Addition of Drain Flow as a State Variable in GWM-2005

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Groundwater discharge to drains is an important component of many water-management systems, particularly in agricultural areas where drains capture excess irrigation water that reaches the water table. Many hydrogeologists simulate groundwater/drain interactions with the Drain Package of MODFLOW (Harbaugh, 2005, p. 6-12 to 6-15), which is one of several head-dependent boundary packages available with the code. The nonlinear groundwater-head/discharge function simulated by the Drain Package is such that a simulated drain can only receive groundwater discharge, which occurs when the groundwater head in the cell in which the drain is located is greater than the elevation of the bottom of the drain. If the groundwater head is less than or equal to the drain elevation, there is no flow between the drain and groundwater system. Because simulated drains do not allow flow from a drain to the groundwater system, the Drain Package also has been used by some modelers to simulate groundwater discharge to streams for conditions in which it can be assumed that streams remain gaining throughout a simulation. Drains might also be used to simulate groundwater discharge to wetlands, as described by Harbaugh (2005, p. 6-15).

Previous releases of the Groundwater Management (GWM) Process for MODFLOW-2005 (Ahlfeld and others, 2005, 2009, and 2011) included the capability to manage groundwater interactions with streams by use of either the STR or SFR streamflow-routing packages. These packages were included in GWM-2005 for a number of reasons, including their ability to account for increases and decreases in streamflow along a stream segment that consists of several connected stream reaches. The Drain Package, however, does not group drain cells into drain segments and, as a result, does not calculate total drain flow within a group of drains that might be of interest to the modeler.

GWM-2005 was recently enhanced to include the use of state variables as components of a groundwater-management problem (Ahlfeld and others, 2011). The initial release of the State Variables Package allowed for three types of simulated state variables—groundwater heads, streamflows, and aquifer-storage changes. This document describes the addition of a fourth type of state variable—drain flow. The document includes a description of the revised input instructions for the State Variable Package to include drains and a sample groundwater-management problem that includes drains as state variables.

Description of Drain Flow as a State Variable in GWM-2005

Drain state variables provide measures of the flow rate or flow volume of groundwater that discharges from the aquifer into a group of drain cells. Drain state variables are specified in the GWM-2005 State Variable input file. Two types of drain flow can be represented: a rate of drain flow at a point in time (input variable DRNTYP = 1) or a volume of flow discharged to a drain over a specified duration of time (DRNTYP = 2). Drain-flow rates are calculated at the end of a specified stress period (input variable SVSP), whereas drain-flow volumes are calculated between the beginning and end of two specified stress periods (input variables SPSTRT and SPEND).

Each drain state variable is given a name (input variable SVNAME) and a specified number of drain cells are associated with the state variable by use of input variable NMDCEL. The location (layer, row, and column) of each drain cell that is associated with a drain state variable must correspond to a drain cell that has been defined in the Drain Package input file during a stress period when either a drain-flow rate or volume will be determined. For drain state variables that have been defined as flow-rate variables, the total flow rate calculated for the drain state variable is the sum of the flow rates to each of the drain cells associated with the drain state variable at the end of the stress period. The flow rate at each cell is calculated as a positive value because, by definition, drain cells cannot have negative flow rates. For drain state variables that have been defined as flow-volume variables, individual flow volumes are computed at every cell and at every time step. The individual flow volumes are computed by multiplying the flow rate at the end of a time step by the duration of that time step. These individual flow volumes are summed to produce the total volume of groundwater that is discharged, during the specified duration, to the specified drain cells associated with the drain state variable. The drain volume is also reported as a positive value.

