A STUDY OF METHODS USED IN

MEASUREMENT AND ANALYSIS OF SEDIMENT LOADS IN STREAMS

REPORT NO. 3

ANALYTICAL STUDY OF METHODS OF SAMPLING SUSPENDED SEDIMENT

NOVEMBER 1941

A Study of Methods Used in

MEASUREMENT AND ANALYSIS OF SEDIMENT LOADS IN STREAMS

Planned and conducted jointly by

Tennessee Valley Authority, Corps of Engineers,

Department of Agriculture, Geological Survey,

Bureau of Reclamation, Indian Service, and

Iowa Institute of Hydraulic Research

Report No. 3

ANALYTICAL STUDY OF METHODS OF SAMPLING SUSPENDED SEDIMENT

Published at

St. Paul U. S. Engineer District Sub-office
Hydraulic Laboratory, University of Iowa
Iowa City, Iowa

November 1941

The cooperative study of methods used in

MEASUREMENT AND ANALYSIS OF SEDIMENT LOADS IN STREAMS

covers phases indicated by the following report titles.

Report No. 1

FIELD PRACTICE AND EQUIPMENT USED IN SAMPLING SUSPENDED SEDIMENT

Report No. 2

EQUIPMENT USED FOR SAMPLING BED-LOAD AND BED MATERIAL

Report No. 3

ANALYTICAL STUDY OF METHODS OF SAMPLING SUSPENDED SEDIMENT

Report No. 4

METHODS OF ANALYZING SEDIMENT SAMPLES

Report No. 5

LABORATORY INVESTIGATIONS OF SUSPENDED SEDIMENT SAMPLERS

SYNOPSIS

In this report is presented an analysis of some of the sampling methods commonly used in determining the magnitude of suspended sediment loads of streams. A study of the various sampling methods has been made to determine their inherent errors when used in a stream where the sediment concentration and velocity vary between the surface and the bed of the stream. The analysis is based upon the assumption that the samples collected represent the true average value of the sediment concentration at every point of observation and that the sediment distribution conforms to the turbulence theory.

A method is presented also whereby the mean sediment concentration at a vertical in a stream can be determined from the concentration and size composition observed at any given point in the vertical.

TABLE OF CONTENTS

Section		Page
	I. INTRODUCTION	
1	Nature and scope of project	9
2	Methods of investigating the accuracy of sediment measurements	10
3	Authority and personnel	12
4	Acknowledgment	13
	II. STREAM FLOW AND SEDIMENT SUSPENSION	
5	Methods of sediment transportation	14
6	Vertical distribution of sediment	16
7	Analysis of accuracy of sampling methods	17
	III. THEORY OF VERTICAL DISTRIBUTION OF SUSPENDED SEDIMENT	
8	Definitions	21
9	General equation for vertical distribution of suspended sediment	22
10	Vertical velocity distribution and its relation to roughness	28
11	Simplified equation for vertical distribution of suspended sediment	31
12	Determination of true mean sediment concentration in the vertical	32
	IV. CONDITIONS OF ANALYSIS OF SAMPLING METHODS	
13	Computation procedure	35
14	Particle sizes and stream conditions considered	35
15	Validity of results	36
16	Manner of presenting results	37
	V. ANALYSIS OF POINT SAMPLE METHODS	
17	Single point methods	40

