lateral extent. The interbeds are also assumed to consist primarily of highly compressible clay and silt beds from which water flows vertically to adjacent coarse-grained beds.

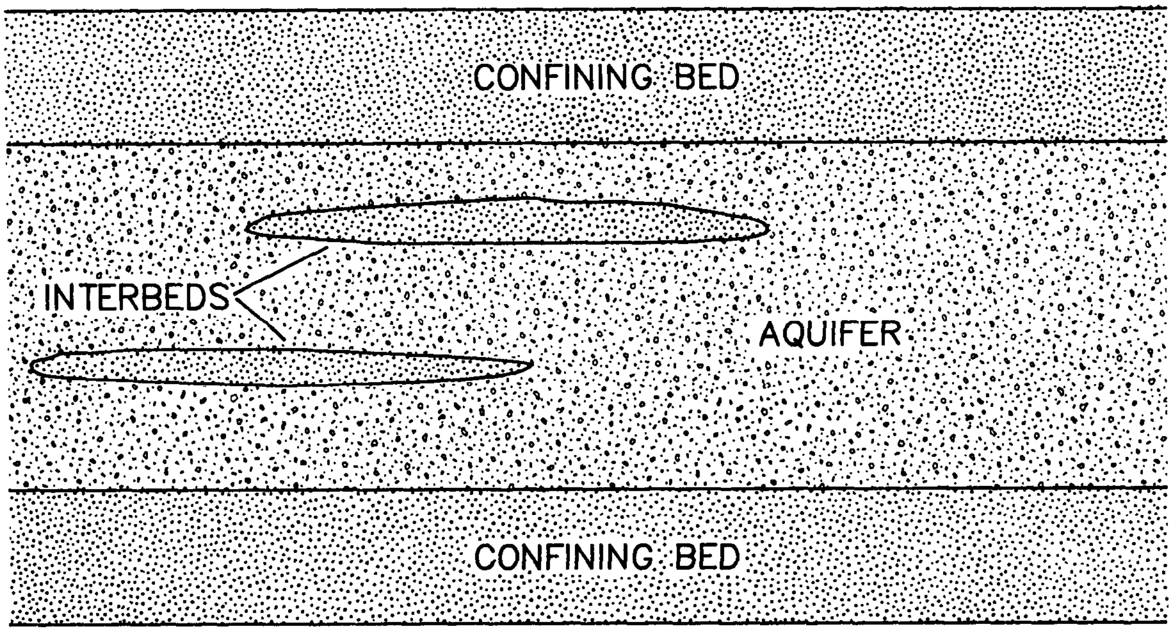

Figure 1.—Types of fine-grained beds in or adjacent to aquifers. Beds may be discontinuous interbeds or continuous confining beds.

Basic Model of Interbed Compaction

Relation of Ground-Water Head Change and Sediment Compaction

The relation between ground-water head change and sediment compaction is based on the principle of effective stress developed by Terzaghi (1925), where effective stress (p') is expressed as the difference between total stress (p), which is the total overburden load or geostatic pressure, and fluid or more pressure (u):

$$p' = p - u. \qquad (1)$$

Effective stress, geostatic pressure, and fluid pressure are commonly expressed in units of force per unit area [FL^{-2}] but may also be expressed as a height of a column of water [L]. That unit may be obtained by dividing quantities of force per unit area by the unit weight of water, γ_w [FL^{-3}].

The geostatic pressure or total overburden load of the sediments and water at a given depth equals the product of the unit weight of moist sediments and the thickness of the unsaturated zone plus the product of the unit weight of saturated sediments below the water table and the thickness of the saturated sediments overlying that depth. Because of the dependency of geostatic stress on the position of the water table, a change in effective stress from a given head change generally is different in confined and unconfined aquifers. If the water table is raised or lowered in an unconfined aquifer, the geostatic pressure will change. The resulting change in effective stress in the unconfined aquifer can be expressed as (Poland and Davis, 1969, p. 195):

$$\Delta p' = -\gamma_w (1 - n + n_w) \Delta wt, \qquad (2)$$

where

Δp is change in effective stress, positive for increase and negative for decrease [FL^{-2}];
n is porosity [dimensionless];
n_w is moisture content above water table as a fraction of total volume [dimensionless]; and
Δwt is change in water table, positive for raising and negative for lowering of the water table [L].

The lowering of head in a confined aquifer however does not change the geostatic pressure, assuming the overlying water table remains constant, but results in an increase of equal amount in effective stress. The increase in effective stress for a given change in head can be expressed as (Poland and Davis, 1969, p. 195):

$$\Delta p' = -\gamma_w \Delta h, \qquad (3)$$

where

Δh is change in head in a confined aquifer, positive for increase and negative for decrease in head [L].

The change in effective stress is less for an unconfined aquifer than for a confined aquifer by the relation ($1-n+n_w$). For example, if n=0.3 and n_w=0.1, the lowering of the water table increases effective stress (p') by a fraction of 0.8 of the corresponding value for an equal lowering of the head in a confined aquifer. This example assumes that the water table that overlies the confined aquifer remains constant. The maximum increase in effective stress for a confined aquifer occurs when the water table rises in an overlying unconfined aquifer while the head declines in a confined aquifer (Lofgren, 1968, p. 225).

Previous studies (Riley, 1969; Helm, 1975) have indicated that elastic compaction or expansion of sediments is proportional or nearly proportional to change in effective stress. That relation can be expressed as

$$\Delta b = \frac{\Delta p'}{\gamma_w} S_{ske} b_o, \qquad (4)$$

where

Δb is change in thickness, positive for compaction and negative for expansion [L];
S_{ske} is skeletal component of elastic specific storage [L^{-1}]; and
b_o is the thickness of the interbed [L].

The product of elastic skeletal specific storage and thickness is the skeletal component of elastic storage coefficient, S_{ke} [dimensionless]. For sediments in confined aquifers in which geostatic pressure is constant, the relation between change in head and change in thickness is

$$\Delta b = -\Delta h S_{ske} b_o. \qquad (5)$$

Equations 4 and 5 generally are applicable to both fine- and coarse-grained sediments. Laboratory consolidation tests and field data indicate that when compressible fine-grained sediments are stressed beyond a previous maximum stress, compaction is permanent (inelastic) and nonrecoverable (Jorgenson, 1980, p. 20; Riley, 1969). Compaction per unit increase in effective stress in the inelastic range is considerably greater than in the elastic range. When effective stress of sediments compacting in the inelastic range is reduced, the sediments again expand and compact with the elastic characteristics until effective stress increases beyond the new maximum effective stress.

Approximate inelastic compaction can be related to increase in effective stress with an expression analogous to equation 4:

$$\Delta b^* = \frac{\Delta p'}{\gamma_w} S_{skv} b_o, \qquad (6)$$

where

Δb^* is approximate inelastic compaction [L] and

S_{skv} is the skeletal component of inelastic specific storage [L^{-1}].

