8766 March, 1972 HY 3

Journal of the

HYDRAULICS DIVISION

Proceedings of the American Society of Civil Engineers

BIAS IN COMPUTED FLOOD RISK

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INTRODUCTION

The probability of exceeding a given flood size obtained from a mathematically fitted flood-frequency curve is a biased estimate of flood risk as demonstrated by Beard (2). This bias, which is due largely to the time-sampling error of a T-year peak estimated from a finite series of annual peaks, was evaluated by Beard for streamflow sites where the population of annual peaks was assumed to be normal or log-normal. In a later report, Beard (3) notes that the same amount of bias is appropriately applicable to Pearson Type III distributions having small skew coefficients. The Work Group on Flood Frequency Methods, Hydrology Committee, Water Resources Council, which established log-Pearson Type III distributions as the uniform method of flood-frequency analysis for federal agencies, concluded that the bias should be given further study (4). According to Stratton and others (9), the unreliability in benefit-cost ratios caused by inaccuracy in hydrologic, hydraulic and economic factors requires that design flood criteria be adjusted upward to assure a reasonable margin of functional safety.

All flood-frequency curves are in error to some extent, whether based on observed annual peaks or estimated from generalized relations. There is no way to tell whether a particular curve is too high or too low. Its accuracy, however can be appraised by considering the standard error of estimated T-year peak flows given by the curve. This accuracy can then be used to compute the flood-risk curve that should be used to compute the average annual cost of potential flood damage or to compute flood insurance rates. Flood-risk curves are sufficiently higher than flood-frequency curves to represent the risk due to inaccuracy as well as that due to the estimated frequency of

Note.—Discussion open until August 1, 1972. To extend the closing date one month, a written request must be filed with the Executive Director, ASCE. This paper is part of the copyrighted Journal of the Hydraulics Division, Proceedings of the American Society of Civil Engineers, Vol. 98, No. HY3, March, 1972. Manuscript was submitted for review for possible publication on June 23, 1971.

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flooding. Computed average annual potential damage therefore, will balance the actual average annual damage over a large number of projects.

This paper shows how flood risk can be computed for both gaged and ungaged sites at which the population of annual peaks can be assumed to follow a log-Pearson Type III distribution. Flood risk is first computed as the average probability of exceedance of T-year peaks estimated from samples of a given size. Then this average probability of exceedance is related to the standard error of the estimated T-year peaks to give a procedure for estimating flood risk at ungaged sites. Details on the statistical problems encountered in evaluating flood risk and the accuracy of flood-frequency curves are omitted so that the paper may focus on general concepts.

BIAS IN FLOOD RISK

Flood risk is commonly appraised by computing the annual premium (without overhead) required to equal average annual flood damage. The average annual damage is computed by estimating the dollar value of damage that

TABLE 1.—COMPUTATION OF AVERAGE ANNUAL FLOOD DAMAGE FOR FLOOD-FREQUENCY CURVE SHOWN IN FIG. 1[a]

Stage, in feet (meters) (1)	Exceedance probability (2)	ΔP (3)	Damage, in dollars (4)	Average damage in range, in dollars (5)	Average annual damage, in dollars (6)
12.5 (3.81)	0.105		0		
		0.038		500	19
13.0 (3.96)	0.067		1,000		
		0.027		1,500	40
13.5 (4.11)	0.040		2,000		
		0.017		2,500	42
14.0 (4.27)	0.023		3,000		
		0.013		3,650	48
14.65 (4.47)	0.01		4,300		
		0.01		5,300	53
15.15 (4.62)	0.005		5,300		
Total average annual flood damage, in dollars					202

[a] Note: ΔP, the probability that the flood stage will be within the indicated range, is the difference between the exceedance probabilities at the limits of the range.

would be caused by floods that reach selected ranges of flood stage. This computation is followed by cumulating the products of the average damage in each range times the associated probability of a flood in that range. The procedure is illustrated in Table 1 in which the exceedance probabilities are those obtained from the flood-frequency curve shown in Fig. 1. In this example, flood damage to a structure in the flood plain starts at stage of 12.5 ft (3.81 m) and increases at the rate of $2,000 per ft of stage. Average damage by floods that exceed the stage corresponding to an exceedance probability of 0.01 is assumed equal to that which would occur at a stage that has an exceedance probability of 0.005. In Table 1, for example, the average annual damage for floods that exceed a stage of 14.65 ft (4.47 m) is assumed to be $5,300, which is the damage that would occur at a stage of 15.15 ft (4.62 m), and the probability of this damage is used as 0.01 because the stage would be above 14.65 ft (4.47 m) in 1 % of the years. The one-to-one relation between stage and discharge implied by the ordinate scale in Fig. 1 is, of course, hypothetical.

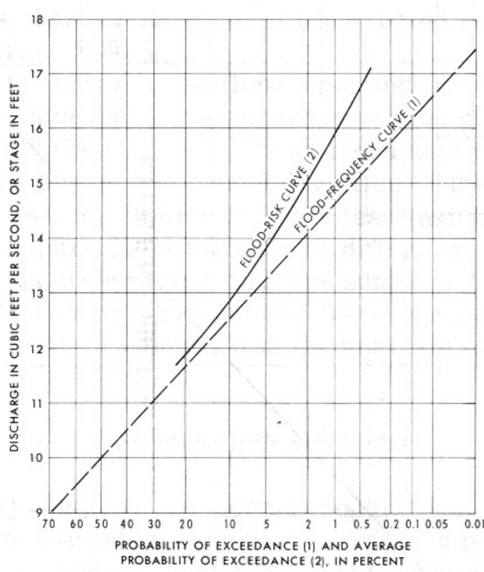

FIG. 1.—FLOOD-RISK CURVE FOR FLOOD-FREQUENCY CURVE BASED ON SAMPLES OF 10 ANNUAL PEAK FLOWS

Computation of average annual flood damage on basis of a flood-frequency curve as shown in Table 1 is incorrect, because the flood-frequency curve gives a biased estimate of flood risk. Average annual flood damage should be computed from the flood-risk curve shown in Fig. 1, which is the flood-frequency curve adjusted for bias using the equation given by Beard (2). When the flood-risk curve is used to compute average annual flood damage as in the previous example, assuming the same relation between stage and damage, the required annual premium is found to be $346, which is 71 % larger than the annual premium of $202 computed in Table 1.