Section Page

18 Surface, mid-depth, and bottom method 40

19 Straub method 44

20 Luby Method 45

21 Sampling errors incident to stream and sediment conditions . . . 47

VI. ANALYSIS OF DEPTH-INTEGRATION METHOD

22 Filling rate equal to stream velocity 51

23 Filling rate constant 53

24 Sampler opened near stream bed 57

25 Depth-integration with an existing sampler 60

VII. METHOD FOR DETERMINING TOTAL CONCENTRATION

26 Sediment gradations and methods used in analysis 64

27 Errors in point sample methods for determining total concentration 66

28 Errors in depth-integration method for determining total concentration 67

29 Coefficients of mean concentration 67

VIII. SUMMARY

30 Basis of analysis 75

31 Comparison of methods 76

LIST OF ILLUSTRATIONS

Figure		Page
1	Vertical sediment distribution for various values of *t*	18
2	Effect of roughness on vertical velocity curve and discharge summation	23
3	Nomograph for determining parameter *t* from stream and sediment conditions	24
4	Evaluation of factor *P* for various values of *t*	25
5	Adjustment of vertical velocity curve to have points of maximum velocity below water surface	30
6	Accuracy of methods of sampling suspended sediment for various types of sediment distribution and a relative roughness 0.010	41
7	Accuracy of methods of sampling suspended sediment for various types of sediment distribution and a relative roughness 0.020	42
8	Accuracy of methods of sampling suspended sediment for various types of sediment distribution and a relative roughness 0.030	43
9	Accuracy of point sample methods for various stream and sediment conditions	48
10	Accuracy of depth-integration method when sampling different portions of the stream depth at a filling rate equal to velocity	54
11	Accuracy of depth-integration method when sampling different portions of the stream depth at a constant filling rate and accuracy when sampler is opened at different depths	56
12	Relation of depth to per cent of sampler capacity filled in initial pressure equalization	58
13	Variation of filling rate of Rock Island sampler with depth for various velocity distributions established by a relative roughness of $n/D^{1/6} = 0.020$	61
14	Accuracy of depth-integration methods for various stream and sediment conditions; relative roughness 0.020	63
15	Sediment gradations used in analysis of accuracy of methods for total concentration determination	64
16	Accuracy of point sample methods for total concentration determination; relative roughness 0.020	65

Figure Page

17 Accuracy of depth-integration methods for total concentration determination; relative roughness 0.020 68

18 Coefficients to determine mean sediment concentration in a vertical from a sample at a single depth for a relative roughness $n/D^{1/6} = 0.010$ 70

19 Coefficients to determine mean sediment concentration in a vertical from a sample at a single depth for a relative roughness $n/D^{1/6} = 0.020$ 71

20 Coefficients to determine mean sediment concentration in a vertical from a sample at a single depth for a relative roughness $n/D^{1/6} = 0.030$ 72

LIST OF TABLES

Table Page

1 Accuracy of point sampling methods 49

2 Accuracy of depth-integration method when sampling different proportions of the stream depth with filling rate equal to velocity . 52

3 Accuracy of depth-integration method when sampling different proportions of the stream depth with filling rate constant 55

4 Accuracy of depth-integration method performed by opening sampler at the stream bed and filling while being raised 59

5 Accuracy of depth-integration method performed by Rock Island sampler filling while being lowered and raised 62

6 Example of computations to correct size distribution for a sample from a single point by the application of a coefficient 74

7 Comparison of Straub, Luby, and depth-integration methods for various sediment sizes and stream velocities 78

ANALYTICAL STUDY OF METHODS OF SAMPLING SUSPENDED SEDIMENT

I. INTRODUCTION

1. Nature and scope of project--This is the third in a series of five reports covering various phases of a study inaugurated in an effort to improve the methods and equipment used in the measurement and analysis of the sediment loads of streams. In this report is presented an analysis and evaluation of errors in the various methods of sampling used to determine the average concentration in a vertical from the surface to the bed of a stream. The titles and brief descriptions of the contents of the four other reports of this series are as follows:

> Report No. 1--"Field Practice and Equipment Used in Sampling Suspended Sediment;" a detailed review of past and present equipment and methods, including a history of their development, the methods of locating the points at which samples are taken, the frequency of sampling, the requirements of an ideal sampling instrument and the adverse features of existing types. The large number of instruments which have been used are also described and classified.
>
> Report No. 2--"Equipment used for Sampling Bed-Load and Bed Material;" a review and classification of the various types of equipment.
>
> Report No. 4--"Methods of Analyzing Sediment Samples;" a presentation and study of the numerous laboratory methods of analyzing sediment samples for particle size and total solids concentration with a view to obtaining the method most suitable for sediment studies.
>
> Report No. 5--"Laboratory Investigations of Suspended Sediment Samplers;" presents the results of an experimental study of the effect of sampler action upon sediment concentration determinations, and of the filling characteristics of a number of slow-filling samplers.

These five reports cover the work completed in 1939 and 1940. However, since a continuation of the project has been authorized, it is expected that additional reports will be issued.

One objective of the study as a whole has been to investigate and, wherever possible, to evaluate quantitatively sources of error inherent in the methods and equipment used to measure suspended sediment. The comprehensive review of field practice and sampling equipment, presented in Report No. 1, indicated that the principal sources of errors occur in: the frequency of sampling, the intake action of the sampler, and the method of determining the average sediment concentration at any point between the surface and the bed of the stream.