For confined aquifers in which geostatic pressure is constant, the expression analogous to equation 5 is

$$\Delta b^* = \Delta h S_{skv} b_o. \qquad (7)$$

Expressions similar to equations 4 and 6 were used by Helm (1975) in development of a one-dimensional compaction model. Meyer and Carr (1979) and Williamson and others (1989) also used analogous relations in regional subsidence models of the Houston area, Texas, and the Central Valley, California, respectively. According to laboratory consolidation tests, inelastic compaction is more nearly proportional to increase in log of effective stress (Jorgenson, 1980, p. 20-21). The validity of assuming that inelastic compaction is proportional to increase in effective stress is discussed in the section entitled "Assumptions and Limitations."

Incorporating Storage Changes into Ground-Water Flow Equation

Storage changes in a saturated aquifer system under conditions of decreasing water levels result from three primary processes: (1) the draining of pore spaces as the water table declines; (2) the compression of the aquifer skeleton caused by increasing effective stresses in the aquifer below the water table; and (3) the expansion of water caused by decreasing fluid pressures. Compression of the aquifer skeleton is elastic if it does not result in a permanent rearrangement of the individual grains. Water removed from storage by this process can be replaced when heads in the aquifer increase. Conversely, compression of the aquifer skeleton is inelastic if it results in a permanent rearrangement of the grains. Water removed from storage by inelastic compaction cannot be replaced when heads in the aquifer increase. In general, layers of compressible clayey beds are the most susceptible to inelastic compaction. Storage changes caused by elastic compression of the aquifer skeleton and expansion of the water are much smaller than those caused by the draining of the pore spaces or by inelastic compaction.

The ground-water flow model of McDonald and Harbaugh (1988) is designed to account for changes in storage caused by water-table fluctuations, elastic compression of the aquifer skeleton, and expansion of water. Their model can be used to simulate inelastic compaction but considerable effort would be required to select and manually specify the proper storage values for given periods of time. The Interbed-Storage Package adds the capability of accounting for storage changes from inelastic compaction of interbeds within an aquifer and the resultant land subsidence without having to manually change the storage value during the model simulation.

Equations that describe ground-water flow include terms that account for flow into or out of storage in response to head changes. The general form of the flow equation used by McDonald and Harbaugh (1988) is

$$\frac{\partial}{\partial x}\left(K_{xx}\frac{\partial h}{\partial x}\right)+\frac{\partial}{\partial y}\left(K_{yy}\frac{\partial h}{\partial y}\right)+\frac{\partial}{\partial z}\left(K_{zz}\frac{\partial h}{\partial z}\right)-W=S_s\frac{\partial h}{\partial t}, \qquad (8)$$

where

x, y, z are cartesian coordinates, aligned along the major axes of the hydraulic conductivity tensor [L];

K_{xx}, K_{yy}, K_{zz} are principal components of the hydraulic conductivity tensor [LT^{-1}];

h is hydraulic head [L];

W is volumetric flux per unit volume of sources and (or) sinks of water [T^{-1}];

S_s is specific storage of aquifer material [L^{-1}]; and

t is time [T].

The term on the right-hand side of this equation describes the flow rate of water into storage in the aquifer per unit volume. The storage term can vary over the modeled area depending on the storage properties of the aquifer materials but the values are assumed constant in time, except for the case where a confined aquifer becomes water table or vice versa (McDonald and Harbaugh, 1988, chap. 5, p. 26). To account for changes in storage caused by compaction of fine-grained interbeds, an additional term has been added to the ground-water flow equation. Assuming that changes in head result in equal but opposite changes in effective stress, the term can be expressed as

$$q_i = S'_{sk}\frac{\partial h}{\partial t}, \qquad (9)$$

where

q_i is rate of flow per unit volume of water flowing into storage in compressible interbeds [T^{-1}] and

S'_{sk} is skeletal component of specific storage of interbeds, a function of previous maximum effective stress [L^{-1}].

Equation 9 represents a volume average flow into or out of storage per unit volume of interbeds.

The skeletal specific-storage value in equation 9 varies between an elastic and an inelastic value depending on the relation of the head in the cell to the preconsolidation head. The preconsolidation head corresponds to the previous maximum effective stress (previous lowest head). The term S'_{sk} is the elastic skeletal specific storage whenever the head in a cell is greater than the preconsolidation head. The term is the inelastic skeletal specific storage whenever the head in a cell is less than the preconsolidation head. The preconsolidation head can also change during the simulation as it is assigned the most recent lowest head value.

For an aquifer with multiple interbeds of differing specific-storage values, a single elastic and a single inelastic skeletal specific-storage value can be determined to account for changes in storage in all the interbeds. For a system with n interbeds with specific-storage values S_{s1}, S_{s2}, ... S_{sn} and with thicknesses b_1, b_2, ... b_n, a single equivalent specific-storage value, $S_{s\,system}$, is given by Jorgenson (1980, equation 69):

$$S_{s\,system} = \frac{S_{s1}b_1 + S_{s2}b_2 + \ldots S_{sn}b_n}{b_1 + b_2 + \ldots b_n}. \qquad (10)$$

Individual thicknesses and elastic skeletal specific-storage values of the interbeds can be combined by equation 10 to compute a single elastic specific-storage value for the S'_{sk} function. Similarly, the thicknesses and inelastic specific-storage values can be combined by equation 10 to compute a single inelastic specific-storage value.

Storage changes related to the compressibility of water in interbeds are not included in the q_i function. The changes are small in relation to changes resulting from inelastic skeletal compressibility and generally can be neglected without significantly affecting estimates of ground-water flow and subsidence. Storage changes from compression or expansion of water in interbeds, however, can be accounted for in the model. To do so, the product of the component of specific storage from compressibility of water, S_{sw}, and thickness of the interbeds is added to the storage coefficient read into the Block-Centered Flow Package (McDonald and Harbaugh, 1988).

The computer program of McDonald and Harbaugh (1988) requires specification of dimensionless storage-coefficient values for each layer. The value for a confined layer is the specific storage assigned to a model cell multiplied by the thickness of the cell. The storage coefficient is then multiplied by the cell area in the program to create a storage capacity. The storage-capacity value for each cell is used in the basic finite-difference equation for ground-water flow. The program described in this report also requires specification of the storage coefficient. Storage coefficients for individual layers of sediments can be combined into an equivalent value in a manner analogous to combining specific-storage values in equation 10. The expression for computing the equivalent storage coefficient, S_{system}, is given by Jorgenson (1980, equation 67) as

$$S_{system} = S_{s1}b_1 + S_{s2}b_2 + \ldots S_{sn}b_n. \qquad (11)$$

The skeletal storage coefficients (elastic and inelastic) are used to estimate elastic and inelastic components of subsidence in addition to the flow of water into and out of storage. The dimensionless storage coefficient is the constant of proportionality between compaction and head change (equations 5, 7). This proportionality is used in the calculation of storage changes and compaction in the computer program.

