Techniques of Water-Resources Investigations
of the United States Geological Survey

CHAPTER D1

APPLICATION OF SURFACE GEOPHYSICS TO GROUND-WATER INVESTIGATIONS

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BOOK 2
COLLECTION OF ENVIRONMENTAL DATA

Gravimetry

By G. P. Eaton

Gravimetry is the geophysical measurement of the acceleration of gravity and has, as its basis, two well-known laws of elementary physics. The Law of Universal Gravitation states that every particle of matter exerts a force of attraction on every other particle that is directly proportional to the product of their masses and inversely proportional to the square of the distance between them. Thus,

$$F = G\,\frac{m_1 m_2}{r^2} \qquad (1)$$

where G is a constant of proportionality (the gravitational constant), m_1 and m_2 are the particle masses, and r is the distance between the particles. The other law is Newton's second law of motion, which may be stated in the form: when a force is applied to a body, the body experiences an acceleration that is directly proportional to the force and inversely proportional to the body's mass. Thus,

$$a = F/m \qquad (2)$$

where a is the acceleration of the body in the direction in which the force is acting.

Because the Earth is approximately spherical and because the mass of a sphere can be treated as though all of it were concentrated at a point at the center, any object with mass m_o, resting on the Earth's surface, will be attracted to the Earth by a force.

$$F \cong G\,\frac{m_o m_e}{R^2} \qquad (3)$$

where m_e is the mass of the Earth and R, its average radius. This force of attraction between the object and the Earth is the object's weight.

If the object is lifted a short distance above the Earth and allowed to fall, it will do so with a gravitational acceleration,

$$g^* = F/m_o \cong G\,\frac{m_e}{R^2}. \qquad (4)$$

This acceleration is the force per unit mass acting on the object. It is a function of both the mass of the Earth and the distance to its center. The principle is the same, however, when the attracting body is something other than the Earth as a whole, and it is on this principle that gravimetry, as a geophysical method, is based.

In gravimetric studies, the local vertical acceleration of gravity (the standard cgs unit of which is the gal, after Galileo) is measured. A gal is equivalent to an acceleration of 1 cm/sec/sec. Most gravity variations associated with geologic bodies in the outer several miles of the Earth's crust are measured in mgals (milligals). The maximum gravity difference between the Earth's normal field (the main gravity field of the reference spheroid) and that actually observed on the surface and corrected for altitude and latitude is of the order of several hundred mgals. This difference, known as a gravity anomaly, reflects lateral density variations in rocks extending to a depth of several tens of miles.

Two types of instruments are used in making gravity measurements in the field: (1) the gravity pendulum, which operates on the principle that the period of a free-swinging pendulum is inversely proportional to a simple function of gravitational acceleration, and (2) the gravity meter, or gravimeter, which is a highly sensitive spring balance with which differences in acceleration are measured by weighing, at different points, a small internal mass suspended from a spring. Because this mass

* Most textbooks of elementary physics denote acceleration with the symbol a, as in equation 2. It is customary in geophysics, however, to use the symbol g to signify gravitational acceleration.

does not change, differences in its weight at different points on the Earth reflect variations in the acceleration of gravity (eq. 4).

The gravity meter rather than the gravity pendulum is used in exploration geophysics because it is light in weight, easily portable, highly accurate, and rapidly read. The modern gravity meter measures, with great accuracy and precision, differences in gravity between points, but does not measure the absolute value of gravity itself at any point. What is measured is the vertical component of the acceleration of gravity rather than the total vector, which may depart slightly from the vertical. The last point is illustrated in figure 64 and discussed below.

The total gravitational attraction of a body M, at point P, can be calculated by subdividing it into a series of vanishingly small elementary masses (fig. 64). One of these is shown in figure 64 as dm. The summed effect of all of the elemental masses contained within body M represents the total attraction.

The gravitational acceleration due to mass dm measured in the direction of r, is

$$dg_r = G\frac{dm}{r^2}. \qquad (5)$$

An instrument designed to measure the vertical component of this attraction will experience an acceleration, dg_z, that is a function of the angle ϕ

$$dg_z = G\frac{dm}{r^2}\cos\phi. \qquad (6)$$

A summation of dm over the entire body yields the vertical component of the total attraction due to M at point P:

$$g_z = G\int_m \frac{dm}{r^2}\cos\phi. \qquad (7)$$

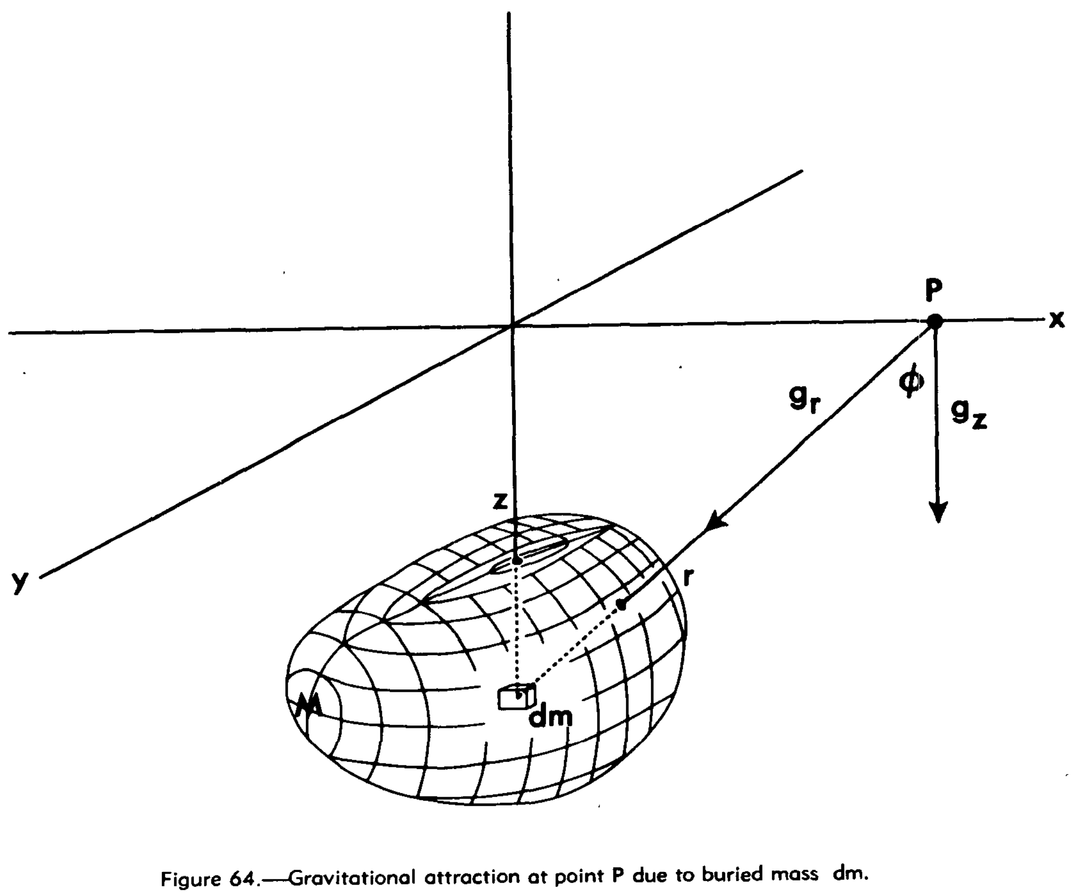

Figure 64.—Gravitational attraction at point P due to buried mass dm.

If the density of body *M* is homogeneous and has the value ρ, we can rewrite equation 7 as:

$$g_s = \rho G \int_v \frac{dv}{r^2} \cos \phi \qquad (8)$$

where *dv* represents the volume of *dm* and the integration is performed over the entire volume. Equation 8 is the basic equation of gravimetry. An exact solution for the integral can be obtained if the body has a simple analytical shape; for example, a sphere, a right circular cylinder, or an infinite, uniformly thick plate. If, however, the body is irregular in shape, as most geologic bodies are, then the total attraction must be calculated by graphical integration or by numerical summation, using a digital or analog computer.

Although the gravitational attraction of any geologic body is a function of its mass, the total gravitational attraction measured by a gravimetric device on the surface above it represents the sum of the attractions of both the body and the rest of the Earth, as a whole. In geophysical prospecting, we are interested only in that part of the gravity field due to the body; therefore, we generally need be concerned only with the excess or deficiency of mass that the body represents, rather than with its absolute mass. Under these circumstances, the body can be described quantitatively in terms of its density contrast with its surroundings. Observed gravity variations, when corrected for nongeologic effects, reflect contrasts in density within the Earth, particularly, lateral contrasts in density. The symbol ρ in equation 8 can be taken to represent the density contrast between a geologic body and its surroundings, rather than the actual density of the body.

Reduction of Gravity Data

Several corrections must be applied to raw gravity data collected in the field before they can be used for geological interpretation. Some of these corrections have a practical effect on the design of a gravity survey and the applicability of gravimetry to the hydrogeological problem at hand.

The theoretical foundations for gravity data reduction have been worked out in rigorous detail but need not be presented here. The interested reader is referred to Dobrin (1960) or Grant and West (1965) for the details and mathematical derivation of the corrections.