In Report No. 1 was pointed out the importance of taking frequent samples of sediment, especially during floods on flashy streams, if accurate estimates of the total quantity of sediment carried by a stream is desired. A laboratory study of the effect of sampling action upon sediment concentration and composition, and a quantitative evaluation of errors for various particle sizes and sampler intake conditions at widely varying velocities, are presented in Report No. 5. These studies were made primarily on the intake action of slow filling samplers. The instantaneous trap type of sampler, in which the primary consideration is the number of samples necessary to secure a satisfactory average due to the magnitude and frequency of fluctuations in sediment concentration, will be studied as one phase of the future investigation.

2. Methods of investigating the accuracy of sediment measurement-- There are two approaches available for investigating the accuracy of the various methods used in determining the average concentration of sediment in a stream. One is an analytical method and the other is based upon the results of analysis of a large number of sediment samples taken under a wide variety of conditions. Each approach has disadvantages and limitations;

neither alone will solve the problem completely. The analytical method can be applied to evaluate only that part of the error which arises due to variation in the sediment distribution from the surface to the bottom of a stream. This method is based on the turbulence-suspension theory of sediment distribution which is unfamiliar to most engineers and, although experimental evidence indicates that it is substantially accurate under ideal conditions, the experimental verification of the theory does not cover a wide enough range of conditions to establish definitely the fact that it always applies. The analytical approach is limited to material carried in suspension and does not take into account that carried by saltation, because the theories covering the movement of material by saltation are not sufficiently developed for analysis, but samplers for suspended load sample the saltation load also when it is present.

A study based upon field measurements also has a number of disadvantages. Considerable time and expense are required to collect and to analyze the large number of samples necessary. The data thus obtained would be subject to errors from several possible sources such as those resulting from improper intake conditions in the sampler and from variations in sediment distribution. Without a fundamental framework with which to classify the results, the collecting of numerous field samples would yield only a mass of unrelated data. Consequently, a comprehensive study of the accuracy of sampling methods would require both qualitative and quantitative analyses.

In this research program it was planned to first make a laboratory study and analysis of the problem in order to develop the fundamental relationships with which to classify future field work. These studies

would then be augmented by an extensive field program in which other phases of the problem could be determined. This report is a quantitative analytical study of the problem and has been fully developed as far as present knowledge of the principles involved permit. However, since it is based upon the assumption that a perfect sample is collected under conditions in which there exists a vertical distribution of sediment according to the turbulence-suspension theory, and since it does not include the saltation load, the results will be valid only for those conditions. This analysis cannot be applied directly to determine the accuracy of measurements made in the past, because, in most cases, sufficient data are not available.

This study will be of considerable value as it provides a general view of the relations involved and will serve as a basis for classification of future work. It also points out conditions under which sediment measurements are likely to be erroneous and thus serves as a criterion for future use. In many cases this report will aid in the proper selection of equipment to best fit any situation. Where conditions closely approximate those assumed in the analysis, the methods herein described should give reasonably accurate quantitative measures of the errors involved. This report also indicates a need for determining the basic conditions at all important sediment measurement stations in order to reduce the sources of error to a minimum.

3. Authority and personnel--The cooperative project in the investigation of the methods and equipment used in the measurement of sediment loads in streams, of which this study is a part, was planned and conducted jointly by the following agencies of the United States Government: Corps

of Engineers, War Department; Geological Survey, Bureau of Reclamation, and Indian Service, Department of Interior; Flood Control Coordinating Committee, Department of Agriculture; and the Tennessee Valley Authority, and also the Iowa Institute of Hydraulic Research. The studies were conducted at the Hydraulic Laboratory of the Iowa Institute of Hydraulic Research, State University of Iowa, under the supervision of Professor E. W. Lane. The representatives of the collaborating agencies, engaged in the preparation of this report, are Victor A. Koelzer, U. S. Geological Survey; Clarence A. Boyll, Tennessee Valley Authority; Cleveland R. Horne, Jr., U. S. Engineer Department; and Vernon J. Palmer, U. S. Soil Conservation Service.

4. Acknowledgments--Professor A. A. Kalinske of the Iowa Institute of Hydraulic Research has been consulted extensively with regard to the theory upon which the analytical study of the sampling methods was based. He reviewed Chapter III of this report and checked the general computation procedure.

The facilities of the U. S. Engineer Sub-Office at Iowa City, the Iowa City District Office of the U. S. Geological Survey, and the Iowa Institute of Hydraulic Research were used for the administrative details and in the preparation of this report. This report was edited by Mr. Martin E. Nelson, Engineer in charge of the U. S. Engineer Sub-Office.