The reason the flood-risk curve in Fig. 1 differs from the flood-frequency curve is obvious from a study of Fig. 2 in which the distribution of estimated 50-yr flood peaks computed from samples of 10 peaks is shown by points plotted to the right of the 2 % probability line. The estimated 50-yr peak flow, $\hat{X}_{50}$, for each of 100 samples of size 10 was computed as

$$\hat{X}_{50} = \overline{X} + ks \quad \text{. . . . .} \quad (1)$$

in which $\overline{X}$ is the mean of each 10-item sample; k = the standardized deviate (7)(k = 2.054 for $\hat{X}_{50}$ in a normal distribution was used in this example); and s = the sample standard deviation computed as

$$s = \sqrt{\frac{\sum_{i=1}^{N} (X_i - \overline{X})^2}{(N - 1)}} \quad \text{. . . . .} \quad (2)$$

in which X_i denotes an annual peak discharge and N is the number of annual

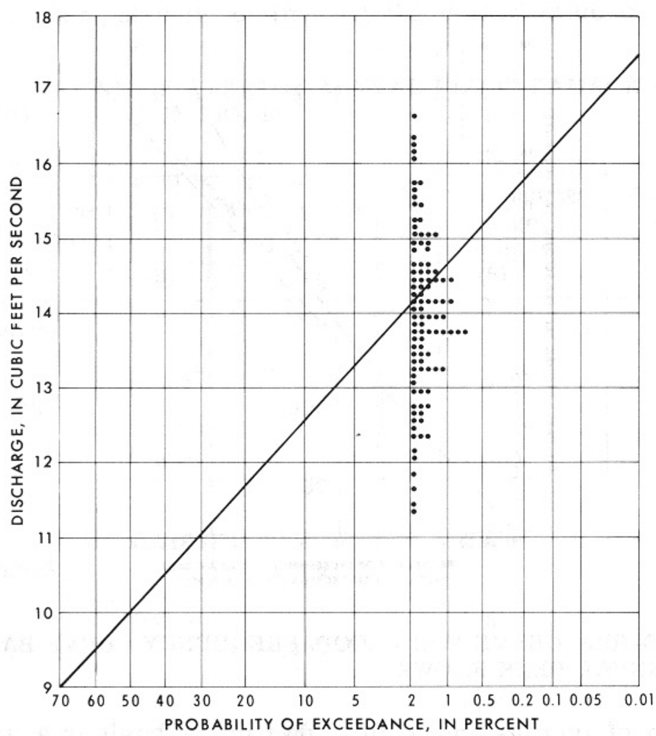

FIG. 2.—DISCHARGE OF 50-YR PEAKS BASED ON SAMPLES OF 10 FROM NORMAL POPULATION OF ANNUAL PEAKS REPRESENTED BY LINE

peaks in the sample. The population of annual peaks flows from which the samples were drawn has a mean of 10 cfs (0.28 m^3/s) and a standard deviation of 2 cfs (0.057 m^3/s).

The discharge of the highest estimated 50-yr peak flow shown in Fig. 2 has a true exceedance probability of 0.048 % and that of the lowest estimated 50-yr peak flow is 25 %. The average probability of exceedance of these two estimates, each of which is equally likely to be computed, is about 12.5 % as compared with the 2 % given by the reciprocal of the recurrence interval.

The average exceedance probability of all 100 points shown in Fig. 2 is 4.11 %, while the average discharge of the 100 points (13.96 cfs or 0.395 m^3/s) has an exceedance probability of only 2.4 %.

The term average exceedance probability used in this paper for the abscissa scale of the flood-risk curve in Fig. 1 is the same as expected probability defined by Beard (2) as being the estimated probability for each project such that the estimated and true probabilities will average out properly over a large number of projects. Thus, the concept of average exceedance probability should provide the basis for calculating insurance rates for flood risk and for comparing the true cost of alternate designs. The term T-year peak is defined in the normal manner as the peak discharge corresponding to the exceedance probability of the frequency curve, and T is the reciprocal of the exceedance probability.

The theoretical average exceedance probability for 50-yr peaks computed by Eq. 1 from samples of size 10 drawn from a normal population is 4.08 % as compared to the 4.11 % given by the points in Fig. 2. For 50-yr peaks computed from samples of size 50, the scatter about the mean of the peaks would be considerably less than for samples of size 10 and the theoretical average exceedance probability is 2.36 %.

Part of the difference between the flood-risk and the flood-frequency curve in Fig. 1 is due to the fact that the standard deviation of small samples computed by Eq. 2 tends to be too small in comparison with the standard deviation of the population. The average standard deviation of the 100 samples used in Fig. 2, for example, is 1.936, which is 3.2 % less than the standard deviation of the population as compared to a theoretical difference (8) of 2.8 %. The fact that the bias in standard deviation and the scatter of the estimated peaks about their mean both tend to increase the average exceedance probability of estimated T-year peak flows complicates the analysis of bias in flood risk.