The Drain Package allows multiple drains to be defined in a single cell of the MODFLOW grid. If a managed drain is present on a cell that has multiple drains, then a method must be available to distinguish among the drains on that cell. GWM-2005 accomplishes this by using the AUXILIARY variable option in the Drain Package. The user defines the name of the AUX variable in data set 2 of the Drain Package input. The name used for GWM auxiliary variables must be ‘GWM-DR’. This name can appear with other auxiliary variables, which can be listed in any order. The user then defines a value for the auxiliary variable in data sets 4 and 6 of the Drain Package input (using input variable ‘xyz’). These values should be positive integer numbers; they serve as an index to identify the drain on that cell. In the GWM state-variable input file, each drain cell is identified by its layer, row, column location, and, optionally, its auxiliary variable index number. When multiple drains are present on a single cell, GWM-2005 determines the correct drain by matching the auxiliary-variable values in the GWM state-variable input file and the Drain Package input file. This is demonstrated in the sample problem (fig. 1), in which three drains are present at the cell at row 14, column 22. Each of these drains has a different AUX value in the Drain Package input file. In the GWM state-variable input file, these AUX values are used to identify the matching drain locations. If no managed drains are present on a cell that has multiple drains, then auxiliary variables are not needed and can be left out of both the Drain Package and State Variable Package input files. If auxiliary variables are needed for any of the managed drains, then they must be used for all managed drains.

Drain state variables can be specified as part of the objective function or as part of the constraint set. For example, the user might want to maximize the amount of drain flow to a group of drains (as illustrated in the sample problem below). When drains are part of the objective function, the objective-function type (input variable FNTYP) will typically be specified as USDV, which indicates that none of the decision or state variables are multiplied by the duration of their activity (see equation 2a in Ahlfeld and others, 2011). Function type USDV is used because (1) if the drain state variables have been defined as flow rates (DRNTYP = 1), then the optimal flows to the drains and their contributions to the objective function are reported in the GWM output file as rates of flow; and (2) if the drain state variables have been defined as flow volumes (DRNTYP = 2), then GWM-2005 automatically calculates the total discharge volume to each managed drain over the specified stress periods during which the drain state variable is active (that is, from SPSTRT to SPEND), and reports the optimal values and their contributions to the objective function as volumes of flow.