II. STREAM FLOW AND SEDIMENT SUSPENSION

5. Methods of sediment transportation--Sediment moved by flowing water may be classified as bed-load, suspended load, and saltation load. Bed-load is defined as that part of the sedimentary load of the stream which is moving in almost continuous contact with the stream bed, being rolled or pushed along the bottom by the tractive force of the moving water. Suspended load is defined as sediment which remains in suspension for a considerable period of time without contact with the bottom. It may also be defined as sediment which is being transported by a stream out of contact with the bed and banks, supported by the upward components of the turbulent flow while being carried forward by the horizontal components. Saltation load may be defined as sediment which is intermittently out of contact with the bed and banks of the stream, being bounced or lifted into the stream by the action of forces other than the vertical components of flow and being carried forward by the horizontal components before settling back to the bottom.

Experiments indicate that the laws governing these three types of sediment transportation differ. The laws for bed-load and suspended load movement have been developed partially, but very little has been done to quantitatively evaluate the movement of saltation load in water, although some work has been done on saltation movement of sand in air. When measuring the sediment load carried by a stream, it is not possible to separate suspended load and saltation load as they are intimately mixed together. If the bed-load is composed of coarse material, it may be measured separately, but if it is fine, it is difficult to separate the bed-load from the saltation and suspended loads. Ordinarily, in measuring

bed-load, the material moving in a given width of the river in a known time, is sampled, and the quantity moving is expressed as weight of sediment per unit of time per unit of stream width. Suspended sediment is measured ordinarily in units of weight of solid sediment per unit of weight of the water-sediment mixture. To obtain the amount carried in a unit time, it is necessary to multiply this value by the weight of the water-sediment mixture flowing in a unit of time. Saltation load in water has never been measured separately. Sampling instruments and methods are, therefore, divided into two classes: bed-load samplers, which collect a sample of the sediment discharge of a known width of the stream, and suspended load samplers, which obtain a certain volume or weight of the water-sediment mixture. This report covers problems involved only in suspended sediment sampling.

Unfortunately, the physical laws which govern the transportation of sediment in water have not been fully developed, and therefore only a partial analysis of this phenomenon is possible. The present analysis will be limited to the vertical distribution of sediment in a stream, for which the theory has been fairly well substantiated. However, the concentration at a point may vary with time due to turbulence, change in discharge, or variations in the character of watershed, none of which can be evaluated analytically at present. Consequently, this report is confined to a treatment of the errors which arise in the various sampling methods due to those conditions which influence the vertical distribution of suspended sediment. Since the various types of sampler intakes in use will cause varying degrees of error in the sample collected, that variable is eliminated from this analysis by assuming that the sampler obtains

a true average sample of the water-sediment mixture at the sampling point.

6. Vertical distribution of sediment--For many years it has been known that the concentration of suspended sediment in a stream increases from the surface to the bottom. A correlation between the sediment concentration and turbulence was first advanced by O'Brien in 1933 (7) and has been partially checked by observations in certain streams and channels by Christiansen (2) and Richardson (8). Rouse (9) checked the correlation in the laboratory using artificial turbulence.

The theory of this correlation has been reduced to a mathematical expression by Lane and Kalinske (6) which allows prediction of the sediment distribution curve that will exist for individual sizes of sediment under specific conditions of stream depth, slope, and roughness. If the stream conditions and the concentrations at any point in the vertical are known, it is possible to calculate with this expression, the concentration at every other point in the vertical and the sediment discharge represented by the vertical. This method of analysis, which for simplicity, incorporates a number of assumptions, will be used in this report in comparing various sampling methods.

In the determination of the sediment distribution curves it was first necessary to establish the vertical velocity curve that would exist under given stream conditions of depth, slope, and roughness. Von Karman's equation for velocity distribution in rough conduits, which appears to conform reasonably well to the average velocity distributions observed in several typical streams, was used in this analysis. The equation, and its application in the development of the basic relation of stream conditions to sediment distribution, are presented in Section 10.

7. Analysis of accuracy of sampling methods--In determining the accuracy of any method of sampling a stream vertical, the turbulence-suspension theory is used to establish the vertical sediment distribution for given stream conditions. The sediment distribution curve is combined with the corresponding vertical velocity curve to determine the rate of sediment transportation and the true mean sediment concentration in the vertical. The concentration of sediment in representative samples taken at the points used in the particular sampling method under study is determined also by use of the turbulence-suspension theory. If more than one sample is collected in a vertical, the samples are combined and the composite sample is analyzed. The error attributed to the sampling method in question is the per cent difference between the concentration indicated by the composite sample and the true mean concentration in the vertical.