Regional subsidence models by Meyer and Carr (1979) and Williamson and others (1989) used approaches similar to that outlined in equation 9 to account for storage changes in compressible sediments. In approximating the storage term in the finite-difference equations, they computed the storage function explicitly from the relation of head to preconsolidation head at the end of a time step in a manner analogous to

$$q_i^m = \frac{S_{sk}^m}{\Delta t}\left(h^m - h^{m-1}\right) \tag{12}$$

$$S_{sk}^m = \begin{cases} S_{ske}, h^{m-1} > H^{m-1} \\ S_{skv}, h^{m-1} \le H^{m-1} \end{cases},$$

where

q_i^m is rate of flow per unit volume of water flowing into storage in compressible interbeds during time step m [T^{-1}];

S_{sk}^m is specific storage for time step m [L^{-1}];

Δt is length of time step [T];

h^m is head at finite-difference node at end of time step m (h^0 is starting head [L]), and

H^m is preconsolidation head at node at end of time step m (H^0 is starting preconsolidation head [L]).

In this explicit method, if the computed head is less than the preconsolidation head at the end of the step, an inelastic storage value is selected for the next time step and the preconsolidation head value is reset to the new lowest head. Similarly, if computed head at the end of a time step is greater than the preconsolidation head, an elastic storage value would be selected for the next time step.

The explicit method is relatively easy to formulate in a ground-water model program but may lead to significant errors unless time steps are small. The problem arises from the large differences between the elastic and inelastic storage values. When computed head values are in the elastic range but are declining toward the preconsolidation head, storage values are relatively small and head-decline rates may be relatively large. Consequently, the computed head values may overshoot the preconsolidation head value by a significant amount. This overshoot causes the new preconsolidation head value to be set too low and may result in incorrect calculations of water released from compressible sediments. The problem is most significant in systems with cyclical head fluctuations that result in repeated switching between the elastic and inelastic regimes. In recognition of this problem, Williamson and others (1989) made the model time steps as small as was feasible with the available computing resources.

For the Interbed-Storage Package, a more complex but more accurate method of approximating q_i is used. The method implicitly selects the appropriate specific storage at the end of a time step and apportions storage changes to either elastic, inelastic, or both components within a time step. The finite-difference expression of the implicit method is

$$q_i^m = \frac{S_{sk}^m}{\Delta t}\left(h^m - H^{m-1}\right) + \frac{S_{ske}}{\Delta t}\left(H^{m-1} - h^{m-1}\right), \tag{13}$$

$$S_{sk}^m = \begin{cases} S_{ske}, h^m > H^{m-1} \\ S_{skv}, h^m \le H^{m-1} \end{cases}.$$

With this formulation, time steps need not be particularly small. The correct value of specific storage at the end of a time step is easily determined within the iterations of solution schemes provided in the model by McDonald and Harbaugh (1988).

Compaction during a time step can be computed by multiplying equation 13 by the length of the time step, Δt, and by interbed thickness, b_0. That rearrangement of equation 13 results in an expression for total compaction that is equivalent to the sum of expressions for elastic and inelastic compaction in equations 5 and 7.

In addition to using the preceding formulation to simulate release of water and compaction of thin interbeds, it could be used to simulate compaction of thicker confining beds (fig. 1). That simulation could be done by representing the confining layer with a number of adjacent finite-difference layers, each with the formulation for the nonlinear q_i function. The degree to which the vertical head distribution can be approximated is dependent on the number of finite-difference layers used to simulate the confining layer. Several finite-difference layers may be required to simulate the vertical head distribution. Although this approach is valid, it does not make efficient use of computer resources because the model would solve for horizontal components of flow and store horizontal-conductance values for layers in which flow is dominantly vertical.

Assumptions and Limitations

The Interbed-Storage Package does not consider changes in effective stress caused by changes in geostatic pressure but assumes that changes in effective stress are a function only of the head changes. Permanent compaction may be overestimated for an unconfined aquifer unless estimates of inelastic storage coefficients are decreased in proportion to the amount that effective stress would be overestimated. However, the specific yield in an unconfined aquifer generally is greater than the inelastic storage coefficient unless the total thickness of the interbeds exceeds 300 meters. In this case it may be better to model the thick unconfined aquifer with more than one model layer. In a confined aquifer overlain by an unconfined aquifer, the method will overestimate effective stress if the water table declines in the unconfined aquifer or underestimate effective stress if the water table increases in the unconfined aquifer. The error in the estimate of effective stress depends on the amount of change in the geostatic pressure.

In adding the function q_i to the ground-water flow equation, the assumption is made that head changes within a model time step in aquifer material also take place throughout fine-grained interbeds. If time steps are not large enough to allow for drainage of all excess pore pressure in the interbeds, the flow from the interbeds will be unrealistically large for the time step. A measure of time required for excess pore pressure to dissipate in a doubly draining interbed following an instantaneous step load is given by Riley (1969, equation 1):

$$\tau = \frac{S_s (b_0/2)^2}{K}, \tag{14}$$

where

τ is time constant of interbed [T];
S_s is specific storage of interbed [L^{-1}];
b_0 is thickness of interbed [L], and
K is hydraulic conductivity of interbed [LT^{-1}].

The time constant τ is the time required for 93 percent of excess pore pressure to dissipate following an instantaneous change in head at the boundaries of the interbed. The variable S_s in equation 14 includes components that account for the compressibility of the sediment skeleton and of water; however, the inelastic skeletal specific storage, S_{skv}, may be substituted into the expression to obtain an estimate of the upper limit of τ. The resulting time constant will apply when the entire layer is compacting inelastically. For that situation, the component of specific storage that accounts for the compressibility of water is negligible. The upper limit of τ can be compared with the length of model time steps to evaluate the validity of the assumption that all excess pore pressure is dissipated in a time step. If this assumption is not valid, the model results may overestimate storage changes and compaction in early time and underestimate those quantities in later time.

Another assumption mentioned previously is that inelastic compaction is proportional to change in effective stress. Laboratory consolidation tests indicate that inelastic compaction of many fine-grained sediments is proportional to the increase in logarithm of effective stress. A relation of compaction to increase in logarithm of effective stress can be derived from equations 45a and 50 in Jorgenson (1980). That relation is

$$\Delta b' = b_0 C_c \frac{(\Delta \log_{10} p')}{(1+e_0)}, \tag{15}$$

where

$\Delta b'$ is inelastic compaction [L];
b_0 is initial thickness of interbeds [L];
C_c is compression index [dimensionless]; and
e_0 is initial void ratio [dimensionless].

The skeletal component of inelastic specific storage is not a constant but rather is a function of effective stress. Jorgenson (1980, equation 59) relates the two quantities by

$$S_{skv} = \frac{0.434 C_c \gamma_w}{p'(1+e_0)}. \tag{16}$$

The error in assuming that S_{skv} is a constant can be expressed as

$$\text{error} = 100 \frac{(\Delta b^* - \Delta b')}{\Delta b'}, \tag{17}$$

where

error is the percentage by which compaction will be overestimated by assuming S_{skv} is constant [dimensionless].

If S_{skv} is assumed to be a constant selected on the basis of the initial state of effective stress, p'_0, equations 6, 15, 16, and 17 can be combined to express the percentage error in computed compaction:

$$\text{error} = 100\left(\frac{0.434\ \Delta p'}{p'_0 \Delta \log_{10} p'} - 1\right). \tag{18}$$

This relation indicates that the percentage error in computed compaction is less than one-half of the percentage increase in effective stress (fig. 2). The analysis points out the merit of application of equation 1 to estimate initial effective stress and changes in effective stress from changes in pore pressure or head. For sediments relatively deep below the land surface, a given decline in head will result in a smaller percentage increase in effective stress than for shallower sediments.