EVALUATION OF BIAS

For samples from normal or log-normal populations of annual peak flows, the theoretical average exceedance probability of flood peaks computed by Eq. 1, is given in Table 2, under the column heading for a zero coefficient of skew. In the absence of a method for computing similar theoretical results for skewed distributions, the average exceedance probability of T-year peaks estimated from samples of size N drawn from Pearson Type III populations of known skew was evaluated numerically by a random sampling procedure. The average exceedance probabilities were obtained by averaging the true probabilities of exceedance of T-year peaks computed by use of Eqs. 1 and 2, and the results are summarized in Table 2 under the column headings for the appropriate coefficient of skew. The k values used in Eq. 1 were the k values associated with the skew coefficients of the population of annual peaks. All results in Table 2 include the effect of bias in the standard deviation computed by Eq. 2 as well as the bias due to the scatter of the estimates about their mean.

In Table 3, the effect of bias in standard deviation has been removed to show the average exceedance probability due solely to the scatter (time-sampling error) of estimated T-year peak flows about their mean. These adjusted probabilities approximate what would be obtained by a random sampling

TABLE 2.—AVERAGE EXCEEDANCE PROBABILITY OF T-YEAR PEAKS ESTIMATED FROM SAMPLES OF ANNUAL PEAKS FROM PEARSON TYPE III POPULATIONS OF KNOWN SKEW[a]

T	N	Average Exceedance Probability, $\overline{P}$, for Indicated Coefficient of Skew of Population, C_s						
		-1.5	-1.0	-0.5	0 (normal)	0.5	1.0	1.5
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
5	5	25.01	24.20	24.01	24.20	25.07	25.73	26.46
5	10	23.17	22.72	22.40	22.24	22.26	22.78	23.14
5	25	21.44	21.31	21.00	20.84	20.80	20.90	21.02
5	50	20.93	20.70	20.56	20.41	20.37	20.44	20.54
5	100				20.25			
5	∞	20.00	20.00	20.00	20.00	20.00	20.00	20.00
10	5	17.59	16.02	15.39	15.34	15.78	16.44	17.12
10	10	14.80	13.48	12.94	12.61	12.78	13.20	13.60
10	25	12.32	11.56	11.24	11.03	10.99	11.20	11.44
10	50	11.40	10.88	10.66	10.53	10.48	10.60	10.80
10	100				10.26			
10	∞	10.00	10.00	10.00	10.00	10.00	10.00	10.00
20	5	14.16	11.84	10.76	10.29	10.50	11.00	11.64
20	10	10.83	8.85	7.98	7.49	7.62	7.92	8.32
20	25	8.01	6.74	6.22	5.97	5.95	6.09	6.30
20	50	6.77	5.95	5.64	5.49	5.47	5.53	5.63
20	100				5.22			
20	∞	5.00	5.00	5.00	5.00	5.00	5.00	5.00
50	5	10.62	8.90	7.35	6.58	6.63	6.92	7.38
50	10	8.33	5.76	4.61	4.08	4.05	4.22	4.53
50	25	5.39	3.67	2.99	2.78	2.71	2.80	2.95
50	50	4.03	2.91	2.50	2.36	2.34	2.39	2.48
50	100				2.20			
50	∞	2.00	2.00	2.00	2.00	2.00	2.00	2.00
100	5	11.02	7.62	5.84	4.92	4.83	5.04	5.40
100	10	7.35	4.47	3.21	2.70	2.58	2.70	2.98
100	25	4.43	2.51	1.84	1.59	1.53	1.58	1.69
100	50	3.06	1.79	1.40	1.28	1.25	1.28	1.34
100	100				1.14			
100	∞	1.00	1.00	1.00	1.00	1.00	1.00	1.00
200	5	10.46	6.75	4.82	3.79	3.62	3.76	4.04
200	10	6.78	3.65	2.40	1.81	1.70	1.76	2.01
200	25	3.87	1.83	1.17	0.93	0.88	0.91	0.99
200	50	2.51	1.17	0.82	0.69	0.67	0.69	0.73
200	100				0.60			
200	∞	0.50	0.50	0.50	0.50	0.50	0.50	0.50

TABLE 2.—*CONTINUED*

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
1,000	5	9.46	5.52	3.32	2.28	2.00	2.02	2.19
1,000	10	6.12	2.62	1.28	0.80	0.68	0.69	0.83
1,000	25	3.27	1.04	0.44	0.29	0.25	0.26	0.29
1,000	50	1.92	0.55	0.24	0.18	0.16	0.17	0.19
1,000	100				0.14			
1,000	∞	0.10	0.10	0.10	0.10	0.10	0.10	0.10

[a] Note: T = recurrence interval, in years; N = number of annual peaks in each sample; and $\overline{P}$ = average exceedance probability of many estimated T-year peaks.

procedure if Eq. 2 were adjusted to remove the bias in standard deviation. The theoretical standard errors of T-year peaks (5) are also shown in Table 3 to define relations between standard error and average exceedance probability for given values of T and of coefficient of skew. Such relations hold also for log-Pearson Type III distributions if the logarithmic standard deviation and skew coefficient of the population are used in entering the table. The theoretical values of SE_{X_T}/σ in Table 3 were obtained by dividing the R values from Table 4 by $\sqrt{N}$.

Relations between standard error of estimated 50-yr peak flows and their average exceedance probability obtained from Table 3 are shown in Fig. 3 as an example. As the average exceedance probability given in Table 3 is due to scatter of the estimated peak flows about their mean, the relations shown in Fig. 3 are applicable to standard errors from any source so long as the distribution of the errors is approximately the same as that of the time-sampling error.