The equations which express the distribution and the mean concentration of sediment in a vertical are developed in terms of the concentration at some reference point in the vertical. The computations in this analysis were simplified by choosing the bottom as the reference point. Equations 1 and 2, therefore, apply only to this study or to a study in which a similar comparison is made.

The equation for determining the sediment concentration of a given particle size at any specific point in the vertical is

$$\overline{N} = \overline{N}_0 e^{-16tz} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad 1$$

where z = ratio of height of any point above the bottom to the total depth of stream.

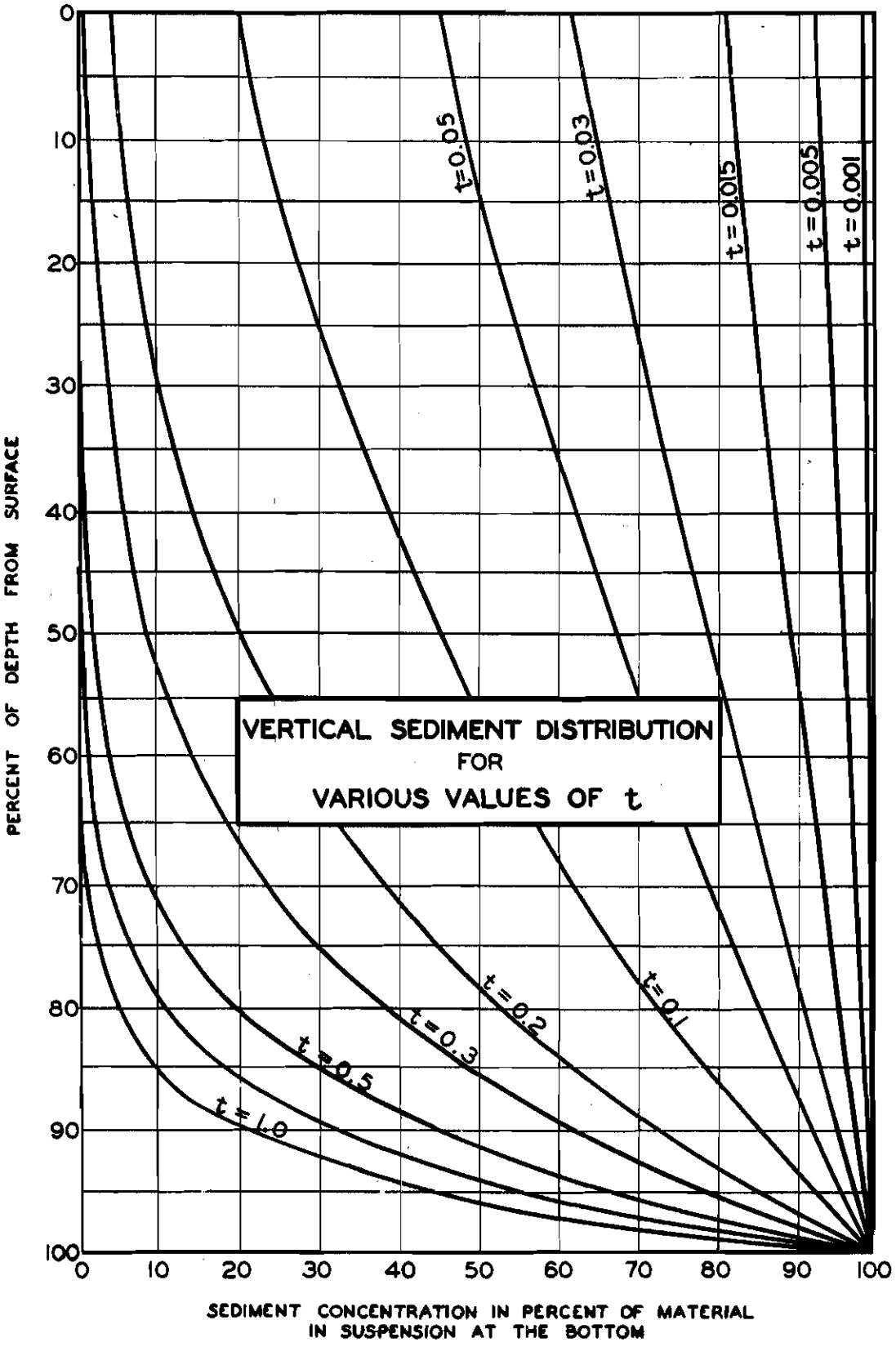

VERTICAL SEDIMENT DISTRIBUTION
FOR
VARIOUS VALUES OF t
t=0.05
t=0.03
t=0.015
t=0.005
t=0.001
t=0.1
t=0.2
t=0.3
t=0.5
t=1.0
PERCENT OF DEPTH FROM SURFACE
0
10
20
30
40
50
60
70
80
90
100
SEDIMENT CONCENTRATION IN PERCENT OF MATERIAL
IN SUSPENSION AT THE BOTTOM

FIG. 1

$\bar{N}$ = mean sediment concentration of a given particle size at point z .