For many ground-water flow systems, increases in effective stress are a relatively small percentage of the initial state of stress. Additionally, errors in compaction computed using equation 7 can be minimized by selecting the constant S_{skv} on the basis of an effective stress in the center of the range of stress change, rather than at the beginning. Alternatively, specific-storage values used in the package can be changed with time, if necessary, by restarting a simulation with new storage values.

Applicability of the Interbed-Storage Package

The Interbed-Storage Package is designed for simulation of ground-water flow in aquifers where permanent compaction of compressible interbeds may be a significant source of pumped water. The amount of permanent compaction of compressible interbeds depends on a number of factors including the total stress borne by the system (preconsolidation stress); variations in the amount, compressibility, and bedding of clayey or silty interbeds; and variations in the applied stress that tends to compact the interbeds. The package is also designed to estimate the amount of land subsidence caused by compaction. In general, the package would not be applied for flow systems in which water-level declines do not cause the preconsolidation stress to be exceeded, unless there is interest in calculating elastic compaction and (or) rebound.

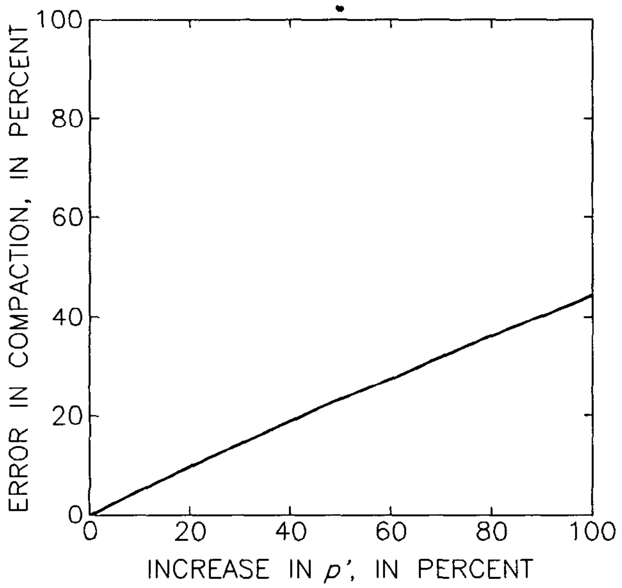

Figure 2.—Relation between increase in effective stress, p', and error in computed compaction. Error is the amount by which computed compaction will be overestimated if inelastic skeletal specific storage is assumed to be a constant selected on the basis of the initial state of effective stress.

Holzer (1981, p. 693) estimated the preconsolidation stress in four areas where permanent compaction was caused by pumping of ground water. The estimated stresses are equivalent to water-level declines of 16 to 63 m below the predevelopment water level. Bull (1975) noted that the interlayering of numerous thin-bedded compressible clayey sediments with coarse-grained sediments resulted in a system that compacted more rapidly than one with fewer but thicker beds of clayey sediments. The amount of compaction also is dependent on the type of clay minerals present. Meade (1967) noted that montmorillonite is more susceptible to compact than either illite or kaolinite and was the predominant clay mineral in the major subsiding areas in the San Joaquin Valley of California.

As ground-water reservoirs in the United States and elsewhere continue to be developed, permanent compaction of sediments will undoubtedly become an increasing problem. Poland (1981, p. 116) identified 17 areas in the United States where ground-water withdrawal has resulted in permanent compaction of sediments and accompanying land subsidence. In several alluvial basins in California and Arizona and in the coastal plain around Houston, Texas, compaction has resulted in more than 3 m of land subsidence. Subsequent investigation by Chi and Reilinger (1984, p. 155) identified more than 30 new areas in the United States where land subsidence may have been caused by ground-water pumping. Their identifications were made on the basis of repeated level surveys and ground-water pumping histories. Many of the new areas identified are along the Mississippi Valley. They also suggested that land subsidence may have occurred in additional ground-water basins but remains undetected because of a lack of repeated levelings.

Test Problems

Storage-Depletion Test Problem

A simulation was run to test the results of the Interbed-Storage Package given a specified uniform decline in head. The finite-difference grid consists of two layers with 10 rows and 12 columns of cells. The first and last column of cells in both layers are constant-head boundaries, so that the other 10 active columns of cells are a flow region subject to head and storage changes. The grid dimensions for each cell are 1,000 m along both rows and columns. Each layer has a transmissivity of 1,000 m^2/d and a storage coefficient of 0.0001. The starting head in the active area specifies a uniform gradient of 0.001 along the rows. The starting head values in the constant-head columns also specify a gradient of 0.001 along the rows over the entire grid but at a level exactly 10 m lower than the starting heads in the active area of the model. Thus, if the transient solution is allowed to run until steady-state conditions are reached, the head at each interior cell will be exactly 10 m below the corresponding starting head.

For this test problem, compressible interbeds are assumed to occur in the aquifer represented by the upper layer. The preconsolidation head is assumed to be 5 m below the starting head at each cell. The sum of the products of thickness and elastic skeletal specific storage for interbeds in the upper layer is assumed to result in an elastic storage coefficient of 0.0001 at each cell. Similarly, the inelastic storage coefficient is assumed to be 0.001 at each cell. Data sets for this test problem, referred to as the storage-depletion test problem, including input for all model packages, are given in Appendix A.

Within 1,000 days of simulation time, head values at all cells will be within about 0.001 m of the final steady-state value. The volumetric budget for the simulation is shown in table 1. The correct volume of water released by the interbeds can be computed as the product of total area excluding boundary cells (1x10^8 m^2), storage coefficient (0.0001 for elastic and 0.001 for inelastic), and head decline (5 m under elastic conditions and 5 m under inelastic conditions). The correct amount of water released from interbed storage therefore is 5x10^4 m^3 for declines in the elastic range and 5x10^5 m^3 for declines in the inelastic range. The sum of the two values is consistent with the 5.5x10^5 m^3 release of water from interbed storage calculated by the model (table 1). For this test problem, the model computes 5.5x10^{-3} m of compaction. That value is consistent with the sum of elastic and inelastic compaction computed from equations 5 and 7.