Reference to figure 65 should provide a qualitative understanding of the origin and nature of the various effects necessitating the corrections. In figure 65A is a spherical geologic body, the center of which lies 6.1 km (3.8 mi) below the Earth's surface. This surface, which is perfectly flat in our example, bounds a rigid, stationary, homogeneous Earth of semi-infinite extent. The buried body, with a density that is 0.50 gm/cm^3 greater than that of the rest of the Earth, represents the only departure from homogeneity affecting the total gravitational field. The gravity anomaly associated with the buried sphere is shown immediately above the model. It represents a local departure from the otherwise featureless gravity field associated with the hypothetical Earth, and is what we would see if we were to make a series of measurements with a gravity meter in the area and plot them, without modification, on a sheet of graph paper. The maximum amplitude of the anomaly is 3.7 mgals. Although spherical masses such as this one are an imprecise representation of most real geologic bodies, they are ones for which the analytical computation of gravity anomalies is relatively simple, hence in the pages that follow the sphere will be used to illustrate several properties of gravity fields. Actually the Earth model we have chosen is far more unrealistic than is the sphere, insofar as a representation of nature is concerned. The real Earth is not flat, it is spheroidal, and its surface is far from plane. In addition, it is neither rigid, stationary, nor homogeneous.

A more accurate representation of the real Earth is shown in figure 65B. The Earth depicted there is a rotating, nonrigid, spheroid within the gravitational influence of other

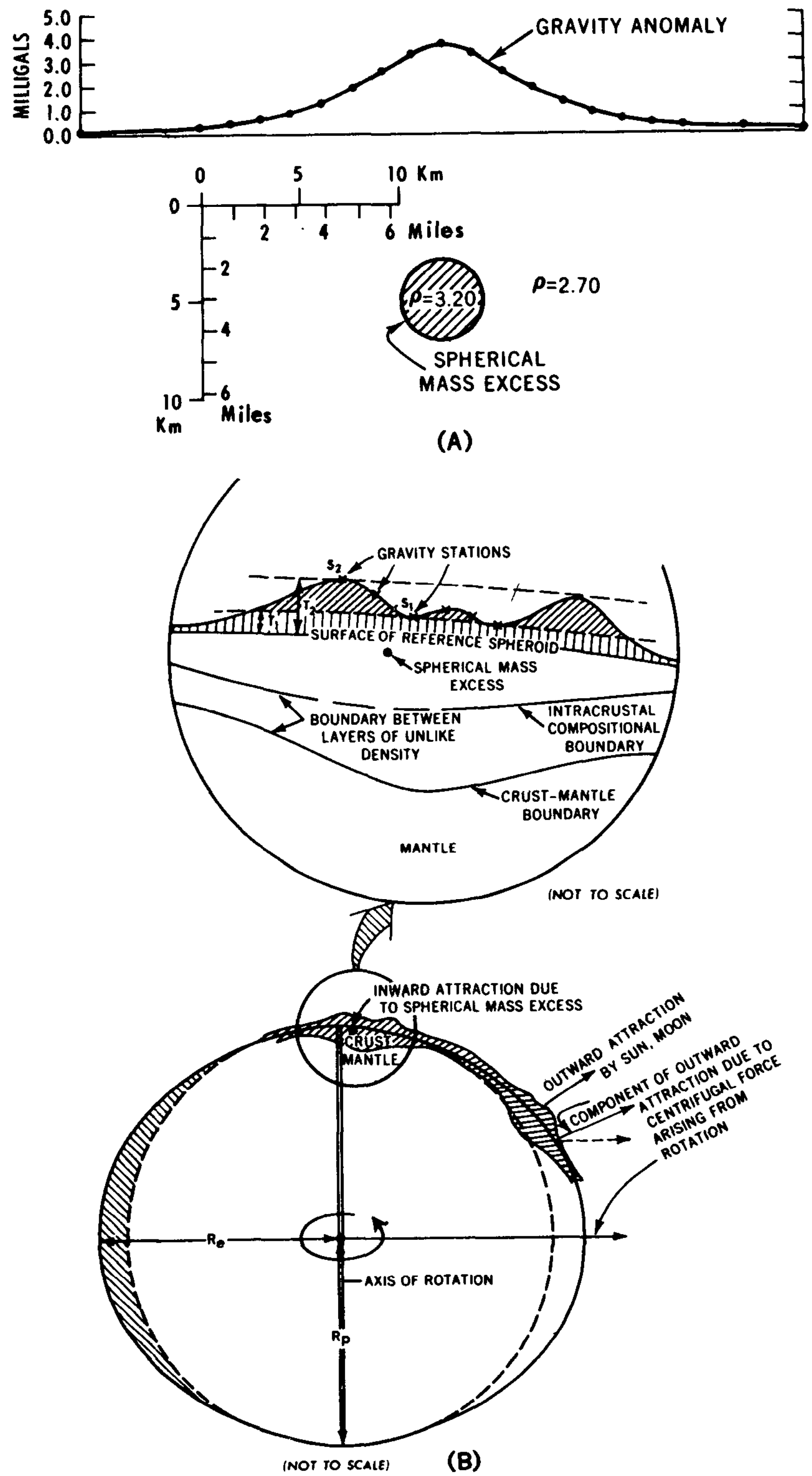

Figure 65.—A, Observed gravity profile for a buried sphere in a homogeneous rigid nonrotating Earth. B, Sources of variation present in gravitational measurements made in the search for a buried sphere in a schematic, but real, Earth model.

celestial bodies, with a compositionally nonhomogeneous crust of geographically varying thickness, and with a topographically rugged surface. Gravity measurements made on the surface of this Earth over a buried sphere would, if plotted as observed, display a scatter of points seemingly distributed without reason or order.

The reduction of gravity data refers to the removal of all unwanted effects that tend to mask or distort the gravity field caused by the object of interest. Several steps in the reduction process can be treated as mathematical routines, making them mechanically simple to execute. Others require judgment based on a knowledge of the local geology.

Latitude Correction

Gravitational acceleration measured on the Earth's surface varies systematically with latitude because the Earth rotates, is not perfectly rigid, and its shape is not precisely that of a sphere. At the poles the distance to the center of the Earth (radius R_P) is less than it is at the equator (radius R_e), and there is no component of centrifugal force, as there is at the equator, acting outward. Both these effects tend to reduce gravity at the equator relative to that at the poles. The effect of a somewhat greater thickness of rock (with consequent greater mass) between the equator and the Earth's center tends to reduce very slightly the effect of the first two factors, but the net result is that gravity at the poles is approximately 5 gals greater than it is at the equator. This latitudinal variation can be expressed as a trigonometric function of latitude. For this reason the latitude correction is both simply and routinely determined, either from table of values at discrete increments of latitude or by high-speed machine computation.

If an accuracy of 0.1 mgal in reduced gravity data is desired, the latitude of each station must be known to within 150 meters (500 feet) of its actual location. If an accuracy of 0.01 mgal is needed (which is approaching the limits of precision of the modern field instrument), locations must be known to within 15 meters (50 feet). With most modern topographic maps published at a scale of 1:62,500 or larger, this is not a serious problem. The correction is made by subtracting from the value for observed gravity, the value of theoretical gravity on the reference spheroid at sea level at the same latitude. For gravity surveys of limited latitudinal extent, the variation of gravity with latitude can be treated as though it were a linear function of surface distance north or south of an arbitrary base line drawn through the area of study. For the continental United States this variation of gravity ranges from approximately 0.6 mgal/km (0.96 mgal/mile) to 0.8 mgal/km (1.29 mgal/mile) and is greatest at 45° north latitude.

Tidal Correction

The Sun and Moon both exert an outward-directed attraction on the gravity meter, just as they do on large bodies of water as evidenced by tides. This attraction varies both with latitude and time. Although its magnitude is small, there are some hydrologic applications of gravimetry where tidal variations must be taken into account. The maximum amplitude of the tidal effect is approximately 0.2 mgal and its maximum rate of change is about 0.05 mgal/hour. If accuracy of this order of magnitude is not required in a gravity survey, the tidal effect may be neglected.

Several routes are open to the geophysicist in making tidal corrections; perhaps the one most commonly used is to monitor tidal variations, empirically, along with instrument drift, by returning every 2 hours or so to a gravity base station. Details of this approach are discussed under the heading "Drift Correction."

Altitude Corrections

Two corrections for station altitude must be made in the data-reduction process. One of these is the free-air correction and the other is the Bouguer correction.

Free-Air Correction

As the gravity meter is moved from hill to valley over the irregular surface of the Earth, the distance to the center of mass of the Earth varies. Equation 4 indicates that variation in the value of R (the distance to the center of the Earth's mass) will cause variations in the measured acceleration of gravity. This effect is known as the free-air effect.

The average value for the free-air gradient of gravity is —0.3086 mgal/m (—0.0941 mgal/foot). This value varies with both latitude and altitude but the variations are very small—less than one percent over most of the Earth's surface from sea level up to altitudes as great as 9,000 meters (31,000 feet). Variations in the free-air gradient of gravity also may be caused by large gravity anomalies arising from the outer part of the Earth. Departures in the measured value of this gradient have been found, under exceptional circumstances, to exceed 10 percent of the average value of —0.3086 mgal/m (—0.091 mgal/foot). Knowledge of the exact local free-air gradient of gravity is not important in most gravity surveys. For some hydrogeologic applications, however, it may be necessary to measure the local value. Measurement of the local free-air gradient, should it be required at any locality, is not an insurmountable problem, but the necessity for doing so should be thoroughly evaluated by the geophysicist.

Bouguer Correction

The Bouguer correction is necessitated by the presence of rock between the gravity station and the elevation datum (commonly mean sea level) to which the observations are to be reduced. Referring to 65B, there is, at gravity station S_1, a mass of rock of thickness T_1, between the station and the elevation datum, which causes an additional downward attraction that would not be sensed had we been able to suspend the gravity meter in free space at the same elevation. This attraction varies with station elevation and has a value at station S_2 different from that at station S_1.