APPLICATION TO UNGAGED SITES

At sites where no record of annual peaks has been obtained but where T-year peak flows have been estimated by multiple regression analyses, the average probability of exceedance of the generalized estimates can be obtained from Tables 2 and 3. If the T-year peaks used to define the regressions are computed using a standard deviation adjusted for bias, the standard error of the estimates can be used with Table 3 or with curves such as those shown in Fig. 3 to obtain the average probability of exceedance. If, however, the T-year peaks are computed using Eqs. 1 and 2, the generalized estimates will tend to be too low and the average exceedance probability shown in Table 3 will need to be increased by $(N_U/N_G)^{1.1}$ times the amount by which $\overline{P}$ in Table 2 exceeds that in Table 3 at an N value equal to N_U, in which N_G is the average length of record at the gaged sites used in the regression and N_U is the accuracy in equivalent years of record for peaks estimated from the regression. If, for example, 50-yr peaks estimated from a regression based on records with an average length of 25 yr have a standard error of prediction (6) equal to that of 50-yr peak flows based on samples of 10 annual peaks observed at the site, N_U would be 10 and N_G would be 25. Then if the population of the annual peaks is

TABLE 3.—RELATION OF AVERAGE EXCEEDANCE PROBABILITY TO TIME-SAMPLING STANDARD ERROR OF ESTIMATED T-YEAR PEAK FLOWS[a]

T	N	AVERAGE EXCEEDANCE PROBABILITY, $\overline{P}$, AS A PERCENTAGE, AND SE_{X_T}/σ FOR INDICATED COEFFICIENT OF SKEW, C_s									
		-1.0		-0.5		0		0.5		1.0	
		SE_{X_T}/σ	$\overline{P}$	SE_{X_T}/σ	$\overline{P}$	SE_{X_T}/σ	$\overline{P}$	SE_{X_T}/σ	$\overline{P}$	SE_{X_T}/σ	$\overline{P}$
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
5	5	0.410	21.73	0.456	22.10	0.520	22.96	0.594	23.42	0.665	23.90
5	10	0.290	21.45	0.323	21.35	0.368	21.62	0.420	21.62	0.470	22.04
5	25	0.183	20.73	0.204	20.66	0.233	20.60	0.266	20.55	0.297	20.61
5	50	0.130	20.43	0.144	20.41	0.165	20.30	0.188	20.24	0.210	20.31
10	5	0.460	13.00	0.513	13.25	0.604	13.80	0.728	13.95	0.875	14.05
10	10	0.325	12.10	0.363	11.97	0.427	11.94	0.515	12.02	0.618	12.31
10	25	0.206	11.06	0.230	10.92	0.270	10.80	0.326	10.67	0.391	10.89
10	50	0.146	10.67	0.162	10.54	0.191	10.41	0.230	10.32	0.277	10.40
20	5	0.507	8.75	0.571	8.77	0.686	8.78	0.859	8.88	1.084	8.86
20	10	0.359	7.53	0.403	7.10	0.485	6.90	0.607	7.02	0.764	7.22
20	25	0.227	6.30	0.255	5.93	0.307	5.78	0.384	5.73	0.483	5.85
20	50	0.160	5.77	0.180	5.51	0.217	5.40	0.272	5.35	0.342	5.38
50	5	0.555	6.27	0.641	5.62	0.788	5.28	1.023	5.07	1.344	5.19
50	10	0.392	4.62	0.453	3.86	0.557	3.63	0.723	3.63	0.950	3.68
50	25	0.248	3.34	0.287	2.79	0.353	2.66	0.458	2.59	0.601	2.66
50	50	0.176	2.76	0.203	2.42	0.249	2.32	0.323	2.28	0.425	2.32
100	5	0.587	5.24	0.688	4.23	0.861	3.84	1.142	3.49	1.537	3.55
100	10	0.415	3.45	0.486	2.68	0.609	2.36	0.808	2.20	1.087	2.26
100	25	0.263	2.22	0.308	1.67	0.385	1.52	0.511	1.42	0.688	1.49
100	50	0.186	1.68	0.217	1.35	0.272	1.24	0.361	1.21	0.486	1.24
200	5	0.612	4.38	0.731	3.33	0.929	2.90	1.256	2.44	1.727	2.46
200	10	0.432	2.78	0.516	1.96	0.657	1.58	0.888	1.43	1.221	1.45
200	25	0.273	1.60	0.327	1.06	0.416	0.86	0.562	0.80	0.772	0.84
200	50	0.193	1.10	0.231	0.78	0.294	0.67	0.397	0.65	0.546	0.66
1,000	5	0.651	3.12	0.813	2.02	1.075	1.40	1.509	1.13	2.107	0.87
1,000	10	0.460	1.98	0.575	0.98	0.760	0.63	1.067	0.54	1.490	0.50
1,000	25	0.291	0.90	0.364	0.40	0.481	0.27	0.675	0.20	0.942	0.21
1,000	50	0.206	0.50	0.257	0.22	0.340	0.17	0.477	0.14	0.666	0.15

[a] Note: T and N are as in Table 2; $\overline{P}$ = average exceedance probability from Table 2 adjusted to remove effect of bias in sample standard deviation; SE_{X_T} = theoretical standard error of the estimated peak flows; and σ = standard deviation of the population of annual peak flows.