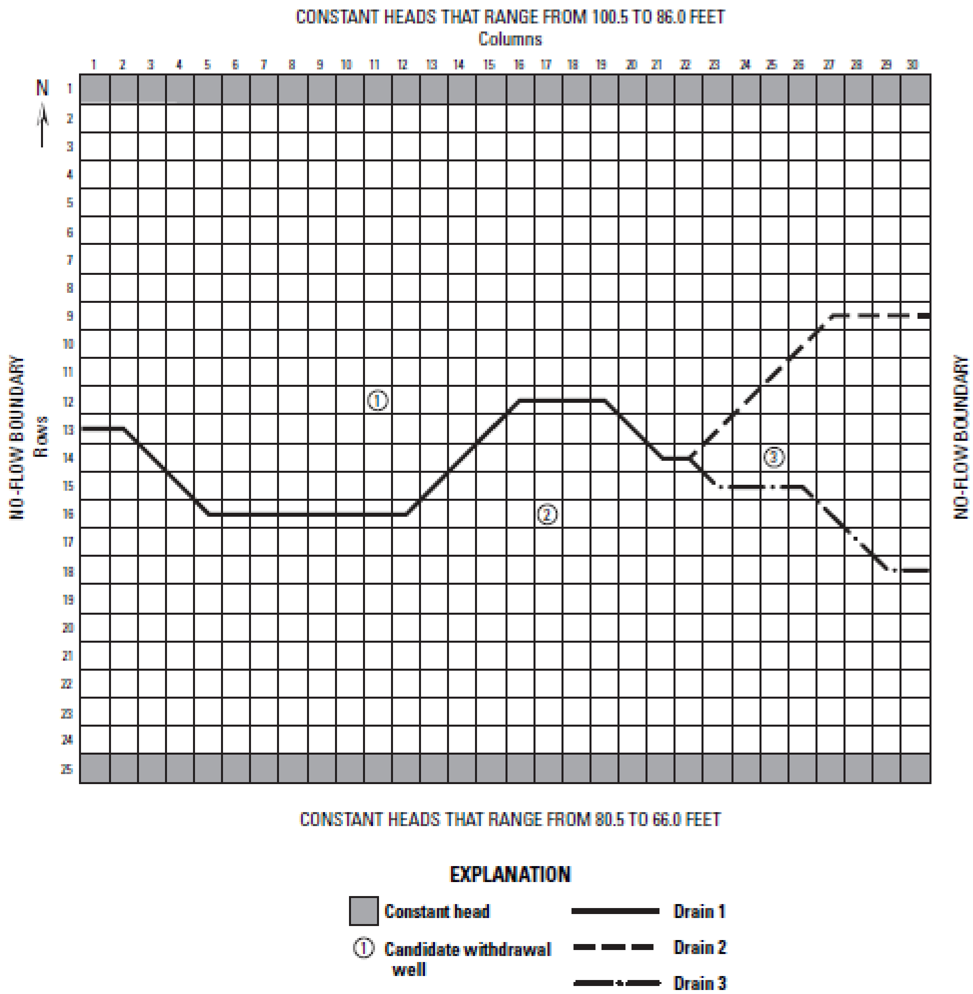

Figure 1. Schematic diagram showing model grid for DRAIN sample problem.

Drain state variables can be defined as part of the constraint set by use of linear-summation constraints. For example, the user might require that the sum of drain flow from two or more separate drains be greater than or equal to a minimum specified value. An example of the use of drain state variables in linear-summation constraints is provided in the sample problem below.

A new type of external variable (ETYPE), which is defined in the Decision Variable Package input file, also has been introduced with drain state variables. ETYPE specified as DR defines the external variable as a drain type. As with the other types of external variables that can be defined for a GWM-2005 simulation, DR type variables are all positive-valued variables. See the discussion on page 7 of Ahlfeld and others (2011) related to the definitions of external variables.

Revised Input Instructions for the State Variables Package

The input instructions for the State Variables Package were initially described in Ahlfeld and others (2011). The revised instructions below include the option for drain state variables. If the problem is multimodel (that is, uses local-grid refinement), then the set of drain cells that are associated with a given state variable must all be located on the grid on which the drain state variable is defined.

The **STAVAR** file consists of six input items:

0. [#Text]
   Item 0 is optional—# must appear in column 1. Item 0 can be repeated multiple times.
1. IPRN
2. NHVAR NRVAR NSVAR NDVAR

3. The following record is read for each of the NHVAR head state variables:
   SVNAME LAY ROW COL SVSP

4. The following record is read for each of the NRVAR streamflow state variables:
   SVNAME SEG REACH SVSP

5a. The following record is read for each of the NSVAR storage state variables:
   SVNAME SPSTRT SPEND CZONE

5b. The following record is read if CZONE is assigned a value of "ZONE":
   NSVZL

5c-d. The following two records are read NSVZL times:
   LNUM
   SVZONE (Read using the MODFLOW U2DINT utility subroutine; see Harbaugh, 2005, p. 8-57 through 8-59)

6a. The following record is read for each of the NDVAR drain state variables:
   SVNAME NMDCEL DRNTYP SVSP SPSTRT SPEND

6b. The following record is read NMDCEL times:
   LAY ROW COL [Aux-value]

The variables are defined as follows:

Text—is a character variable up to 199 characters long that starts in column 2. Any characters can be included in Text. Lines beginning with # are restricted to the first lines of the file. Text is printed when the file is read.

IPRN—is an integer variable that describes the amount of output written to the **GWM OUT** file. IPRN must be specified as either 0 or 1. When IPRN equals 0, a minimum amount of information about the decision variables is written to the GWM output file; when IPRN equals 1, detailed information about the decision variables is written to the GWM output file.

NHVAR—is an integer variable equal to the number of head-type state variables. NHVAR must be greater than or equal to 0.

NRVAR—is an integer variable equal to the number of streamflow-type state variables. NRVAR must be greater than or equal to 0.

NSVAR—is an integer variable equal to the number of storage-type state variables. NSVAR must be greater than or equal to 0.

NDVAR—is an integer variable equal to the number of drain state variables. NDVAR must be greater than or equal to 0.

SVNAME—is a character variable up to 10 characters long that is a unique name designated for the state variable. For head, streamflow, and drain state variables, each name must be unique (that is, the same name cannot be used for more than one variable, or in more than one model). For storage state variables applied to multimodel problems (that is, those using local grid refinement), the same name may appear in more than one **STAVAR** file in order to define a storage state variable that extends over multiple grids. However, within a given **STAVAR** file on one grid of a multimodel problem, the state variable name must be unique. No spaces are allowed in the name. The end of the name is designated by a blank space.