The standard procedure for making the Bouguer correction is to assume that an infinite slab of rock, of thickness equal to the height of the station above the datum, is present beneath the station. For a station in relatively flat country this approximation is a reasonable one, but for areas of rugged topography it is not. For example, in figure 65B, the infinite slab approximation is good for station S_1, but poor for station S_2. An adjustment is made for the relatively poor fit of the infinite slab model in topographic situations such as that of station S_2 when one makes the terrain correction described below.

The gravitational acceleration due to an infinite horizontal slab of uniform density ρ is:

$$g_z = 2\,\pi\,G\rho\,T \tag{9}$$

where T is the thickness of the slab. Note that the gravitational acceleration is not dependent on the distance of the point of measurement from the surface of the slab, but only on the slab's thickness and density. Thus the gravitational acceleration caused by a given slab of homogeneous rock would be the same whether measured on its surface, or on a tower several hundred feet above its surface. This apparent peculiarity of the gravitational field of an infinite slab has great utility in gravity exploration, both for data reduction and data interpretation, as will be apparent later.

Two parameters in equation 9 are needed to make the Bouguer correction, density and thickness. In many gravity surveys, particularly those of regional extent, mean sea level is chosen as the elevation datum. The value of T is then the elevation of the gravity station. Likewise, ρ routinely is assigned a constant value of 2.67 gm/cm^3. These choices, though they have some theoretical and practical foundation, are essentially arbitrary and may not be appropriate for use in some areas or in certain hydrogeologic studies. When subtle gravitational variations are being sought it is important to use true

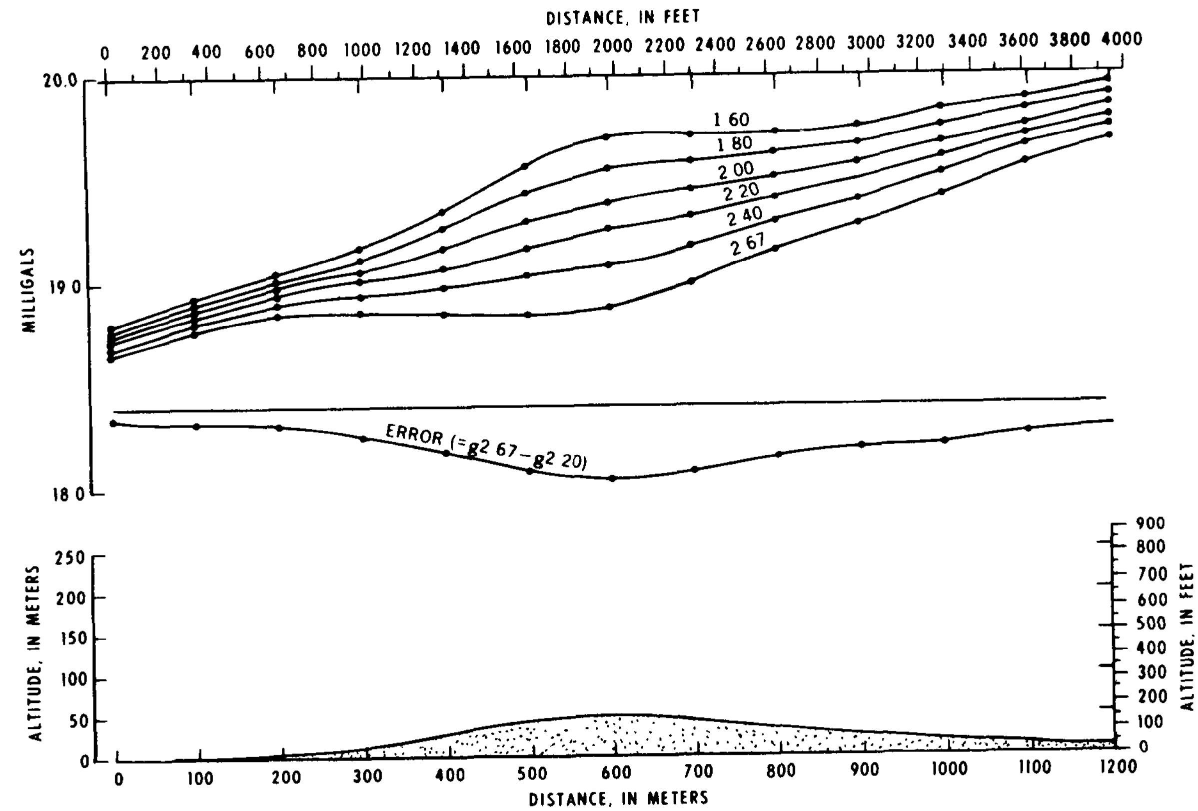

Figure 66.—Bouguer gravity profiles across a low ridge based on six different densities employed in calculating the Bouguer correction. The proper value for density is 2.20 gm/cm³.

rock density values and to use a local elevation datum.

The effect of the use of an incorrect density value in making the Bouguer correction is shown in figure 66. In the lower part of figure 66 is a topographic profile of a broad ridge. This ridge is underlain by young sedimentary rocks that have a uniform density of 2.20 gm/cm³. A regional gravity gradient slopes downward across the area from right to left. It is caused by a deep-seated density variation. The gravity anomaly curve labelled 2.20 is the one that would be obtained if tidal, latitude, free-air and Bouguer corrections were made, using in the Bouguer correction a density of 2.20 gm/cm³.

If there were no data on local rock densities and an assumed value of 2.67 gm/cm³ were used, the reduced gravity data would provide the curve labelled 2.67. This curve, which mirrors the topography, is in error. It displays an artificial local anomaly, superimposed on the regional gradient. The curve labelled "ERROR" represents the algebraic difference between the correct Bouguer gravity curve, based on the true density 2.20 gm/cm³, and the erroneous one created by assuming a density of 2.67 gm/cm³.

If the density data were based on cores recovered in an area a few miles away, where the local near-surface density was 1.60 gm/cm³, and this value were used to make the Bouguer correction, an artificial anomaly in the form of an upward convexity superimposed on the regional gradient (curve 1.60) is created.

In summary, knowledge of the correct local rock density is essential to the correct reduction of gravity data. If incorrect values are used, artificial gravity anomalies related to topography are created. Hills or ridges produce artificial gravity highs if the density value used is smaller than the actual value and they produce gravity lows if the density value is too high.

In many regions the geology is sufficiently complex that the assumption of a single uniform density is not warranted. When seeking targets with very small differences in density, variable density values must be used in making the Bouguer correction. In effect, this amounts to making a correction for the near-surface geology. The more that is known about the local distribution of rock types and their densities, the less chance there is of introducing artificiality and error in the result. For regional surveys of a reconnaissance nature this kind of sophistication usually is not justified. For highly detailed studies, with closely spaced gravity stations and subtle targets, it is.

If local rock densities are poorly known, or if the densities vary vertically, it is advisable to use a datum as close to the great bulk of the station elevations as possible. Either of two options can be employed. A frequency diagram of all station elevations can be plotted, and the modal elevation value for the datum selected or the elevation of the lowest station in the survey area can be used as datum. Doing either minimizes the chance of errors resulting from imperfect knowledge of the geology between the station and the datum.

Because the free-air and Bouguer corrections are both simple functions of the elevation of the gravity station above the datum, they are combined, for computation, into a single correction referred to as the combined elevation correction. The algebraic form of the combined elevation correction is Kh, where h is the height of the station above the datum and K is a function of the free-air gradient and the rock density. Examination of the magnitude of K for varying values of density and a fixed value for the free-air gradient of —0.3086 mgal/m (—0.0941 mgal/foot), illustrates the magnitude of the errors incurred when station elevations are imperfectly known:

Rock density (gm/cm³)	*K value*	*Approximate error, in mgal, created by an elevation error of 3 meters (9.84 ft)*
1.7	0.2375	0.71
1.9	.2291	.69
2.1	.2207	.66
2.3	.2123	.64
2.5	.2040	.61
2.7	.1956	.59

The errors in the right-hand column are larger than can be tolerated in certain kinds of ground-water investigations. In those studies where anomalies of several hundredths to a few tenths of a milligal are sought, elevations must be known to the nearest 3 cm (0.1 ft). Precision levelling is required for station elevations of this accuracy.

Terrain Correction

It is apparent that some correction must be made for the topographic masses situated above the level of the gravity station. Hills that project above that level exert an upward gravitational attraction, reducing the gravity value read at the station. Similarly, valleys represent topographic depressions which are filled with rock computationally in making the Bouguer correction (fig. 65). Just as a correction is needed for positive masses projecting above the station, one also is needed for negative masses that are created artificially below it. The algebraic sign of the terrain correction therefore is always positive, whether for hills or valleys.

A terrain correction generally requires the existence of good topographic maps. If they are not available, the cost of obtaining the necessary topographic information is generally prohibitive. The topographic detail required depends on the accuracy sought in the reduced values and on the proximity of topographic irregularities to the stations. For example, if the topography enclosed within two concentric circles about a given station, the inner circle, with a radius of 17 m (56 feet), and the outer one, with a radius of 53 m (174 feet), differs in elevation from the station by an average of 8 m (26 feet), and if the rock density is 2.67 gm/cm^3, a terrain correction of approximately 0.13 mgal is required. To estimate this elevation difference accurately, a topographic map at a scale of close to 1 : 25,000 or better and with a contour interval of 3–5 m (10–15 feet) is required. In the absence of this kind of topographic detail it is better not to locate the station where the terrain is varied enough to create effects of this size when high accuracy is sought. Balance between the detail and accuracy sought from the survey and the topographic detail available must be considered in designing the survey. For a study of an intermontane valley with dimensions of 25 by 65 km (15.5 by 40.4 miles), and filled with 2,000 m (6,500 feet) of late Cenozoic sediments, the expectable maximum amplitude of the associated gravity anomaly will be several tens of milligals. If one is interested only in the gross configuration of the buried bedrock floor of this valley, and in a quantitative estimate of the depth to bedrock, errors of a milligal or so can be tolerated. This means that the topographic detail needed for the terrain corrections is not nearly as limiting as it is for a buried outwash channel only 160 m (525 feet) deep and 1 km (0.62 mile) across. The maximum amplitude of the anomaly for the buried channel will not exceed 5 mgal. If the channel contains clay-rich glacial till, the anomaly may be only a few tenths of a milligal. Here the accuracy of each correction must be kept as high as possible and errors should not be allowed to exceed a few hundredths of a milligal.