TABLE 4.—VALUES OF R FOR USE IN APPRAISING ACCURACY OF *T*-YEAR PEAK ESTIMATED FROM OBSERVED ANNUAL PEAKS FROM PEARSON TYPE III POPULATION OF KNOWN SKEW[a]

T	*R* for Indicated Coefficient of Skew of Population								
	-1.5	-1.0	-0.5	-0.2	0	+0.2	+0.5	+1.0	+1.5
2	0.845	0.933	0.983	0.997	1.000	0.997	0.983	0.933	0.845
5	0.819	0.916	1.020	1.102	1.164	1.229	1.328	1.486	1.638
10	0.926	1.029	1.148	1.258	1.350	1.454	1.629	1.956	2.325
20	1.006	1.134	1.276	1.414	1.534	1.674	1.921	2.416	3.010
25	1.026	1.163	1.316	1.500	1.591	1.747	2.013	2.560	3.228
50	1.075	1.246	1.433	1.608	1.763	1.950	2.288	3.006	3.903
100	1.107	1.313	1.538	1.742	1.925	2.146	2.554	3.438	4.574
200	1.130	1.367	1.633	1.868	2.078	2.334	2.809	3.861	5.239
1,000	1.156	1.456	1.819	2.129	2.403	2.739	3.374	4.712	6.760

[a] Note: $R = SE_{X_T}\sqrt{N}/\sigma$ in which SE_{X_T} is the standard error of the estimated *T*-year peak, σ = the standard deviation of the population of annual peaks, and N = their number.

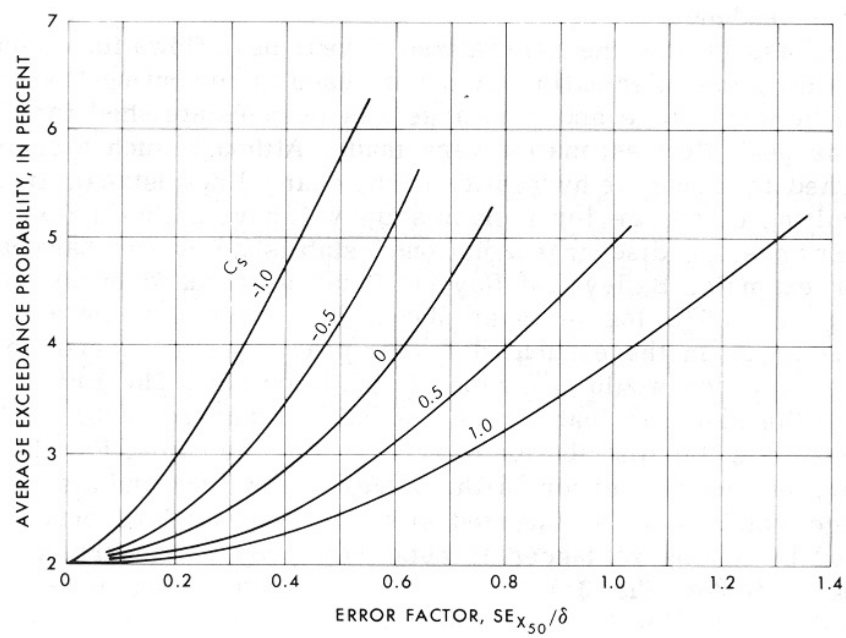

FIG. 3.—EFFECT OF STANDARD ERROR, 50-YR PEAKS

normal or log normal, the average exceedance probability of 3.63 given in Table 3 at N equal to 10 should be increased by (4.08 - 3.63) times $(10/25)^{1.1}$ = 0.45 (0.365) = 0.16 to give 3.79 for the average probability of exceedance. To use Tables 2 and 3, the population of annual peaks at the ungaged site must be assumed to follow a Pearson (or log-Pearson) Type III distribution and the skew coefficient and standard deviation must be estimated.

For T-year peaks obtained by regression of the logarithmic mean, standard deviation, and skew coefficient with basin characteristics, the generalized logarithmic standard deviation can be used as σ for the purpose of computing the error factor, SE_{X_T}/σ, with which to enter Table 3, and the generalized logarithmic skew coefficient can be used as the known skew. A procedure would have to be developed, however, for estimating the standard error (SE_{X_T}) of the T-year peaks estimated from the generalized statistics.

For T-year peaks obtained by regression with basin characteristics, the standard error of estimate can be obtained from the standard error of estimate of the regressions by giving proper attention to the relation between time-sampling error and space-sampling error (6). Thus for such estimates it is only necessary to select a combination of logarithmic skew coefficient and logarithmic standard deviation to give a flood-frequency curve reasonably compatible with the T-year peaks. This can be done by selecting a reasonable logarithmic skew coefficient and by then finding what logarithmic standard deviation defines a flood-frequency curve that gives the best fit to all the T-peaks estimated for a given site. The selection of the skew coefficient can be refined by comparing the fit of several trial values. The logarithmic skew coefficient and logarithmic standard deviation of such a curve of best fit can be used with curves such as those shown in Fig. 3 to obtain the average probability of exceedance.

Regardless of how the generalized T-year peak flows for an ungaged site are obtained, the information cannot be used in computing flood risk until a relation between stage and discharge has been established for the site for which the peak flow estimates were made. Although such a relation can be established by using the hydraulics of the channel downstream from the site, the resulting discharge for a given stage will have a standard error of estimate. From stage-discharge relations established by step backwater analysis, for example, Bailey and Ray (1) found a standard error in estimated discharge of 0.075 log units or about 18 %. Even if there were no time-sampling error in the estimated T-year peak flows, this error in the stage-discharge relation would cause bias in the flood risk. The amount of the bias would be the same as that for an equivalent standard error in T-year peak flows assuming the distribution of errors to be the same. For the usual condition where there is error in the T-year peak flow and also in the stage-discharge relation at an ungaged site, the two standard errors should be combined by adding variances to obtain the proper standard error to use in entering Table 3 or Fig. 3.