Ramp-Load Test Problem

Another more complex test problem involves comparing the results of the Interbed-Storage Package with those of one-dimensional compaction model COMPAC1 (Helm, 1975, 1984). The problem is similar to the non-declining ramp-load problem given by Helm (1975, p. 470-473). Model COMPAC1 computes compaction of compress-

Table 1.—*Volumetric budget for storage-depletion test problem at end of time step 10 in stress period 3*

CUMULATIVE VOLUMES L**3	RATES FOR THIS TIME STEP L**3/T
IN:	
STORAGE = 0.20000E+06	STORAGE = 0.00000
CONSTANT HEAD = 0.59683E+08	CONSTANT HEAD = 20000.
INTERBED STORAGE = 0.55000E+06	INTERBED STORAGE = 0.00000
TOTAL IN = 0.60433E+08	TOTAL IN = 20000.
OUT:	
STORAGE = 0.25808E-01	STORAGE = 0.00000
CONSTANT HEAD = 0.60433E+08	CONSTANT HEAD = 20000.
INTERBED STORAGE = 0.43711E-01	INTERBED STORAGE = 0.00000
TOTAL OUT = 0.60433E+08	TOTAL OUT = 20000.
IN - OUT = -104.00	IN - OUT = -0.35156E-01
PERCENT DISCREPANCY = 0.00	PERCENT DISCREPANCY = 0.00

ible beds given effective stress changes at the boundary by solving a one-dimensional partial-differential equation by finite differences. The ramp-load test problem presented here involves cumulative compaction of 100 identical interbeds with the following properties:

Property	Value
Thickness (m)	10
Hydraulic conductivity (m/yr)	$1\text{x}10^{-3}$
Elastic specific storage (m^{-1})	$1\text{x}10^{-6}$
Inelastic specific storage (m^{-1})	$1\text{x}10^{-4}$

With these properties, the elastic and inelastic time constants are 0.025 and 2.5 years, respectively. The head at the boundaries of the interbeds is set to successively decline linearly for 180 days to 10 m below the starting value. Following the declines, the head recovers linearly for 180 days to the original value. Successive cycles of declines and recoveries are approximated with 180 1-day steps in each ramp (fig. 3*A*). The preconsolidation head throughout the interbeds is assumed to be equal to the starting head so that compaction is initially in the inelastic range.

The problem can be solved by COMPAC1 and by the Interbed-Storage Package by simulating compaction in one-half of a single doubly draining bed and multiplying those results by 200 to obtain the total compaction for all 100 interbeds. For model COMPAC1, a grid spacing of 0.2 m was selected, thus requiring 25 finite-difference cells to simulate one-half of a doubly draining bed. To apply the Interbed-Storage Package to a problem of flow and compaction in a confining bed, one approach would be to discretize the confining bed into a number of model layers. Flow in the confining bed is assumed to be one-dimensional; therefore, the problem could also be solved with a single row of finite-difference cells in one model layer. The single-row approach is less awkward than the multiple-layer approach because input to and output from the flow model generally are carried out layer by layer. A disadvantage of orienting the grid along a row is that the Interbed-Storage Package sums compaction in all layers to compute total compaction. The package is incapable of summing compaction along a row to calculate total compaction; however, the total compaction can be obtained by dividing the net cumulative volume of water released from interbed storage by the cross-sectional area of the grid. The finite-difference grid for the Interbed-Storage Package is shown in figure 4. A cross-sectional area of 1 m by 1 m was chosen so that the net volume of water from interbed storage in the volumetric budget of the model would directly relate to total compaction for half of the doubly draining bed. A cell dimension of 0.25 m was selected thereby requiring 20 finite-difference cells to represent one-half of the doubly draining bed. An additional cell was required to impose the specified-head boundary condition of cyclical ramp loading.

Computed interbed-storage values from the volumetric budget were converted to total compaction for all 100 layers for comparison with the results of model COMPAC1 (fig. 3*B*). The results of the two models are virtually the same. The input for all model packages for the ramp-load test problem is given in Appendix B.

In the ground-water model by McDonald and Harbaugh (1988), the only provision for changing specified head at boundary cells is by use of the River Package, Drain Package, or General-Head Boundary Package. Those packages allow specification of a different boundary head at each stress period. If associated river-bed conductances are large enough, the head in the aquifer will almost equal the boundary head. For the ramp-load test problem, difficulties may arise from using these packages to approximate a specified aquifer head. First, the flow from the boundary to the aquifer is computed as the product of conductance and head difference between the boundary and the aquifer. If the intent is to make that difference small, mass-balance errors result because of imprecision in computing the head difference across the boundary. Precision is lost when two nearly equal numbers are subtracted. Second, a large amount of input to and output from the model are associated with each stress period. Simulation of one complete cycle of the loading to the precision shown in figure 3*A* would require 360 stress periods. In an attempt to use the River Package to simulate one complete cycle of loading with 1-day stress periods, more than 28,000 lines of output were generated by the model.

As an alternative way of simulating a specified boundary head, a new model package was developed to simulate variable-head boundaries. The new package is named the Time-Variant Specified-Head Package. It makes use of the capability of the model to incorporate constant-head cells in a simulation. At the start of a simulation, the Time-Variant Specified-Head Package reads in the maximum number of constant-head cells that may be specified for any stress period.