Terrain corrections are made by arbitrarily subdividing the region about the station into a series of rectangles or curvilinear cells and estimating the average topographic elevation of each. Mathematical computations are then made to determine the correction for each cell and the results summed to obtain the total correction for the station. Either of two schemes may be used. One, a manual method, consists of centering a transparent graticule on the station, subdividing it into compartments by radii, and estimating the compartment elevations by eye. The other, usually justified only by a relatively large number of gravity stations, consists of digitizing the topography of the surrounding region on a rectangular grid, and performing the necessary calculations with a high-speed digital computer. The computer program in use in the Geological Survey allows terrain correction computations to be extended to a distance of 166.7 km (104 miles) from the station. In most hydrologic applications computations to this distance are unnecessary. Terrain corrections are rarely extended beyond 25 km (16 miles) when the calculations are made by hand. The judgment of a person experienced in making gravity terrain corrections is advisable, although not absolutely necessary for efficient design of the reduction program.

Drift Correction

Because the materials of construction of most, if not all, gravity meters are susceptible to both elastic and inelastic strains when subjected to thermal or mechanical stresses, reoccupation of the same gravity station at different times with a given meter may reveal differences in the readings obtained. The observed differences may be caused by tidal effects, but some result from stresses or shock to the internal components of the instrument. Gravity differences resulting from these stresses are referred to as instrument drift. In practice, instrument drift and tidal effects usually are monitored together by returning to a base station every 2 hours or so during the course of a survey. It is assumed that variations between reoccupations of the base are time-dependent. Corrections for readings at field stations occupied in the interim are scaled from a plot of drift versus time.

Regional Gradients

All the corrections described thus far are designed to eliminate nongeologic effects such as those caused by variations in elevation and latitude, topographic irregularities, or other extraneous sources. The resulting contoured gravity field is known as a complete Bouguer gravity anomaly map and displays features that theoretically are due only to lateral variations in rock density below the elevation datum. An analysis of the gravity field in terms of this geology is presumably the reason for making the survey in the first place. From a practical standpoint, things are not quite so simple. Usually a target of geologic interest is quite specific at the outset and the gravity field arising from it is the objective sought. The problem which arises results from the fact that rarely do we see the gravity field of a given geologic body in isolation. Usually, the anomaly of interest is distorted or partly masked by the gravity fields of other bodies. As a result, the geophysicist is faced with the problem of sorting out those parts of a total gravity field caused only by the object of immediate interest. Basically he knows only the magnitude and shape of the total Bouguer gravity field, but he hopes to be able to subtract from it the contributions caused by geologic bodies of unknown shape, density, and location, in order to isolate the residual anomaly of interest. A simple example, and one for which the isolation process is usually rather simple, can be seen in figure 65B. The target here is the spherical body. Interfering with the gravity field of the sphere is another which arises from variation in density between the lower part of the crust and the mantle beneath it. The interface between the crust and mantle is not concentric with the reference spheroid and hence it constitutes a lateral density contrast that will be sensed by the gravity meter. Because it is a broad deep-seated feature, its gravitational effect will be that of a gentle areally-extensive undulation. If the center of the anomaly sought is well up on one flank of this undulation, the regional effect will be that of a continuous gradient extending across the survey area for a distance many times greater than the width of the target anomaly. We refer to this part of the total field as the regional gradient and in order to make a quantitative interpretation of the anomaly caused by the target alone, we must somehow subtract the effect of the regional gradient. (See fig. 71 and related text for an actual example of a regional gradient). Many schemes have been proposed for doing this. The interested reader may want to read Nettleton (1954) for a nonmathematical discussion of the methods in use today.

Bouguer Anomaly

If the value of absolute gravity is known at a station by virtue of having tied it directly, or indirectly with a gravity meter, to a base station where pendulum measurements of gravity have been made, the calculated corrections can be added algebraically to this value to obtain what is known as the complete Bouguer gravity anomaly. This anomaly is defined as follows:

Observed gravity plus drift and tidal correction plus combined elevation correction plus terrain correction minus theoretical gravity on the reference spheroid (latitude correction).

If the terrain corrections have not been made, the results are referred to as simple Bouguer anomaly values.

In gravimetric prospecting it is not necessary to know the value of absolute gravity at any point in the survey area. The concern is principally with variations in Bouguer gravity from point to point and an arbitrary value can be assigned to the base station. The resulting field differs from the true Bouguer anomaly field by a constant amount everywhere. Knowing the value of absolute gravity at the base provides the means of tying the gravity survey to others and for this reason it is common practice to relate each survey to the same absolute datum.

Interpretation of Gravity Data

Ambiguity

In its simplest form, the interpretation of gravity data consists of constructing a hypothetical distribution of mass that would give rise to a gravity field like the one observed. Models are constructed graphically or mathematically and their gravity effects calculated from equation 8 by numerical summation or graphical integration. The difficulty lies in the fact that a large number (theoretically, an infinity) of hypothetical models will produce the same gravity anomaly. The known quantity g_z is a complex function of three unknowns: density, shape, and depth of the causative body. It is apparent, even without knowledge of an exact solution for the volume integral in equation 8, that one could substitute, simultaneously, a variety of values for the parameters ρ, r, ϕ, and $\int_v dv$ in such a way as to maintain a constant value for g_z at point P on the surface.

If we had enough information in a given

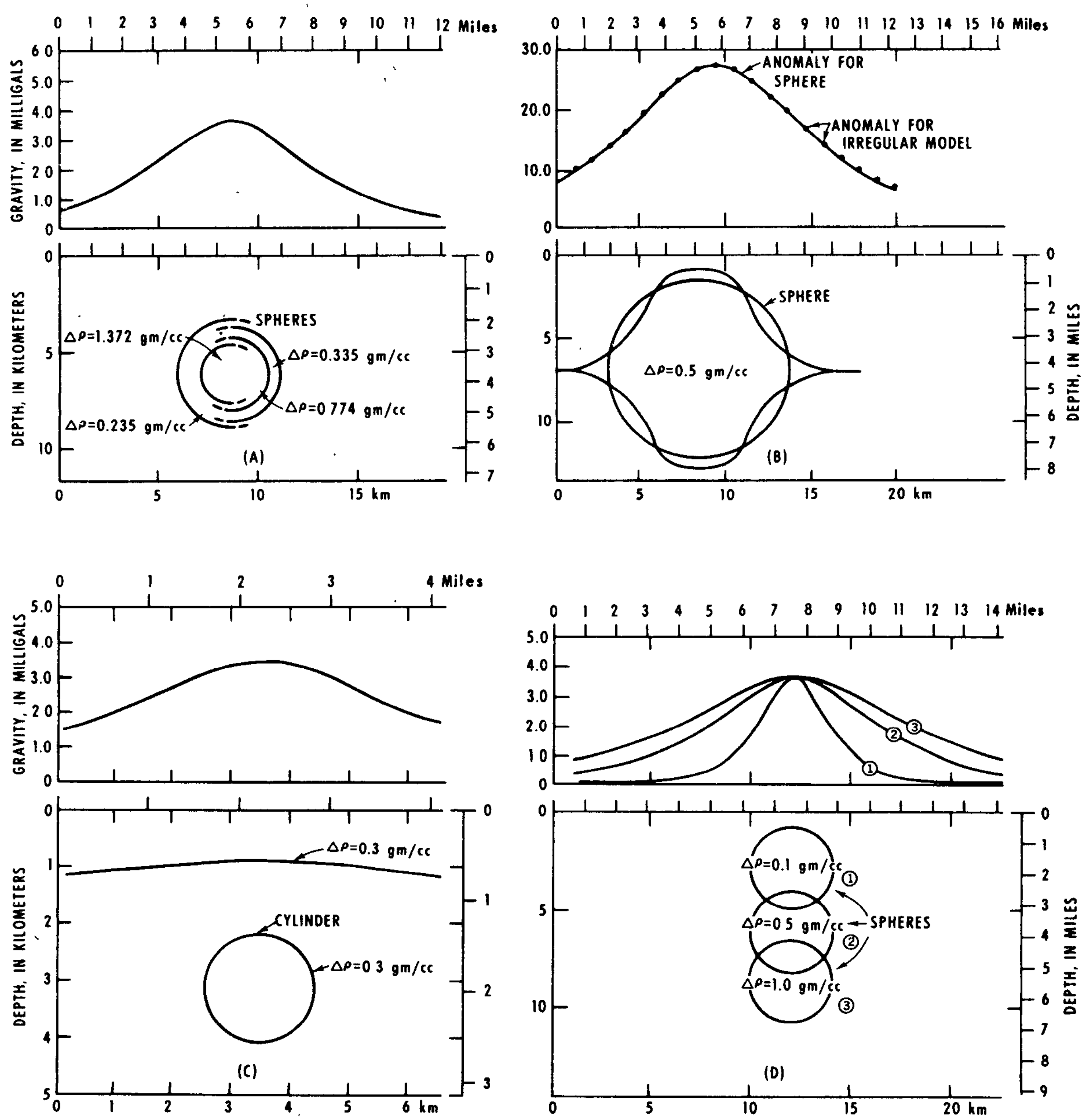

Figure 67.—Schematic models and associated Bouguer gravity anomalies for idealized geologic bodies.

situation to know that we were dealing with a spherical body with its center buried at a specific depth, we still could not make a unique interpretation of the gravity anomaly in terms of size and density (fig. 67A). The gravity anomalies for these four spheres are identical. This results from the fact that the mass of a sphere can be treated as though it were concentrated at a point at the center. In figure 67A, the radius and density of the spheres have been adjusted to keep the total mass constant. The geologic implication is clear.