APPLICATION TO GAGED SITES

For gaged sites at which T-year peak flows are computed by fitting log-Pearson Type III distribution curve to the statistics of a sample of annual peaks, the average exceedance probability depends on whether or not the skew coefficient of the population is substituted for the sample skew coefficient and on whether the bias has been removed from the standard deviation. If an average or generalized skew coefficient is used to determine k in Eq. 1 and if the standard deviation is computed by Eq. 2, the average exceedance probability of the T-year peak flows thus estimated is as given in Table 2. If the standard deviation is adjusted for bias, the average exceedance probability would be as given in Table 3.

If the sample skew coefficient is used to define k in Eq. 1, the average exceedance probability of T-year peaks will be somewhat different than that given in Tables 2 and 3. Although it is not the purpose of this paper to propose a procedure for evaluating the average exceedance probability of T-year peaks computed using sample skew coefficients, an idea of this difference is given in Table 5, which is based on computations using standard deviations computed by Eq. 2.

TABLE 5.—EFFECT OF USING SAMPLE SKEW INSTEAD OF POPULATION SKEW TO COMPUTE T-YEAR PEAK FLOWS

T	N	Adjustment to Values of Average Exceedance Probability Shown in Table 2 for Indicated Skew Coefficient of Population				
		-1.0	-0.5	0.5	1.0	Average
10	10	-1.25	0	0.58	0.51	-0.04
10	50	-0.27	-0.01	0.17	0.23	0.03
50	10	-1.93	-0.12	1.14	1.37	0.12
50	50	-0.39	0	0.26	0.34	0.05
1,000	10	-0.77	0.62	1.18	1.30	0.58
1,000	50	0.47	0.27	0.15	0.17	0.26

Whether it would be better to use a generalized estimate of the skew coefficient of the population in computing T-year peak flows from observed annual peaks or to use the sample skew coefficient is outside the scope of this paper. The main point in regard to flood risk at gaged sites is that 50-yr flood peaks estimated from 10 annual peaks at each of many sites have flood risks that average about twice as large as that given by the reciprocal of 50 regardless of the procedure used in computing the 50-yr peak flows.

CONCLUSIONS

Flood-frequency curves fitted mathematically to observed annual peak flows or based on generalized estimates of T-year peaks at ungaged sites give too small an estimate of flood risk. This bias in flood risk is due to the inaccuracy inherent in all such curves. For ungaged sites the amount of the bias can be estimated from relations between it and the standard error of the estimated T-year peaks if the population of annual peaks at the site is assumed to follow a log-Pearson Type III distribution and if the logarithmic standard deviation and skew coefficient of the population are assumed to be the same as for the estimated flood-frequency curve. For these sites the standard error of the estimated T-year peak flow should be combined with the standard error in the estimated stage-discharge relation before the bias in flood risk is appraised. For gaged sites where T-year peak flows are estimated from short records of annual peaks, the amount of bias can be estimated from the number of years of record if a generalized skew coefficient is used as being that of the population of annual peaks. The estimated bias for gaged or for ungaged sites should be applied as an adjustment to the exceedance probability obtained from a flood-frequency curve to define a flood-risk curve, and the flood-risk curve should be used to compute the annual premium required to balance expected flood damage at a large number of sites.

In view of the relation between bias and standard error, it is imperative that the accuracy of all procedures used in deriving the data used in the evaluation of flood risk be appraised in terms of estimated standard error so that the variances can be added to obtain the standard error that should be used in appraising flood risk.

The fact that the amount of bias in flood risk is independent of the standard deviation of the population of annual peaks shows that 10 years of record at a site with a large variability in annual peak flows is as good as 10 years of record at a site with small variability even though the standard error of estimated T-year peaks is greater. The same amount of additional record at both sites would give the same reduction in flood risk and thus both sites would have the same relation between flood risk and cost of collecting records of flood peaks.

ACKNOWLEDGMENT

Publication of this paper was authorized by the Director, U.S. Geological Survey.

APPENDIX I.—REFERENCES

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APPENDIX II.—NOTATION

The following symbols are used in this paper:

C_s = coefficient of skew of annual peak flows or of their logarithms;
k = number of standard deviation units from sample mean;
N = number of annual peak flows;
N_G = average length of record, in years, at gaged sites;
N_U = accuracy in equivalent years of record for T-year peak flows at ungaged sites estimated from regression;
P = probability of exceedance; $\overline{P}$ is average probability of exceedance;
R = factor relating SE_{X_T} to σ and N; equals $SE_{X_T}\sqrt{N}/\sigma$;
SE_{X_T} = standard error of estimated T-year peak flow;
s = standard deviation of sample of annual peak discharge or of their logarithms;
T = recurrence interval, in years;
X = discharge of annual peak;
$\overline{X}$ = mean of annual peak discharges;
X_T = T-year peak discharge, $\hat{X}_T$ is X_T estimated as $\overline{X} + ks$; and
σ = standard deviation of population of annual peak discharges or of their logarithms.

8766 BIAS IN COMPUTED FLOOD RISK

KEY WORDS: **Benefit-cost ratios; Economic analysis; Flood control;** Flood plain zoning; Floods; **Hydraulics; Hydrology;** Risks; **Statistical analysis;** Statistical distributions; Streamflow

ABSTRACT: Flood damage computed from flood-frequency curves fitted mathematically to observed annual peak flows or estimated by regression with basin characteristics, is a biased estimator of flood risk. The inaccuracy inherent in any flood-frequency curve increases the annual premium that would have to be charged to break even over a large number of projects. For ungaged sites where the population of annual peaks can be assumed to follow a log-Pearson Type III distribution, the true risk is evaluated by relating it to the standard error of estimate of the regression used to define the flood-frequency curve. In view of this relation between bias and error, the accuracy of all procedures used in evaluating flood frequency should be appraised in terms of standard error so that the proper flood risk can be obtained.