LAY, ROW, and COL—are integer variables equal to the layer, row, and column number of the model cell in which the head-type or drain-type state variable is located. For drain state variables, the LAY, ROW, and COL numbers must correspond to a valid drain location as specified in the MODFLOW DRN input file.

SVSP—is an integer variable that indicates the stress period during which the head, streamflow, or drain flow rate state variable is to be evaluated. To evaluate a head, streamflow, or drain flow rate state variable for multiple stress periods, define multiple state variables.

SEG and REACH—are integer variables equal to the segment and reach numbers of the model cell in which the streamflow-type state variable is located. The SEG and REACH numbers must correspond to a valid segment and reach as specified in either the STR or SFR input files.

SPSTRT and SPEND—are integer variables equal to the stress periods at which the evaluation of the storage or drain volume state variables will start and end. For storage state variables, the change in storage associated with the state variable will be computed by subtracting the volume of water in a specified portion of the model domain at the beginning of stress period SPSTRT from the volume of water in the same portion at the end of stress period SPEND. For drain volume state variables, the volume of discharge to the drain state variable will be computed for the time extending from the beginning of the specified stress period SPSTRT to the end of the specified stress period SPEND.

CZONE—is a character variable that describes the portion of the aquifer domain to be included in the storage state variable. Two options are allowed:

ALL—the storage state variable will record the change in water stored in the entire model domain.

ZONE—the storage state variable will record the change in water stored in a portion of the model domain that is defined in subsequent records in the file.

NSVZL—is an integer variable equal to the number of model layers included in the storage state variable zone. The zone array will be read one layer at a time.

LNUM—is an integer variable equal to the layer number for the storage state variable zone array.

SVZONE—is a two-dimensional (one layer) zone array that is read using the U2DINT utility subroutine of MODFLOW (see Harbaugh, 2005, p. 8-57 through 8-59). A value will be read for each cell in the model layer. A value of zero indicates the cell is not included in the storage state variable; a value greater than zero indicates the cell will be included in the definition of the storage state variable.

NMDCEL—is an integer variable equal to the number of managed drain cells associated with the state variable.

DRNTYP—is an integer variable that takes a value of 1 or 2:

If DRNTYP = 1, the state variable represents a drain flow rate at the end of stress period SVSP (the values of SPSTRT and SPEND are ignored).

If DRNTYP = 2, the state variable represents a volume of drain flow between a starting time (SPSTRT) and ending time (SPEND). The value of SVSP is ignored (but a value must be specified)

AUX-value—is an optional positive-integer variable that serves as an index to identify the drain on the cell. When multiple drains are present on a single cell, GWM-2005 determines the correct drain by matching the auxiliary-variable values defined by AUX-value with the auxiliary variables defined by auxiliary-variable GWM-DR in the Drain Package input file.

Sample Problem

As mentioned previously, modelers sometimes use drains to model streams when it can be assumed that the groundwater-discharge rate to the streams always remains greater than or equal to zero. In this sample problem, drains are used to simulate a stream network. The drains are grouped into three drain state variables (fig. 1). In the first formulation, the objective is to maximize flow rates in the drains during a single stress period; in the second formulation, the objective is to maximize the volume of drain flow between the beginning and end of a stress period.

All of the input and output files for this sample problem are provided in the DRAIN subdirectory within the data directory of the GWM-2005 distribution. Most of the input files are common to both formulations, but there are separate **NAME**, **GWM**, **STAVAR**, and **SUMCON** files for the two formulations. Two batch files to run the problems are provided in the test-win directory (files ‘drain.rate.bat’ and ‘drain.volume.bat’).

The sample problem is based on the STREAMFLOW problem described in Ahlfeld and others (2011). A confined aquifer is in hydraulic connection with three streams, which are simulated by use of the MODFLOW Drain Package (fig. 1; input file ‘drain.drn’). The total conductance of the drains in each cell is assumed to be 20,000 ft^2/d, based on data provided for the STREAMFLOW sample problem (streambed thickness and hydraulic conductivity of 1 ft and 5 ft/d, respectively, and stream width and length in each grid cell of 20 ft and 200 ft, respectively). For all but three of the specified drains there is only a single drain in each cell; however, there are three drains in cell 14, 22 with a combined conductance of 20,000 ft^2/d (see input file ‘drain.drn’). Elevations of the drain cells range from 47.5 ft along the western boundary of the simulated aquifer to 30.0 ft along the eastern boundary. Auxiliary variable GWM-DR is defined in the Drain Package input file so that GWM-2005 can calculate individual flows to the three drains specified in cell 14, 22.