In addition, the gravity field of a sphere does not have a unique configuration (fig. 67B). Thus the shape of a body cannot be determined from its gravity anomaly alone, even when the density contrast and center of gravity are known. In figure 67B the anomaly arising from the sphere is shown as a smooth curve and the field due to an irregular body of different rotational shape, with coincident center and density, is shown by dots. The two curves match one another very closely.

Bodies of other shape also produce non-unique anomalies (fig. 67C). The gravity anomaly of a horizontal right circular cylinder buried at a depth slightly in excess of 3 km (1.9 miles) can be matched by that of a gently convex basement surface at a depth of approximately 1 km (0.6 mile) when the density contrast between basement and overburden matches that of the sphere and its surroundings.

In summary, the non-uniqueness is pronounced. The fact that gravimetry has been successfully employed as an exploration technique for many decades indicates that ambiguity is not an insurmountable interpretation problem. For example, the individual masses and gravity fields of the spheres of different size shown in figure 67A were kept constant by holding the product ρR^3 constant. The maximum range of bulk densities for common, naturally occurring consolidated rocks and unconsolidated sediments is well known. Reference to Clark (1966, Sec. 4) and Manger (1963) indicates that the limits of the range are approximately 1.70 and 3.00 gm/cm^3. These limits represent well sorted, unconsolidated clastic sediments of high porosity and massive basalt, respectively. There are a few earth materials with densities outside this range, but they are not common. This range places an upper limit on the magnitude of the density contrast that one might expect to encounter in nature and constitutes the maximum density contrast (1.30 gm/cm^3) that can be used in modelling. In most geologic settings the contrast is less than 1.00 gm/cm^3. Greater restriction can be placed on the density contrast in an actual setting from a knowledge of the local geology.

Other boundary conditions can be imposed as well. Consider a typical valley-fill aquifer. It consists of unconsolidated or semiconsolidated sediments resting uncomformably on older, and usually more consolidated (and therefore, denser), rocks. Geologic mapping determines the approximate surface location of the interface between the aquifer and the rocks on which it rests. If, in addition, the top of the aquifer is coincident with the surface of the ground, this fact constitutes an additional boundary condition. Further limits on the interpretational model can be achieved by making measurements of the average bedrock density and the density of the uppermost part of the valley fill. It can be reasonably assumed that the fill density probably increases with depth and that the walls of the valley probably slope inward in the subsurface. Thus severe limitations have been placed upon the conceptual model. Several different models that will produce the observed anomaly probably can still be constructed, but the differences between the models may not be significant. If they are, however, we might be able to bring other data to bear that would furnish still further constraints and thus allow a more nearly unique interpretation. The greater the amount of geologic data that can be used in establishing limits or constraints on the model, the more unique will be the interpretation.

Another facet of the interpretation process that is of aid in the early stages of data analysis is shown in figure 67D. Three spheres of the same size at different depths have had their densities adjusted so as to keep the maximum amplitude of their anomalies the same. At horizontal distances that are several times the depth of burial of the spheres, all three anomalies asymptotically approach zero because the vertical component of gravity at this distance is negligible. Between the regions of zero and maximum amplitude, however, the three curves are notably different. The greater the depth of burial of a given body, the gentler are the gradients of the flanks of its anomaly. The gradients of any anomaly are also a function of the shape of the producing body because two bodies at distinctly different depths may produce anomalies with the same gradients. There is, however, a limit to the depth to which we can push a model and still maintain anomaly gradients at or above a fixed value. For example, there is no infinitely-long, horizontal body of any cross-sectional shape that can be buried with its upper surface at a depth of 3 km (1.9 miles) or more and still produce an anomaly that has flanking gradients as steep as those shown in figure 67D. There are some general formulas, based on potential theory, that allow determination of the maximum possible depth to the top of anomaly-producing body from the ratio of the maximum amplitude to the maximum gradient of its flanks (Bott and Smith, 1958; Bancroft, 1960). These formulas are useful for a rough fix on maximum depth to the top of a body in the early stages of modelling.

Interpretation Techniques

The basic technique of gravity interpretation is field matching. A model is constructed and its gravity field calculated for comparison with the observed field.

Several methods of calculation are open to the investigator and the one chosen depends on the factors of accuracy and detail sought, the shape and complexity of the model, and the time and equipment available. All of the methods represent some form of integration or summation. Computation of the model field is followed by a comparison of the results with the observed anomaly. The model is then changed and its anomaly recalculated, until the desired fit between observed and theoretical anomalies is achieved.

In its crudest form, the body under study may be assumed to have a constant density and an analytical shape (that is, a sphere, cylinder, or plate), its field being calculated by appropriate substitutions in equation 8. In its most sophisticated form the body can be given an irregular three-dimensional form, with a spatially continuous or discontinuous distribution of density, and its field calculated by digital computer. The computer can be instructed to follow an iterative routine, wherein it makes the comparison between the observed and calculated data, institutes certain changes in the model that will lead to a better fit, recomputes the field, makes a second comparison, and so on.

Presentation of details of the various interpretation methods currently in use is relevant, but not appropriate here. The interested reader is referred to Dobrin (1960, p. 253–262) and Grant and West (1965, p. 268–305). Two points should be stressed however; they are: (1) The solution for a given gravity anomaly is never unique and the use of highly sophisticated and elegant mathematical methods of interpretation does not make it so, and (2) the quality and uniqueness of the interpretation are, in part, a function of the kind and amount of geologic information available to the interpreter.

Significance and Use of Density Measurements

The interpretation of gravity data necessitates accurate knowledge of rock densities in the area surveyed. Because variations in rock density produce the potential field differences we observe after data reduction, this property is of fundamental importance.

There are several ways in which the geophysicist may obtain the density values to be used in handling the data for a given area.

The cost of the method selected should be in rough proportion to the significance of the problem. Eight methods are described briefly below. They are listed approximately in order of increasing significance and accuracy.

1. Assumption of a constant density value of 2.67 gm/cm^3.
2. Assignment of density values on the basis of lithology. Because of the wide variability of rock composition and rock density within a lithologic classification, values assigned on this basis can be in error by as much as 40 percent.
3. Estimates of density based on sound-wave velocities in rocks. Compressional wave velocities and densities of rocks are a function of some of the same lithologic factors. Because of this, they show a pronounced correlation. Approximately three-fourths of the data points in figure 68 fall within 0.1 gm/cm^3 of the regression curve fitted to them.
4. In situ gamma-gamma logging. A gamma-gamma borehole logging device measures radiation that originates from a source in the sonde and travels through a shell of rock adjacent to the borehole. The decrease in strength of the returning signal is approximately proportional to the density of the rock. However, the borehole diameter, the presence of borehole fluids, mudcake on the hole walls, mud-filtrate invasion, and the roughness of the hole all adversely affect the results. A separation of the logging tool from contact with the rock by as little as 0.7 cm (0.3 in) can cause a significant error in the density value.
5. Density measurements on handspecimens collected at the outcrop. If care is taken to procure unweathered material, if the sampling statistics are adequate, and if the samples are large and geologically representative, the results of this method are usually quite accurate. This probably is the method most frequently used today.
6. Density profiling with the gravity meter. If a topographic feature such as a hill or valley is underlain by rocks of laterally homogeneous density and if the topography is not an expression of geologic structure, the data from a gravity profile can be used to measure the average bulk density. The principle is illustrated in figure 66, where the Bouguer anomaly curve computed using the correct density of 2.20 gm/cm^3 shows the least tendency to mirror the topography. The advantage of this method is that it samples, in place, a very large volume of rock.
7. Laboratory measurements of drill-core samples of consolidated rocks. This method provides a means of sampling below the zone of weathering and, if recovery is good, it also provides the basis for computing geologically-weighted means for the section. Recent tests (McCulloh, 1965) indicate that when proper care is taken in handling the cores, the accuracy of this method is high. However, a borehole represents a single vertical traverse of the rocks in an area. If there are pronounced lateral variations in density, cores from a single hole may not suffice.
8. Logging with a borehole gravity meter. A gravity meter lowered in a borehole can be used to measure the in situ density of rocks directly. Its ability to do so stems in large part from the relationship expressed in equation 9. The difference in the acceleration of gravity between two points in a borehole, separated vertically by the distance T, is a function of the product $4\pi G\rho T$. At the top of the interval downward attraction is $2\pi G\rho T$ and at the base, $-2\pi G\rho T$ (the same attraction acting upward, or in a negative sense). T can be measured and the measured value of the gravity difference, Δg_z, can be used to calculate the density, ρ. The radius of the region of rock that is sampled is roughly five