REFERENCE: Hardison, Clayton H., and Jennings, Marshall E., "Bias in Computed Flood Risk," **Journal of the Hydraulics Division,** ASCE, Vol. 98, No. HY3, **Proc. Paper 8766**, March, 1972, pp. 415-427

VOL.99 NO.HY1. JAN. 1973

JOURNAL OF THE HYDRAULICS DIVISION

PROCEEDINGS OF THE AMERICAN SOCIETY OF CIVIL ENGINEERS

BIAS IN COMPUTED FLOOD RISK[a]

Discussion by Bernard W. Gould

BERNARD W. GOULD.[3]—Sampling bias in flood damage risk estimation. The authors have drawn attention to an interesting and important topic. However, the writer disagrees with the authors' main conclusions; he is of the opinion that sampling bias in flood damage estimation not only is insignificant in comparison with sampling errors, but also is of the opposite sign to that suggested by the authors in their paper. If this opinion is correct, the tables given by the authors, and their recommended procedures to increase the value of flood damage risk estimates, are misleading because they would not remove bias, but rather greatly increase the already existing small bias.

This opinion is based on a brief theoretical study and sampling experiments which have been performed by the writer. The limited space available for discussion does not permit presentation of full details of the argument, a copy of which has been sent to the authors. A limited number of copies are available for distribution to interested readers.

Under the heading Bias in Flood Risk the authors describe a sampling experiment in which they appear to demonstrate that, on the average, exceedance values for samples are in excess of the true exceedance values, as given by the population parameters. This would seem to establish that by using parameters estimated from samples the probable damage would be, on the average, overestimated.

However, the authors then state that exceedance values estimated from samples should be further increased to remove bias in flood damage risk calculations.

In order to assess the bias in estimated flood damage risk, it is necessary to examine that particular statistic, and not others, such as exceedance values.

There are two main sources of bias (or systematic error, as distinct from random error) in flood damage risk calculations. First, the use of finite increments of stage introduces computational errors; second, the probable er-

[a] March, 1972, by Clayton H. Hardison and Marshall E. Jennings (Proc. Paper 8766).

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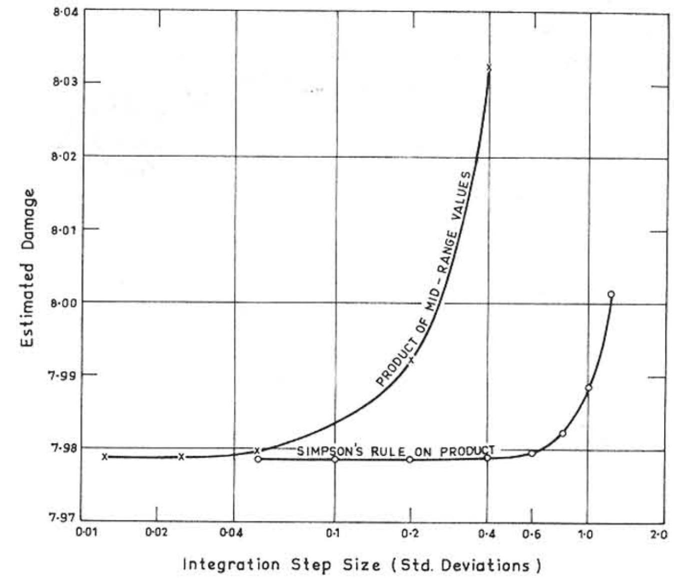

FIG. 4.—DEGREE OF ERROR-SIZE OF INCREMENT

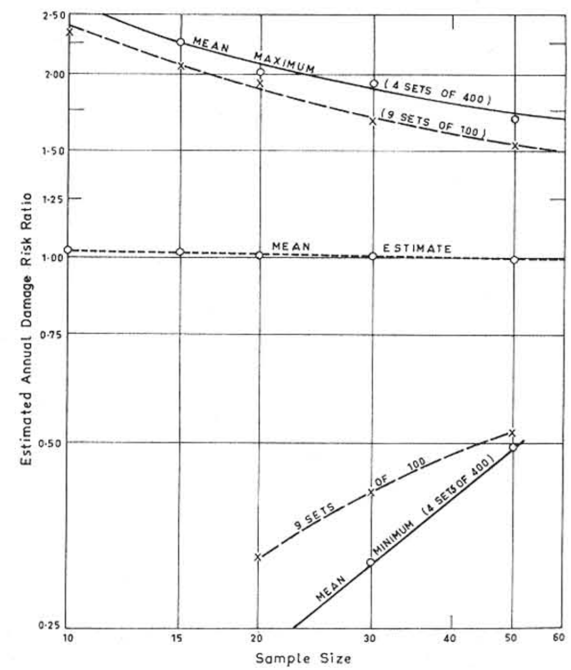

FIG. 5.—SUMMARIZED RESULTS

rors of sampling may cause the statistic to have an inherent bias.

The writer has investigated the degree of error associated with the size of increment used in calculation. Fig. 4 shows the superiority of the Simpson's rule calculation over the product of midvalues calculation, and indicates that an increment size of 0.3 times the standard deviation is likely to give a sufficient degree of accuracy.

The writer has estimated that the theoretically expected bias is approximately $(25/N)$ % (with the sample estimate of risk being, on the average, greater than the true value) when the stage distribution is normal, the damage-stage relationship is linear, and N is the number of years of records included in a sample.