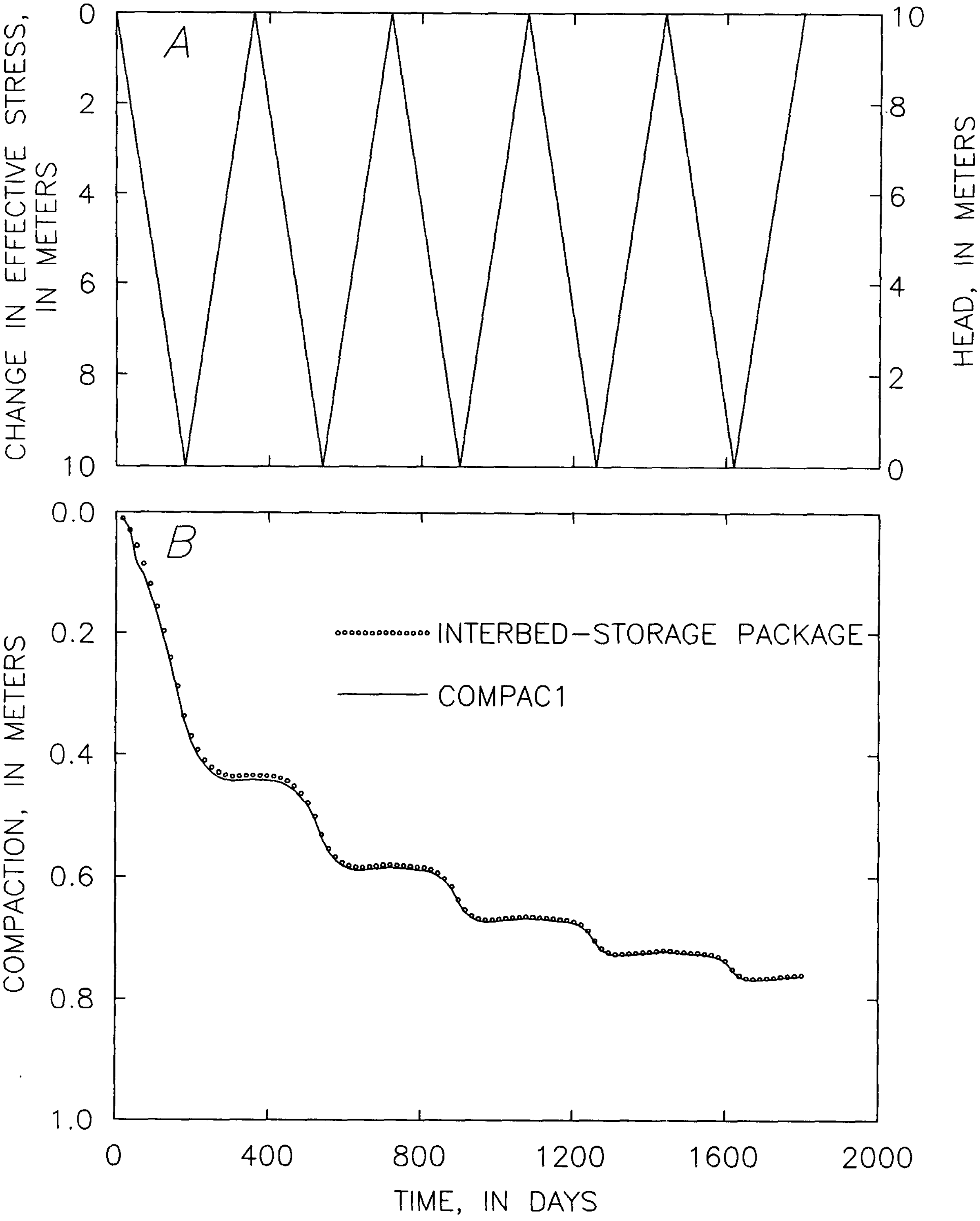

Figure 3.—Stress and compaction for the ramp-load test problem. *A*, Non-declining cyclical ramp load. *B*, Compaction computed by the Interbed-Storage Package and by model COMPAC1 (Helm, 1975, 1984).

For each stress period, the package reads the number of boundary cells at which head will be specified. For each boundary cell, the package reads layer, row, and column indices and corresponding head values at the start and end of the stress period. For each time step within the stress period, the value of each specified head is linearly interpolated from the starting and ending heads of the period and the proportion of elapsed time within the stress period to length of the stress period. The package sets the corresponding elements in the IBOUND array to negative numbers to indicate that the cells are constant-head cells. The Block-Centered Flow Package of the model (McDonald and Harbaugh, 1988) accounts for flow to or from the constant-head cells as though they were specified as such by the Block-Centered Flow Package. The data-input instructions and listings of subroutines of the Time-Variant Specified-Head Package are given in Appendix C.

Implementation of Interbed-Storage Package in the Ground-Water Model

The Interbed-Storage Package is designed for incorporation into the ground-water flow

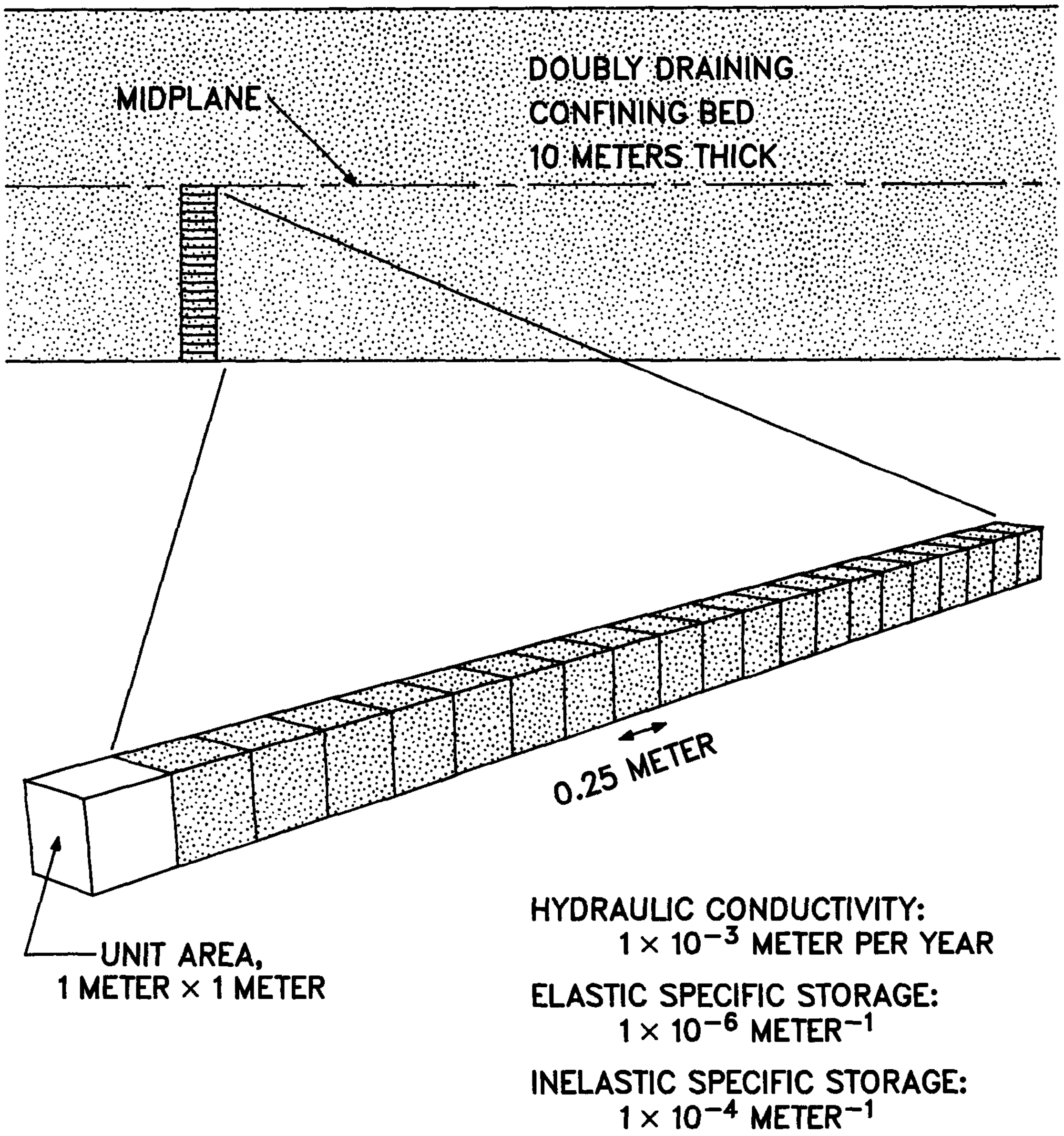

Figure 4.—Finite-difference grid used by the Interbed-Storage Package for the ramp-load test problem.

model by McDonald and Harbaugh (1988). It consists of five FORTRAN subroutines, referred to as modules, that carry out the following procedures: Allocate memory, read and prepare data, formulate finite-difference equations, compute mass-balance components, and output compaction arrays. The modules are named IBS1AL, IBS1RP, IBS1FM, IBS1BD, and IBS1OT, respectively. The first three characters of the names identify the modules as being a part of the Interbed-Storage Package. The next character identifies the version number of the package. The last two characters identify the procedure carried out by the module; for example, "AL" for allocate memory, "RP" for read and prepare data. The procedures used by the Interbed-Storage Package are consistent with procedures used by existing packages in the ground-water flow model (fig. 5). Detailed descriptions of each of the five modules are presented in a following section entitled "Module Documentation for the Interbed-Storage Package."

The main program of the ground-water model must be modified to call the five modules of the Interbed-Storage Package. The procedures on the left side of figure 5 are listed in the order that procedures are carried out in the main program of the model. Calls to modules of the Interbed-Storage Package must be placed in sections of the main program in which the particular procedure is being carried out for other packages. For instance, the IBS1RP module must be called within the section of the main program in which other read and prepare modules such as BAS1RP and BCF1RP are called. The FORTRAN calls to be added to the main program are as shown below. An example of a main program that includes the above calls to the Interbed-Storage Package modules is given in Appendix D.