$\bar{N}_o$ = mean sediment concentration of the same particle size at the bottom.

e = 2.718

t = dimensionless parameter denoting a particular sediment distribution.

The index t is a function of the fall velocity of the particle and the mean velocity, depth, and roughness of the stream. The exact relation expressed in common units is

$$t = \frac{0.2619\ c}{n/D^{1/6}V_m} \quad \text{or} \quad \frac{0.1746\ c\ C}{V_m} \quad \ldots\ldots \quad 2$$

where c = fall velocity of particle, ft./sec. (If c is expressed cm./sec. the constants in equation 2 should be 0.0086 and 0.0057, respectively.)

n = Manning roughness coefficient.

D = depth of stream, ft. (Replaces hydraulic radius R for simplicity. Use R if D is appreciably different.)

V_m = mean velocity in the vertical, ft./sec.

C = Chezy friction coefficient $\frac{1.5}{n/D^{1/6}}$

In Fig. 1 are shown sediment distribution curves for various values of t . The quantity $n/D^{1/6}$, inversely proportional to the Chezy friction coefficient, may be considered an index of relative roughness. For convenience, it will be termed m in this report. The effect of relative roughness upon the vertical distribution of velocity, determined from the Von Karman velocity equation presented in Section 10, is illustrated in Fig. 2

A nomograph which facilitates computation of t from given stream

and sediment conditions is presented in Fig. 3. In using this nomograph it should be observed that certain factors are expressed in English units and others in metric units.

The mean concentration in the vertical indicated by this particular method of sampling in question was referred to and expressed as a decimal fraction of the concentration of suspended sediment at the bottom of the verticals. This ratio was related to another ratio P (See Fig. 4) which expresses the true mean concentration in the vertical as a decimal fraction of this same bottom concentration.

III. THEORY OF VERTICAL DISTRIBUTION OF SUSPENDED SEDIMENT

8. Definitions--The theory upon which is based the correlation of the characteristics of a stream and its suspended sediment load, as used in this report, has been presented in published engineering literature (6). This chapter will be devoted to a brief review of the turbulence-suspension theory and a presentation of the fundamental mathematical relationships involved. In this review the following definitions of terms will be pertinent:

Turbulence is the state at any point within a fluid where the direction and magnitude of the velocity vary irregularly with time. In true turbulence, such fluctuations are not periodic but occur entirely at random, their frequency of occurrence following the normal error law.

Turbulence exchange is the exchange of momentum or material between two layers of a fluid due to velocity fluctuations caused by turbulence.

True mean sediment concentration in a vertical is a quantity such that when multiplied by the mean velocity in the vertical, the value of the actual suspended sediment discharge in a unit width of the stream is obtained.

The characters listed and defined below will be used in the mathematical derivations which follow. No units are shown inasmuch as the equations apply under any system of units used consistently.

c = fall velocity of sediment particle in still water.

C = Chezy roughness coefficient, $\frac{1.5}{n/D^{1/6}}$.

D = total depth of stream.

ϵ = momentum transfer coefficient.

ℓ = Prandtl mixing length.

m = a relative roughness factor, $n/D^{1/6}$.

n = Manning roughness coefficient.

N = sediment concentration at a point in weight per unit volume of water sediment mixture.

P = ratio of true mean sediment concentration in the vertical to sediment concentration in suspension at the bottom.

ρ = unit density of fluid.

S = slope of water surface or energy gradient.

t = a dimensionless parameter denoting a particular sediment distribution, $c/\sqrt{\tau_o/\rho}$

τ = unit shear between two layers of a fluid.

τ_o = unit shear at boundary.

u = vertical component of fluctuating velocity, $\bar{u} = 0$.

$\bar{V}$ = mean velocity in the direction of stream flow (a bar above a symbol denotes the mean of fluctuating values).

w = unit weight of fluid.