In the sampling experiments, repeated samples of a nominated size (e.g., 10, 15, 20, 30, and 50—in turn) were drawn at random from a normal popula-

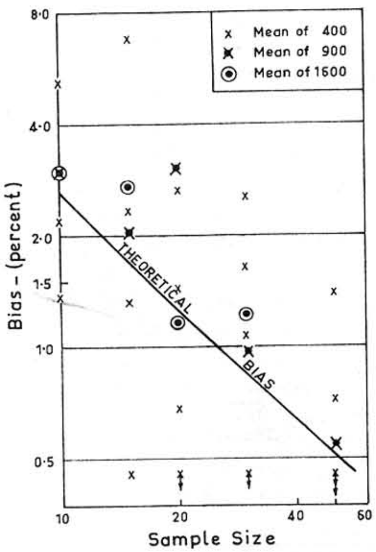

FIG. 6.—THEORETICAL AND EXPERIMENTAL VALUES OF BIAS

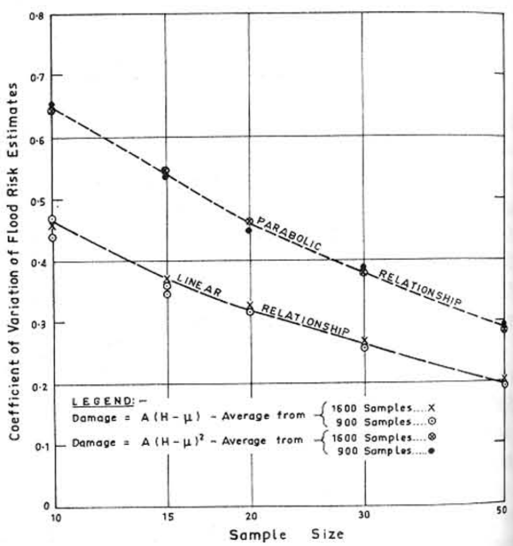

FIG. 7.—COEFFICIENT OF VARIATION OF RISK ESTIMATES

tion having a mean of 100 units, and a standard deviation of 10 units. For each sample, the sample mean and sample standard deviation were calculated, and, from these, the corresponding values of annual flood damage risk were obtained by a Simpson's rule computation.

The summarized results showing the mean values, and average maximum and minimum values are shown in Fig. 5. Fig. 6 shows the theoretical and experimental values of bias for different sample sizes. The estimation of bias from the sampling experiment confirms the theoretical study.

As a matter of interest, Fig. 7 shows the coefficient of variation of risk estimates made from random samples of various sizes for both a linear damage-stage function and a parabolic damage-stage function.

Compared with these estimated sampling errors, the bias of estimate is insignificant, and is of the opposite sign to bias suggested by the authors.

VOL.99 NO.HY7. JULY 1973

JOURNAL OF THE HYDRAULICS DIVISION

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BIAS IN COMPUTED FLOOD RISK[a]

Closure by Clayton H. Hardison,[4] F. ASCE and Marshall E. Jennings,[5] A. M. ASCE

The writers acknowledge with thanks the discussion by Gould. His experiments apparently deal with averaging flood risk over a range of a flood-frequency curve, whereas the writers deal with flood risk averaged over a large number of projects. The writers contend that it is this latter risk that should be used in computing annual costs, and offer the following split-sampling results as evidence that bias in flood risk is essentially as given in the paper. As usual, split-sampling involves using only part of a sample to make an estimate and then using the rest of the sample to check the estimate.

The results of split sampling summarized in Table 6 are based on observed annual peak discharges at 157 stream gaging stations in Parts 1-4 of the U.S. Geological Survey's Water Supply Paper series. These Parts cover most of the United States east of the Mississippi River. A 40-yr period of record for each station was used. The record for the first 20 yr was used to compute a 50-yr peak discharge by the log-Pearson Type III method. This discharge was compared with the annual peaks in the second 20 yr to determine the number of exceedances. Then the procedure was repeated with the second 20 yr being used to define a 50-yr peak for use in counting the exceedances in the first 20 yr. The sums of the exceedances thus determined for each of the stations in each part are shown in the next to the last column of Table 6, and the corresponding percentages of the total years are shown in the last column.

[a] March, 1972, by Clayton H. Hardison and Marshall E. Jennings (Proc. Paper 8766).

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The fact that the exceedance percentages shown in the last column of Table 6 average 3.95% instead of the 2.0% that would be obtained from the reciprocal of the 50-yr recurrence interval is evidence that the risk is considerably larger than that given by a flood-frequency curve. In fact, the values shown in Table 2 represent the minimum amount of bias as they are based on T-year peaks computed using a known skew coefficient. When a skew coefficient based on each sample of annual peaks is used, as it was in the split-sampling experiment, a larger bias is obtained, as evidenced by the fact that the 3.95% from Table 6 for an average skew coefficient of 0.16 is about one percentage point larger than the value obtained from Table 2. This additional bias is due to the larger standard error introduced by the error in estimating the skew coefficient.

TABLE 6.—Exceedances of 50-yr Peak Discharges by Split-Sampling

WSP Part (1)	Number of stations (2)	Total years (3)	Exceedances: Number (4)	Exceedances: As a percentage (5)
1	64	2,560	102	3.98
2	24	960	46	4.79
3	49	1,960	68	3.47
4	20	800	32	4.00
Total	157	6,280	248	3.95

Note: The 50-yr peaks were computed from the statistics of 20 annual peaks using the logarithmic skew coefficient of each sample. The logarithmic skew coefficients average about 0.16.

The writers reiterate that bias in flood risk as defined by them is due primarily to the inaccuracy inherent in any estimate of a T-year peak. The fact that the standard deviation of small samples tends to be too small by about $(25/N)\%$, adds a small amount to the bias in flood risk associated with inaccuracy.

In conclusion, the writers urge that the existence of a sizable bias in flood risk be recognized and that appropriate allowance be made in the economic design of flood related projects.