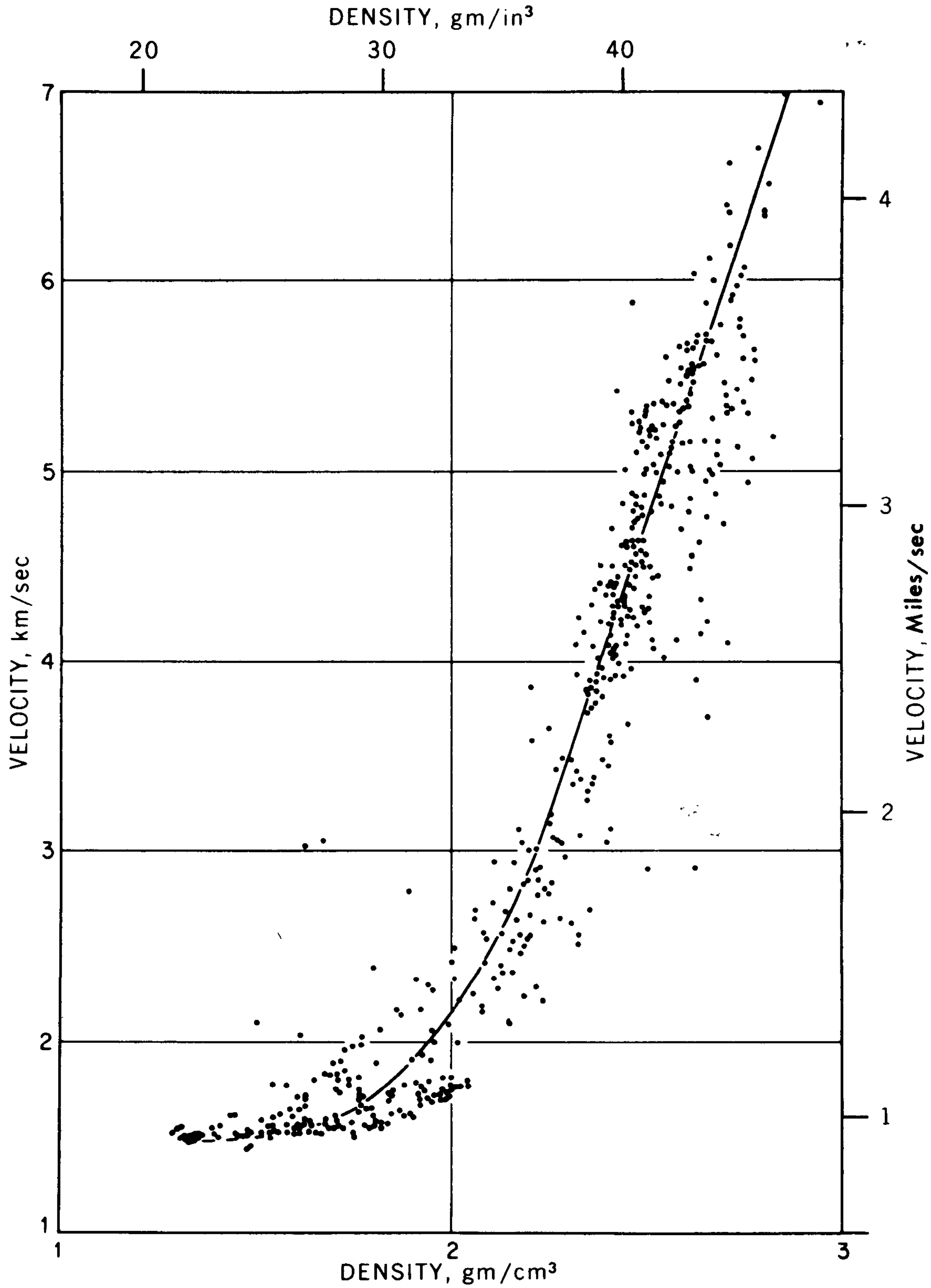

Figure 68.—Plot of observed compressional wave velocities versus density for sediments and sedimentary rocks (after Grant and West, 1965). Reproduced with permission of McGraw-Hill Book Company.

times the length of the vertical interval, T. A typical borehole gravity meter log of a thick section of alluvium is shown in figure 72.

Application of Gravimetry to Hydrogeology

Aquifer Geometry

The gravity method is a rapid, inexpensive means of determining the gross configuration of an aquifer, providing an adequate density contrast between the aquifer and the underlying bedrock exists. It is useful in locating areas of maximum aquifer thickness, in tracing the axis of a buried channel (fig. 69A), and in locating a buried bedrock high that may impede the flow of ground water (fig. 69B).

In figure 69A, the irregular belt of unconsolidated sediments that runs from the northwest corner of the map to the south-central part consists of buried outwash or ice-contact deposits resting in a glacially-overdeepened, preglacial bedrock channel of the Connecticut River. Well data (Cushman, 1964) defined the course of this buried channel, and its axis coincides with the axis of the gravity trough shown. Thus the gravity data reflect the locus of maximum thickness of the unconsolidated sediments. The success of the gravity method in defining the geometry of the aquifer in this area is due to the high density contrast between the unconsolidated fill and the bedrock, which consists of dense Paleozoic metamorphic rocks and Triassic sedimentary rocks. In areas where the contrast is lower, the definition of a narrow buried valley, such as the one shown here, becomes more difficult. If the density contrast is zero, the gravity method is useless for defining or mapping buried channels.

The San Gorgonio Pass area in southern California (fig. 69B) is bounded on the north and south by high mountain ranges consisting of Pre-Cenozoic metamorphic and igneous rocks. These rocks have a relatively high density. Deformed sedimentary rocks of late Tertiary age are exposed east and west of the map area along the north side of the pass. Recent sand and gravel underlie the central part of the area. Water levels measured in the spring of 1961 in two wells (A and B) define a water table sloping gently eastward with a gradient of about 5.7 m/km (30.1 feet/mile), in agreement with other well data west of the map area. In the vicinity of well B, however, the water table drops abruptly from an elevation of 345 m (1,130 ft) to 160 m (525 ft) in well C.

Contours of complete Bouguer gravity reveal that the cause of the discontinuity in the water table is a subsurface continuation of the exposed bedrock ridge which projects northward from the south side of the pass. This ridge rock is virtually impermeable and serves as a ground-water barrier. Aside from its visible expression on the south side of the pass, there is no surface evidence of its presence. The gravity method thus provides a means for recognizing its existence.

Estimating Average Total Porosity

Surface Method

Figure 70A shows the distribution of outcrops of granitic rocks bordering Perris Valley, Calif. Also shown are structure contours on the buried bedrock surface, as defined by well data. The structure contours reveal a large buried channel in the vicinity of Perris Boulevard. The land surface in this area is at an altitude of approximately 1,400 feet, which means that the maximum thickness of the unconsolidated sediments filling the buried valley is approximately 800 feet.

Figure 70B shows a Bouguer gravity map of the same area. The gravity map mimics the bedrock topography of the buried channel almost perfectly. Because of this high degree of correlation and the unusual amount of well control available from the area, estimation of the average in situ sediment porosity from surface gravity measurements

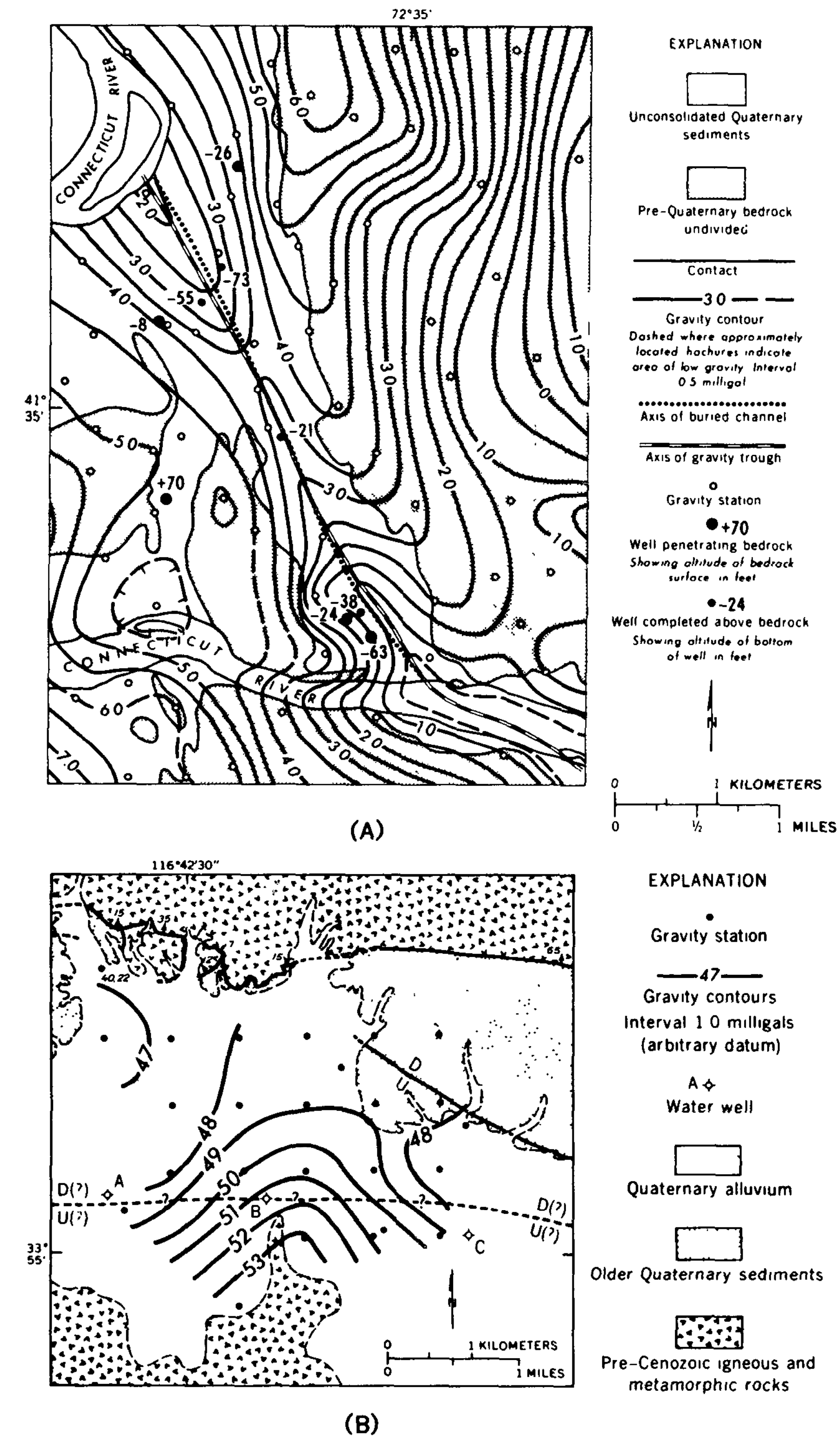

Figure 69.—A, Complete Bouguer-gravity map of a buried pre-glacial channel of the Connecticut River (after Eaton and Watkins, 1970). B, Complete Bouguer-gravity map of part of San Gorgonio Pass, California (after Eaton and others, 1964).