The aquifer is homogeneous and isotropic with a transmissivity of 5,000 ft^2/d and a storage coefficient of 0.05 (dimensionless). The modeled area of interest is 6,000 ft long and 5,000 ft wide. The model consists of a single layer with 25 rows and 30 columns and each model cell is 200 ft by 200 ft. The modeled area is bounded on the east and west by no-flow conditions and on the north and south by constant heads that decrease in elevation from west to east (fig. 1).

A 3-year period of water-supply management is simulated. The 3-year period is divided into 12 seasons (winter, spring, summer, and fall of each year), each of which is represented by a single stress period. The aquifer is recharged at a rate of 0.0005 ft/d in the winter, 0.002 ft/d in the spring, 0 ft/d in the summer, and 0.001 ft/d in the fall. The variable rates of recharge result in groundwater-discharge rates to the drains that are highest during the spring and lowest during the summer.

Model-calculated groundwater-discharge rates to the simulated drains are close in value to those calculated with the SFR Package in the STREAMFLOW sample problem. For example, at the end of the 11^{th} stress period, which corresponds to the end of summer in the third year of simulation, total calculated discharge to the drains is 13.82 ft^3/s, compared with 13.45 ft^3/s of groundwater discharge to simulated streams. The differences are likely due to small differences between the specified drain elevations and the stream-stage elevations computed in the SFR Package.

Maximize Rates of Drain Flow

The objective of the first management formulation is to maximize flow rates in the three drains during the summer of the third year of simulation (stress period 11). Three drain-flow-type state variables are defined for the problem: *DR1*, *DR2*, and *DR3*. *DR1* has 22 drain cells associated with it; *DR2* and *DR3* each have 9 drain cells associated with them. The three drain-flow state variables are identified as DRNTYP = 1 (drain flow rates). The values specified for the auxiliary variable (variable AUX-value) in the State Variables Package input file ('drain.rate.stavar') are consistent with those specified in the Drain Package input file.

The objective function is

$$Maximize\ \ DR1 + DR2 + DR3\,. \tag{1}$$

Dimensionless, uniform weighting coefficients equal to 1.0 are specified for each state-variable coefficient and FNTYP is set to USDV in the **OBJFNC** file indicating that these variables are not multiplied by the duration of the stress period in which they are active.

Groundwater that is pumped to meet water-supply demands can be withdrawn at three candidate well locations, *Q1*, *Q2*, and *Q3* (fig. 1). Because it may be advantageous to have some wells pump at variable rates throughout the year, multiple decision variables are defined for wells *Q2* and *Q3*: Well *Q2* is allowed to have a different withdrawal rate during each of the four seasons (identified as decision variables *Q2win*, *Q2spr*, *Q2sum*, and *Q2fal*), and well *Q3* can be pumped during the spring (*Q3spr*) and summer (*Q3sum*) months. Well *Q1* must have a constant withdrawal rate throughout the year. The minimum and maximum pumping rates at each well are 0 and 50,000 ft^3/d, respectively.

The minimum water-supply demands on the aquifer are variable throughout the year, with rates of 30,000 ft^3/d during the winter and fall, 45,000 ft^3/d during the spring, and 60,000 ft^3/d during the summer. The maximum demands are constant throughout the year and are equal to 80,000 ft^3/d; although the maximum demands are specified in this sample problem for illustrative purposes, it can be expected that they will not be binding in the solution because the overall formulation of the problem will tend to select solutions that pump at the smallest permissible rates. These seasonally variable demands are specified by use of linear-summation constraints.