The package is selected by entering a positive unit number in element 19 of the IUNIT array in the basic package input of the model (McDonald and Harbaugh, 1988, chap. 4, p. 9). If a different element of the IUNIT array is desired for selecting the package, each of the preceding calls to the five modules must be modified so that "19" is changed to the desired element number. The reference to IUNIT(19) occurs in each of the five IF statements as well as within the argument lists of calls to modules IBS1AL, IBS1RP, and IBS1OT.

The Interbed-Storage Package allows a user to select which model layers are to include calculations to account for release of water from compressible interbeds. For each layer with interbed storage, the user must enter several two-dimensional arrays that represent areal distributions of properties within the model layer. The arrays are elastic storage coefficient [dimensionless], inelastic storage coefficient [dimensionless], and initial preconsolidation head [L]. Storage coefficient for a model cell is the sum of the products of specific-storage values and thicknesses of all -compressible interbeds within the cell (equation 11).

```
      IF (IUNIT(19).GT.0) CALL IBS1AL(ISUM,LENX,LCHC,LCSCE,LCSCV,
     1    LCSUB,NCOL,NROW,NLAY,IIBSCB,IIBSOC,ISS,IUNIT(19),IOUT)

      IF(IUNIT(19).GT.0) CALL IBS1RP(X(LCDELR),X(LCDELC),X(LCHNEW),
     1 X(LCHC),X(LCSCE),X(LCSCV),X(LCSUB),NCOL,NROW,NLAY,
     2 NODES,IIBSOC,ISUBFM,ICOMFM,IHCFM,ISUBUN,ICOMUN,IHCUN,
     3 IUNIT(19),IOUT)

      IF(IUNIT(19).GT.0) CALL IBS1FM(X(LCRHS),X(LCHCOF),X(LCHNEW),
     1 X(LCHOLD),X(LCHC),X(LCSCE),X(LCSCV),X(LCIBOU),
     2 NCOL,NROW,NLAY,DELT)

      IF(IUNIT(19).GT.0) CALL IBS1BD(X(LCIBOU),X(LCHNEW),X(LCHOLD),
     1 X(LCHC),X(LCSCE),X(LCSCV),X(LCSUB),X(LCDELR),X(LCDELC),
     2 NCOL,NROW,NLAY,DELT,VBVL,VBNM,MSUM,KKSTP,KKPER,IIBSCB,
     3 ICBCFL,X(LCBUFF),IOUT)

      IF(IUNIT(19).GT.0)CALL IBS1OT(NCOL,NROW,NLAY,PERTIM,TOTIM,KKSTP,
     1 KKPER,NSTP,X(LCBUFF),X(LCSUB),X(LCHC),IIBSOC,ISUBFM,ICOMFM,
     2 IHCFM,ISUBUN,ICOMUN,IHCUN,IUNIT(19),IOUT)
```

PACKAGES

PROCEDURES	BAS	BCF	WEL	RCH	RIV	DRN	EVT	GHB	SIP	SOR	IBS
Define (DF)	BAS1DF										
Allocate (AL)	BAS1AL	BCF1AL	WEL1AL	RCH1AL	RIV1AL	DRN1AL	EVT1AL	GHB1AL	SIP1AL	SOR1AL	IBS1AL
Read & Prepare (RP)	BAS1RP$_U$	BCF1RP$_{US}$							SIP1RP	SOR1RP	IBS1RP$_U$
Stress (ST)	BAS1ST										
Read & Prepare (RP)			WEL1RP	RCH1RP$_U$	RIV1RP	DRN1RP	EVT1RP$_U$	GHB1RP			
Advance (AD)	BAS1AD										
Formulate (FM)	BAS1FM	BCF1FM$_S$	WEL1FM	RCH1FM	RIV1FM	DRN1FM	EVT1FM	GHB1FM			IBS1FM
Approximate (AP)									SIP1AP$_S$	SOR1AP$_S$	
Output Control (OC)	BAS1OC										
Budget (BD)		BCF1BD$_{US}$	WEL1BD$_U$	RCH1BD$_U$	RIV1BD$_U$	DRN1BD$_U$	EVT1BD$_U$	GHB1BD$_U$			IBS1BD$_U$
Output (OT)	BAS1OT$_U$										IBS1OT$_U$

Modified from McDonald and Harbaugh (1988, figure 15)

EXPLANATION

BCF1BD$_{US}$ NAME OF MODULE—Subscript, U, indicates that utility modules are utilized. Subscript, S, indicates that submodules are utilized

Figure 5.—Primary modules of ground-water flow model organized by procedure and package.

The finite-difference equations in the ground-water flow model describe flow rates in terms of volumetric rates [L^3T^{-1}], whereas equation 13 expresses flow into storage as a rate per unit volume [T^{-1}]. To make equation 13 match the dimensionality of the finite-difference equations, specific-storage quantities are replaced by corresponding storage-capacity values in module IBS1RP. Storage capacity is defined by McDonald and Harbaugh (1988, chap. 5, p. 25) as the product of specific storage and volume. The S_{ske} quantity in equation 13 is replaced by elastic storage capacity SCE [L^2], which is the product of S_{ske}, thickness of compacting interbeds, and area of the finite-difference cell. The inelastic storage capacity, SCV [L^2], is computed the same way using S_{skv}. The storage capacity at the end of time step m is expressed as SC^m. The resulting equation is:

$$QI^m = \frac{SC^m}{\Delta t}\left(h^m - H^{m-1}\right) + \frac{SCE}{\Delta t}\left(H^{m-1} - h^{m-1}\right), \quad (19)$$

where

QI^m is rate of flow into storage in compressible interbeds during time step m [L^3T^{-1}].

In this equation, the only unknown head quantity is h^m. The coefficient of this quantity, $SC^m/\Delta t$, must be subtracted from the corresponding element in the array HCOF of the ground-water model. The remaining terms, $-(SC^m/\Delta t)H^{m-1} + (SCE/\Delta t)(H^{m-1}-h^{m-1})$ are added to the corresponding element of the array RHS. In these expressions, the quantity SC^m is unknown because it is a function of h^m. Within each iteration that the model carries out to compute h^m, a value of SC^m is selected in module IBS1FM on the basis of the most recent estimate of h^m.

Interbed-Storage Package output consists of the rate of flow to and from interbed storage during a time step and the accumulated volume to and from interbed storage whenever a volumetric budget is printed for the entire model. Also, at the end of each stress period, the package prints out a single two-dimensional array showing the sum of the compaction of all model layers that have interbed storage. That sum is referred to as SUBSIDENCE in the model output. Optionally, for any particular model time step, the user may print and (or) write to a disk file the sum of compaction for all layers and (or) compaction for each layer that has interbed storage and (or) the current preconsolidation head for each layer that has interbed storage. Additionally, for any time step, a user may write to a disk file an unformatted array containing values of rate of flow from storage for each cell in the finite-difference grid. The disk files of compaction and subsidence are unformatted arrays written with the ground-water model utility subroutine ULASAV (McDonald and Harbaugh, 1988, chap. 14, p. 9-11). The text label written with each array is COMPACTION for individual compaction arrays, SUBSIDENCE for the array that is the sum of compaction for all layers, and CRITICAL HEAD for preconsolidation head arrays. Each of those character strings are left justified in four elements of text arrays, with four characters included in each element. Trailing blanks are included to total 16 characters in each text label. The unformatted cell-by-cell budget array is written with model utility subroutine UBUDSV (McDonald and Harbaugh, 1988, chap. 14, p. 6-8). The text string written out with that array is INTERBED STORAGE.