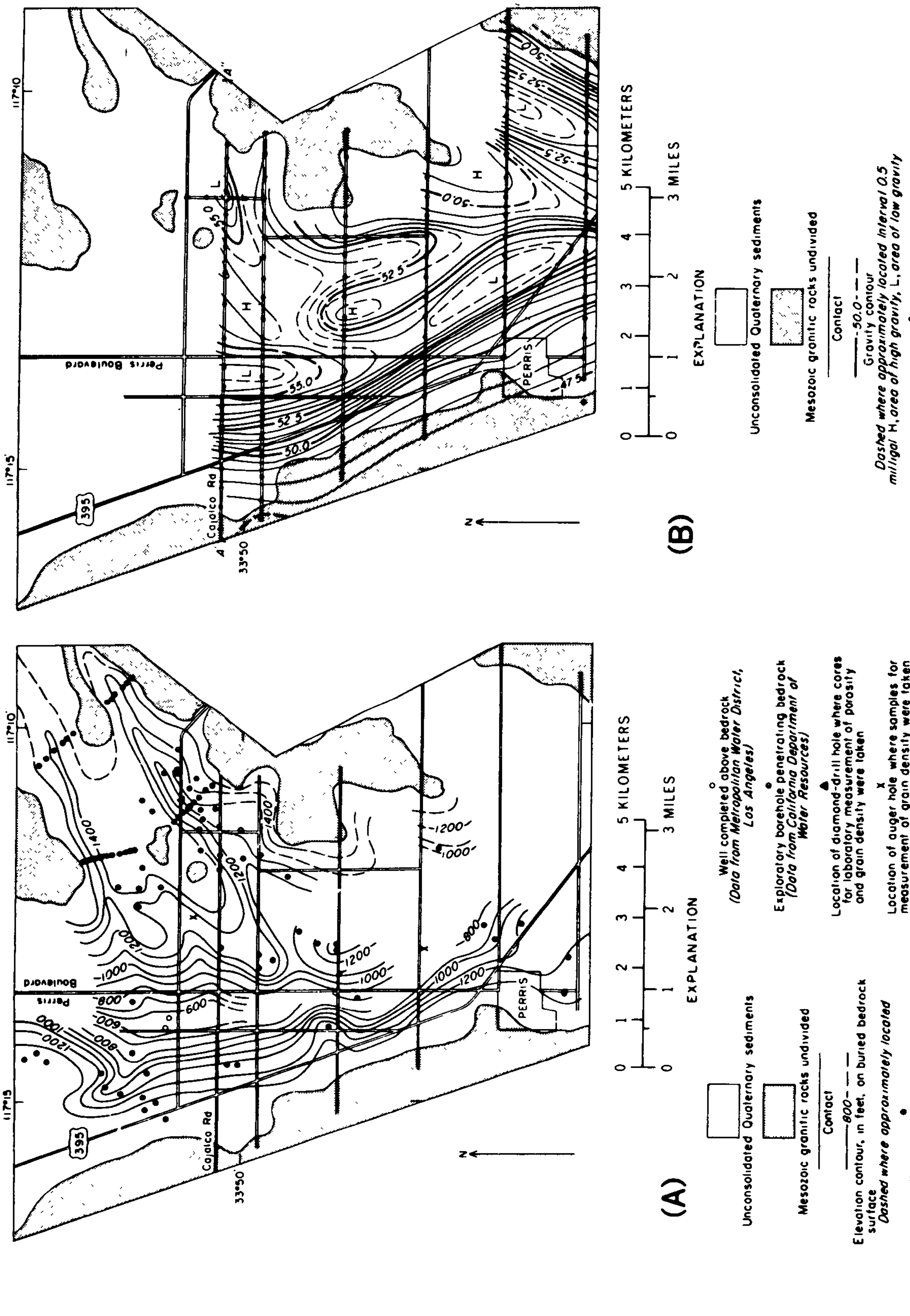

Figure 70.—A, Distribution of outcrops and structure contours on the buried bedrock surface, Perris Valley, Calif. B, Bouguer-gravity map of Perris Valley, Calif. (after Eaton and Watkins, 1970). Reproduced by permission of "Information Canada."

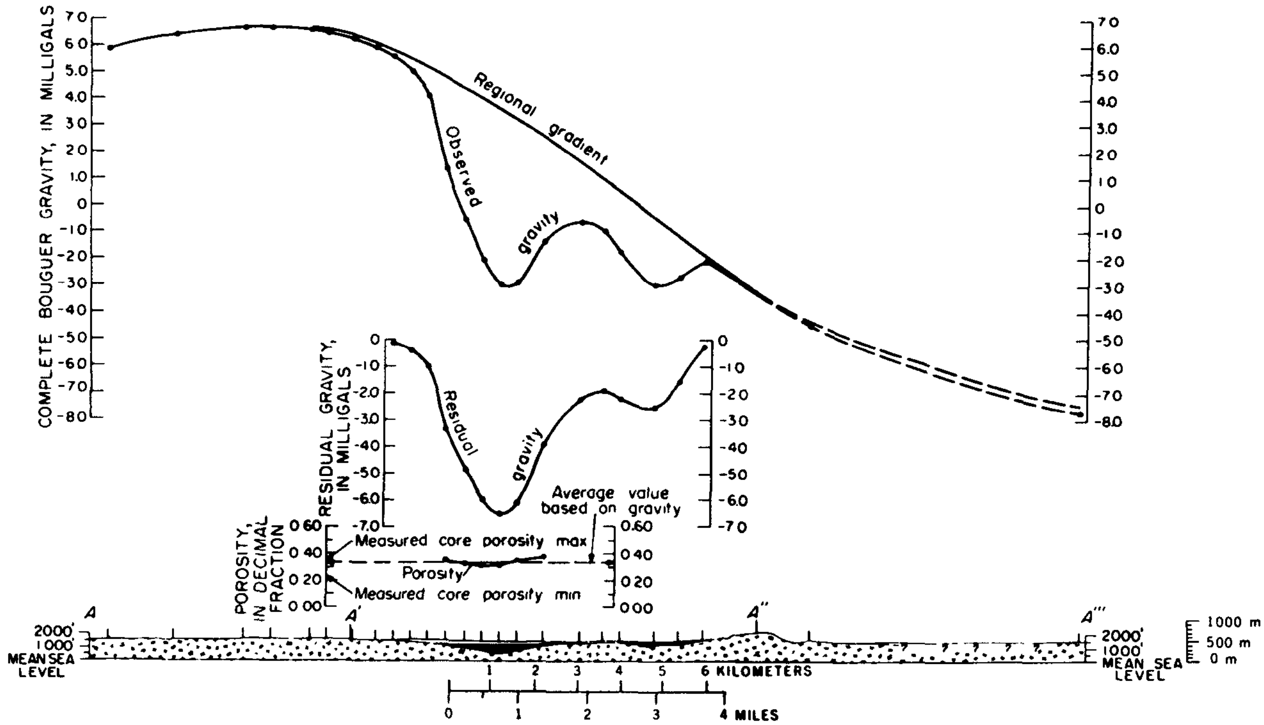

Figure 71.—Profiles of observed Bouguer gravity, residual gravity, and calculated porosity for Perris Valley, Calif. (after Eaton and Watkins, 1970). Reproduced by permission of "Information Canada."

was undertaken (Eaton and Watkins, 1970). A long gravity profile was extended beyond the borders of the map at the latitude of Cajalco Road in order to study the regional gradient. In making this profile (fig. 71), a different datum was employed from that on which the map was based. Hence the gravity values in figures 70B and 71 are different. Bedrock of fairly uniform composition (granitic rock of the southern California batholith) is exposed for many miles east and west of the valley so the eastern and western branches of the observed gravity curve were used for the regional gradient, the residual anomaly due to the low density valley fill being restricted to the central part of the area. If this gravity survey were part of a study of the batholith, or individual lithologic units within the batholith, it would have been necessary to define a different regional gradient and interpret the shape of a residual anomaly that would have included part of the regional gradient as defined here. A regional gradient is defined arbitrarily by the objective or target, which means that one must have at least an approximate idea concerning its size and nature to begin with. All parts of the observed gravity field in figure 71 have geologic origins, but we are interested in focusing our attention only on that part arising from sources close to the surface. Hence we concern ourselves with that part of the curve having the steepest gradients.

The residual gravity curve was calculated by subtracting the regional gradient from the observed gravity curve and was used, in conjunction with the geologic cross section shown below it, to calculate the average total porosity of the alluvial fill. Basically, the fill was weighed by the gravity meter, and, when its average bulk density had been determined from the gravity measurements, its porosity was calculated from the bulk density value and additional measured values of average grain density. Porosity values were calculated at six gravity stations over the central part of this valley. The results are shown in figure 71 on a porosity profile, where the average porosity is seen to be 33 percent. For comparison, 10 samples of the fill were collected at depths ranging from 6 to 82 meters (20 to 270 feet) in a borehole nearby and found to have porosities ranging from 23 to 35 percent. No significance is attached to the convexity of the porosity profile because the resolving power of the method is not great enough to distinguish real differences as small as those shown.