Linear-summation constraints also are used to specify minimum rates of discharge to the drains (see the last three constraints specified in file 'drain.rate.sumcon'). These minimum drain-flow requirements, which are specified for illustrative purposes, are set at 3.47 ft^3/s (shown as 300,000 ft^3/d in the input file), 2.66 ft^3/s, and 2.73 ft^3/s for the three drains, respectively.

Although the aquifer is confined and the objective function is linear, the presence of the drains introduces head-dependent boundary conditions to the simulation and potentially causes the management problem to be nonlinear. As a consequence, sequential linear programming (SLP) is selected in the **SOLN** file to solve the nonlinear formulation. An initial perturbation value of 20 percent of the maximum withdrawal rate of each candidate well is specified (that is, variable DINIT is specified as 0.2), which results in an initial perturbation withdrawal rate of 10,000 ft^3/d at each well. Convergence criteria on the flow-rate decision variables of 1.0×10^{-5} ft^3/d (variable SLPVCRIT) and on the value of the objective function of 1.0×10^{-4} ft^3 (variable SLPZCRIT) of drain flow are specified.

As with the STREAMFLOW sample problem, only two iterations of the SLP algorithm were required to satisfy the two convergence criteria, indicating that the problem is very weakly nonlinear. The optimal annual total withdrawal from the three wells is equal to that calculated for the STREAMFLOW problem but the distribution of withdrawal rates for this DRAIN sample problem is different. Well *Q1* is pumped at a constant rate throughout the year; well *Q2* is pumped during the winter, summer, and fall; and well *Q3* is pumped during the spring. In order to meet peak demand during the summer, well *Q2* is pumped at a rate of 44,514 ft^3/d and well *Q1* at a rate of 15,486 ft^3/d. Not surprisingly, because the objective is to maximize flow in each of the drains, the minimum amount of water is pumped each season to meet the water-supply demands, and each of the four minimum water-supply demands are binding at the optimal solution.

The optimal values for each drain state variable are reported as rates during the 11^{th} stress period. For example, the optimal discharge rate to *DR1* given in the sample GWM output file ('drain.rate.gwmout') is 6.747553E+05 ft^3/d. As shown in the output file, the constraint specified on the minimum discharge rate to drain *DR3* is binding.

Maximize Volumes of Drain Flow

The objective of the second management problem is to maximize the volume of drain flow in each drain during stress period 11. The drain state variables are defined the same as in the previous formulation, except that variable DRNTYP is set to 2 (drain volumes), variable SVSP is set to 0, and variables SPSTRT and SPEND are each set to 11. Although the drain state variables are defined differently, the objective-function file for both management formulations is the same. The linear-summation constraints for the three minimum drain-flow requirements, however, are modified such that the minimum drain flows are specified as volumes of flow during stress period 11; this was done by multiplying each minimum flow-rate requirement by the duration of the stress period (91 days).

The solution to this management formulation is similar in many respects to the first formulation. For example, the optimal pumping rates at each well and the binding constraints of the formulation are the same as for the first formulation. The optimal value of each drain state variable, however, is not the same, because the flow rate to each drain is multiplied by the duration of the 11^{th} stress period (91 days). For example, the optimal discharge volume to *DR1* during the 11^{th} stress period given in the GWM output file ('drain.volume.gwmout') is 6.140273E+07 ft^3.

References

Ahlfeld, D.P., Barlow, P.M., and Baker, K.M., 2011, Documentation for the State Variables Package for the Groundwater-Management Process of MODFLOW-2005 (GWM-2005): U.S. Geological Survey Techniques and Methods 6-A36, 45 p.

Ahlfeld, D.P., Baker, K.M., and Barlow, P.M., 2009, GWM-2005--A Groundwater-Management Process for MODFLOW-2005 with Local Grid Refinement (LGR) capability: U.S. Geological Survey Techniques and Methods 6-A33, 65 p.

Ahlfeld, D.P., Barlow, P.M., and Mulligan, A.E., 2005, GWM-A ground-water management process for the U.S. Geological Survey modular ground-water model (MODFLOW-2000): U.S. Geological Survey Open-File Report 2005-1072, 124 p.

Harbaugh, 2005, MODFLOW-2005, The U.S. Geological Survey modular ground-water model—the Ground-Water Flow Process: U.S. Geological Survey Techniques and Methods 6-A16, variously paginated.