Input Instructions

Input for the Interbed-Storage Package is read from the unit IUNIT(19), specified in the Basic Package input (McDonald and Harbaugh, 1988, chap. 4, p. 9-11).

FOR EACH SIMULATION

IBS1AL

1. Data: IIBSCB IIBSOC
   Format: I10 I10

2. Data: IBQ(NLAY) (Maximum of 80 layers)
   Format: 40I2

   (If there are 40 or fewer layers, use one record; otherwise, use two records)

IBS1RP

The following four arrays are needed to specify the material properties and initial compaction of model layers having interbed storage as indicated in the IBQ array. The four arrays are first read for the uppermost layer with interbed storage, and then continuing downward to lower layers with interbed storage.

FOR EACH LAYER WITH IBQ CODE GREATER THAN ZERO

3. Data: HC(NCOL,NROW)
   Module: U2DREL

4. Data: Sfe(NCOL,NROW)
   Module: U2DREL

5. Data: Sfv(NCOL,NROW)
   Module: U2DREL

6. Data: COM(NCOL,NROW)
   Module: U2DREL

IF IIBSOC IS GREATER THAN ZERO

7. Data: ISUBFM ICOMFM IHCFM ISUBUN ICOMUN IHCUN
   Format: I10 I10 I10 I10 I10 I10

IBS1OT

FOR EACH TIME STEP
IF IIBSOC IS GREATER THAN ZERO

8. Data: ISUBPR ICOMPR IHCPR ISUBSV ICOMSV IHCSV
   Format: I10 I10 I10 I10 I10 I10

Explanation of Fields Used in Input Instructions

IIBSCB	is a flag and unit number. If IIBSCB > 0, it is the unit number on which cell-by-cell flow terms will be recorded whenever ICBCFL (see McDonald and Harbaugh, 1988, chap. 4, p. 14-15) is set. If IIBSCB ≤ 0, cell-by-cell flow terms will not be recorded.
IIBSOC	is a flag for selecting output control for each time step. If IIBSOC > 0, output control will be read each time step for printing and recording subsidence (total compaction of all layers with interbed storage), and compaction and preconsolidation head for all layers with interbed storage. If IIBSOC ≤ 0, subsidence will be printed at the end of each stress period using format (10G11.4). Compaction and preconsolidation head will not be printed and subsidence, compaction, and preconsolidation head will not be recorded.
IBQ	is an indicator to specify which model layers have interbed storage. If IBQ(K) > 0, model layer K has interbed storage. If IBQ(K) ≤ 0, model layer K does not have interbed storage.
HC	is an array specifying the preconsolidation head or preconsolidation stress in terms of head in the aquifer. Preconsolidation head is the previous minimum head value in the aquifer. For any model cells in which specified HC is greater than the corresponding value of starting head (see McDonald and Harbaugh, 1988, chap. 4, p. 9-12), value of HC will be set to that of starting head.
Sfe	is an array specifying the dimensionless elastic storage factor for interbeds present in model layer. The storage factor may be estimated as the sum of the products of elastic skeletal specific storage and thickness of all interbeds in a model layer.
Sfv	is an array specifying the dimensionless inelastic storage factor for interbeds present in model layer. The storage factor may be estimated as the sum of the products of inelastic skeletal specific storage and thickness of all interbeds in a model layer.
Com	is an array specifying the starting compaction in each layer with interbed storage. Compaction values computed by the package are added to values in this array so that printed or stored values of compaction and land subsidence may include previous components. Values in this array do not affect calculations of storage changes or resulting compaction. For simulations in which output values are to reflect compaction and subsidence since the start of the simulation, enter zero values for all elements of this array.
ISUBFM	is a code for the format in which subsidence will be printed.
ICOMFM	is a code for the format in which compaction will be printed.

IHCFM is a code for the format in which preconsolidation head will be printed. Format codes have the same meaning for subsidence, compaction, and preconsolidation head and are the same as codes specified by McDonald and Harbaugh (1988, chap. 4, p. 14–15) for printing head and drawdown. A positive code selects their wrap format and a negative code selects their strip format. The absolute value of the code specifies the printout format as follows:

0 - (10G11.4)
1 - (11G10.3)
2 - (9G13.6)
3 - (15F7.1)
4 - (15F7.2)
5 - (15F7.3)
6 - (15F7.4)
7 - (20F5.0)
8 - (20F5.1)
9 - (20F5.2)
10 - (20F5.3)
11 - (20F5.4)
12 - (10G11.4)

ISUBUN is the unit number to which subsidence arrays will be written if they are saved on disk.

ICOMUN is the unit number to which compaction arrays will be written if they are saved on disk.

IHCUN is the unit number to which preconsolidation head arrays will be written if they are saved on disk.

ISUBPR is the output flag for subsidence printout.

If ISUBPR ≤ 0, subsidence is not printed.
If ISUBPR > 0, subsidence is printed.

ICOMPR is the output flag for compaction printout.

If ICOMPR ≤ 0, compaction is not printed.
If ICOMPR > 0, compaction is printed for each layer with interbed storage.

IHCPR is the output flag for preconsolidation head printout.

If IHCPR ≤ 0, preconsolidation head is not printed.
If IHCPR > 0, preconsolidation head is printed for each layer with interbed storage.

ISUBSV is the output flag for saving subsidence in an unformatted disk file.

If ISUBSV ≤ 0, subsidence is not saved.
If ISUBSV > 0, subsidence is saved.

ICOMSV is the output flag for saving compaction in an unformatted disk file.

If ICOMSV ≤ 0, compaction is not saved.
If ICOMSV > 0, compaction is saved for each layer with interbed storage.

IHCSV is the output flag for saving preconsolidation head in an unformatted disk file.

If IHCSV ≤ 0, preconsolidation head is not saved.
If IHCSV > 0, preconsolidation head is saved for each layer with interbed storage.

MODULE DOCUMENTATION FOR THE INTERBED-STORAGE PACKAGE

The Interbed-Storage Package (IBS1) consists of five modules, all of which are called by the main program. The modules are

IBS1AL	Allocates space for data arrays.
IBS1RP	Reads all data needed by package and initializes the preconsolidation-head and storage-capacity arrays. Initializes flags for output control.
IBS1FM	Calculates terms to add to right-hand side (RHS) and main diagonal (HCOF).
IBS1BD	Calculates flow rates and accumulated flow volumes from interbed storage. Calculates compaction and updates preconsolidation head each time step.
IBS1OT	Reads flags for printing and storing subsidence, compaction, and preconsolidation head. Prints and stores arrays.