Borehole Method

An *in situ* density log (fig. 72) of a section of unconsolidated sediments in Hot Creek Valley, Nev., was made using the U.S. Geological Survey—LaCoste and Romberg borehole gravity meter system (McCulloh and others, 1967) and shows a remarkably systematic increase in bulk density with depth in the alluvium. At a depth of approximately 975 m (3,200 feet) the sediments have a maximum density of 2.34 gm/cm^3 and remain at or near this value to a depth of 1,280 m (4,200 feet), where lake beds underlie the alluvium. The reading interval of the gravity meter in this study was fairly coarse—61 m (200 feet)—which means that the slab of material contributing to each calculation extended horizontally away from the hole to a distance of some 300 m (985 feet). A gamma-gamma log of the same hole would have sampled a zone of sediments surrounding the well that was only a few centimeters thick and it could not have been used in a cased hole. If cores or cuttings had been taken from the well in which the density log of figure 72 was run, a highly detailed, vertical profile of porosity could have been calculated. Such a profile would be clearly superior to a single, averaged value of porosity as determined in the manner shown in figure 71, but the difference in cost between these two methods is considerable.

Surface gravity measurements are used primarily in a regional search and evaluation study. Borehole gravity meter measurements are warranted only in the case of a detailed site evaluation study and require a well or borehole with a diameter of approximately 18 cm (7 in) or more in order to accept the sonde.

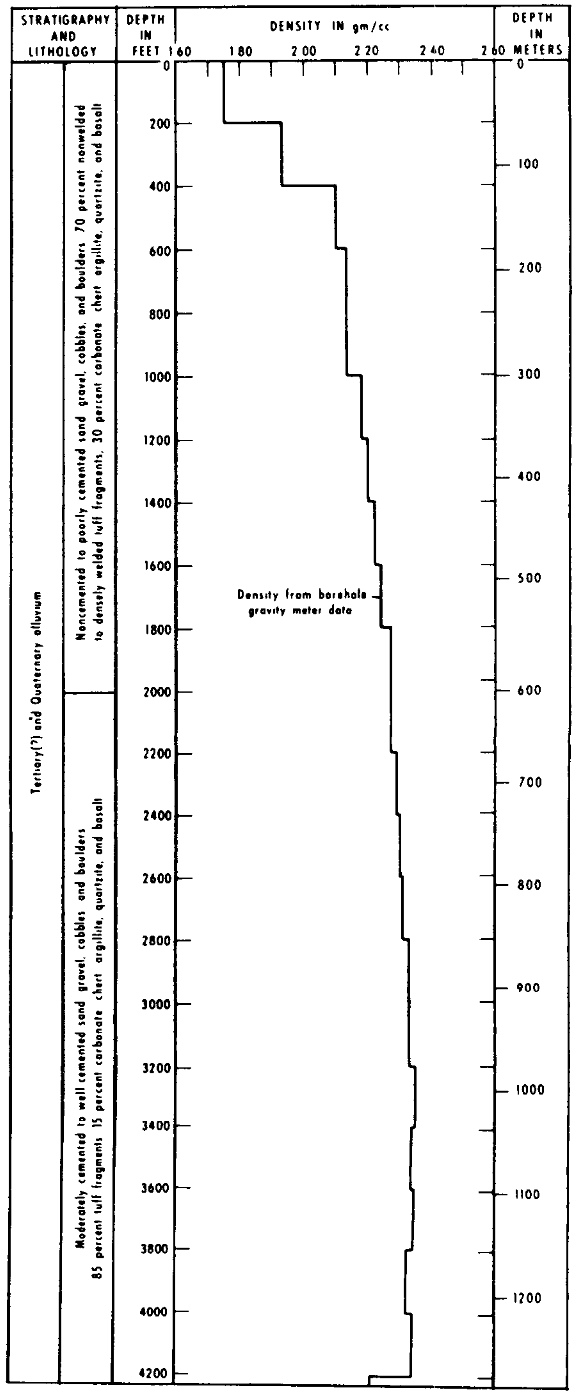

Figure 72.—In situ density log determined with a borehole gravity meter; drill hole UCe-18, Hot Creek Valley, Nev. (after Healey, 1970).

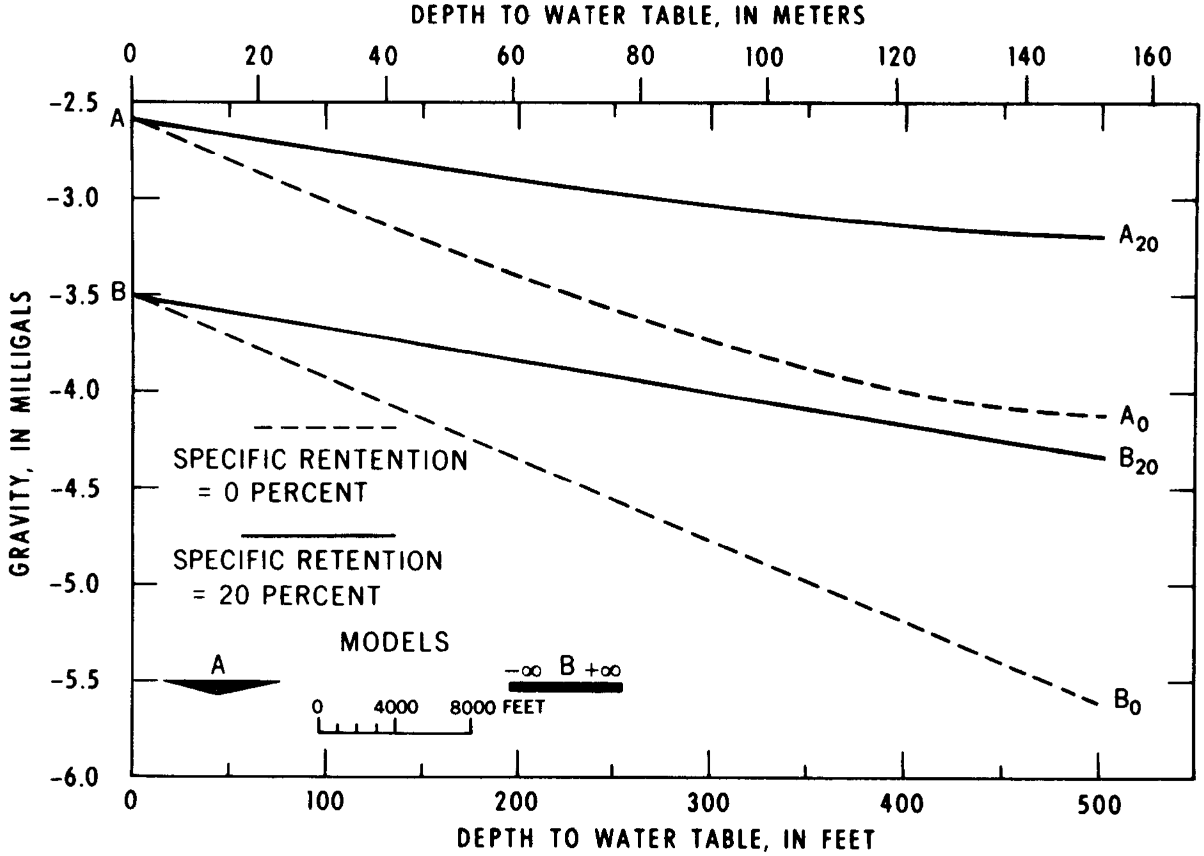

Figure 73.—Plots of gravity values versus depth to the water table for aquifers having a porosity of 33 percent and specific retentions of 0 percent and 20 percent, respectively. A, triangular aquifer, and B, infinitely extensive sheetlike aquifer.

Effect of Ground-Water Levels on Gravity Readings

Water in the interstices of a rock contributes to the total mass of the rock and if porosity is moderate or high, this effect is detectable with a gravity meter. For example, gravity effects resulting from changes of water level in two different aquifers are shown in figure 73. One of these aquifers is an idealized buried stream channel with triangular cross section and the other is a sheetlike deposit of unconsolidated sediment. The gravity effects plotted in this figure are the largest that would be observed, which, for the buried channel, are measured over its center. The physical properties of the rocks employed in calculating the gravity effects displayed by this model were as follows: bedrock density, 2.67 gm/cm^3; bedrock porosity, 0 percent; dry bulk density of the unconsolidated material, 1.79 gm/cm^3; porosity, 33 percent. Curves for two different values of specific retention (0 and 20 percent) in the unsaturated zone are shown. Curves for materials with intermediate values of specific retention fall between the two curves shown in the figure.

A water table decline of approximately 30 meters (100 feet) in a sheetlike aquifer produces a maximum gravity change of 0.42 mgal if the specific retention of the deposit has the limiting value of zero. If the specific retention is 20 percent, a more realistic value, the gravity change is only 0.17 mgal. Because of the peculiarity of the gravitational field of an infinite sheet, its gravity effect is the same regardless of the distance to the point of measurement, that is, the depth to the water table. Furthermore, the slopes of the curves from this model are linear and are a function of the specific yield. If water-level declines in a water-table aquifer of this configuration are monitored with a gravity meter the results can be translated into a measure of the aquifer's specific yield. In areas of long-period water-table decline, repeated gravity measurements, coupled with water-level observations at a few wells, would suffice for a calculation of specific yield, independent of well tests.

This use of the gravity method requires the utmost in precision and accuracy. A gravity difference of 0.17 mgal is a small one to measure accurately and its achievement depends on accuracy at every stage of the data reduction process.

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