W = total load of suspended particles having a fall velocity c , per unit width of stream.

x = ratio of velocity at a point to the mean in the vertical, $\bar{V}/V_m$.

y = height of a given point above the stream bed.

z = ratio of height of any point above the bottom to the total depth of stream, y/D .

9. General equation for vertical distribution of suspended sediment--The instantaneous sediment concentration at a point is equal to the average concentration at that point plus or minus the fluctuation in concentration.

$$N = \bar{N} \pm \Delta N \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad 3$$

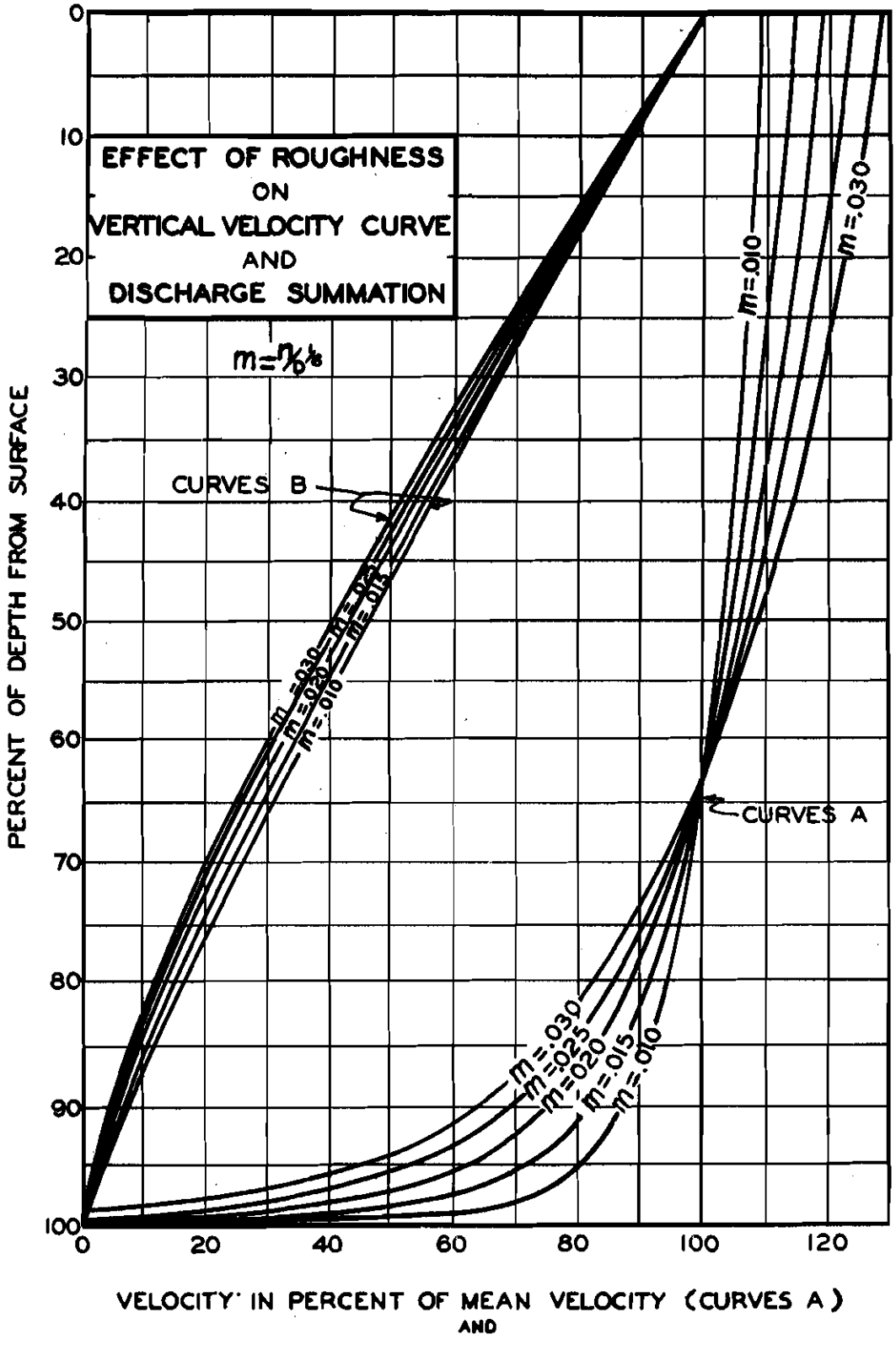
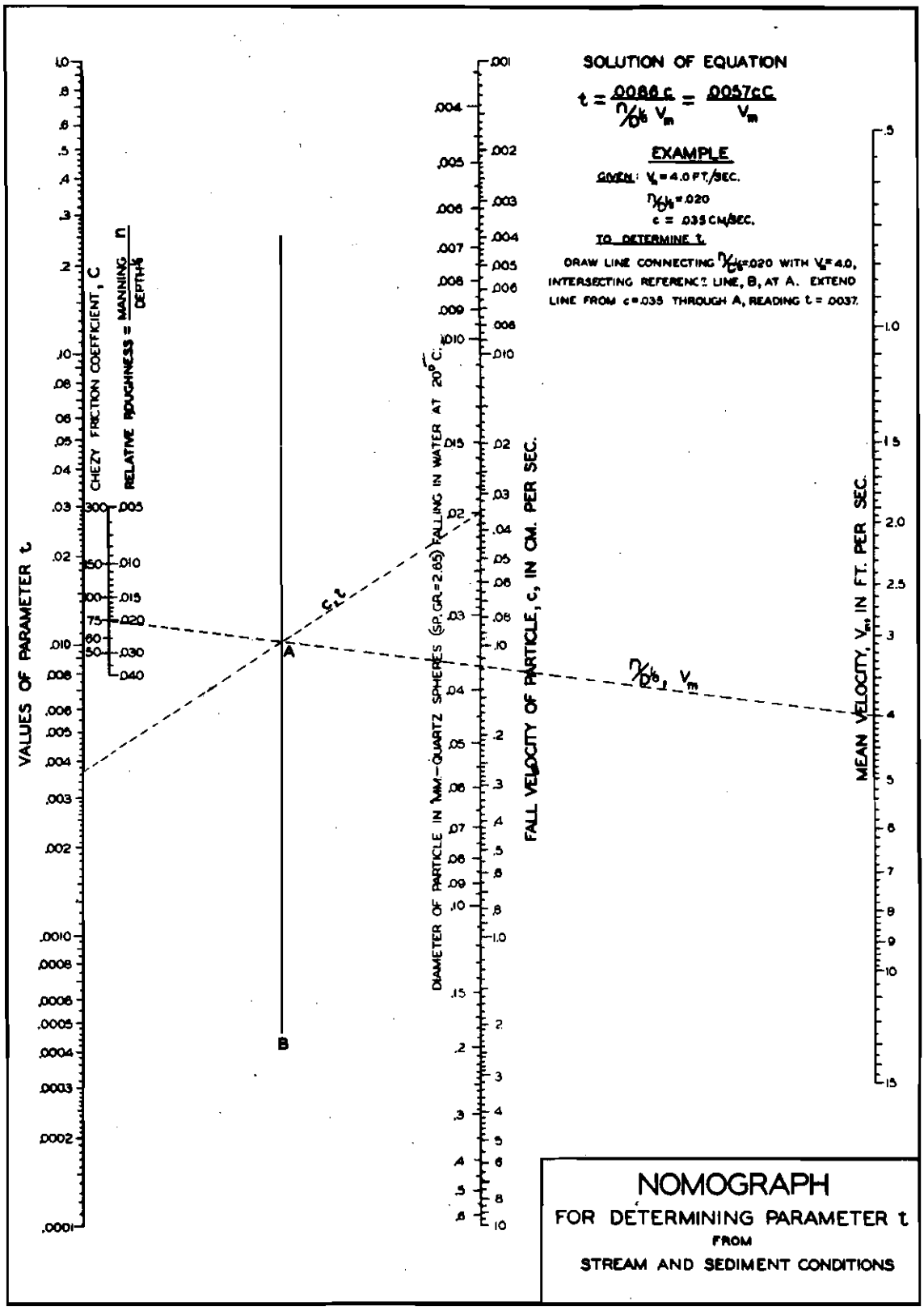

EFFECT OF ROUGHNESS
ON
VERTICAL VELOCITY CURVE
AND
DISCHARGE SUMMATION
m = n/b^1/6
CURVES B
CURVES A
m = .030
m = .025
m = .020
m = .015
m = .010
PERCENT OF DEPTH FROM SURFACE
0
10
20
30
40
50
60
70
80
90
100
0
20
40
60
80
100
120
VELOCITY IN PERCENT OF MEAN VELOCITY (CURVES A)
AND
PERCENT OF DISCHARGE BELOW INDICATED DEPTH (CURVES B)

FIG. 2



NOMOGRAPH
FOR DETERMINING PARAMETER t
FROM
STREAM AND SEDIMENT CONDITIONS

FIG. 3

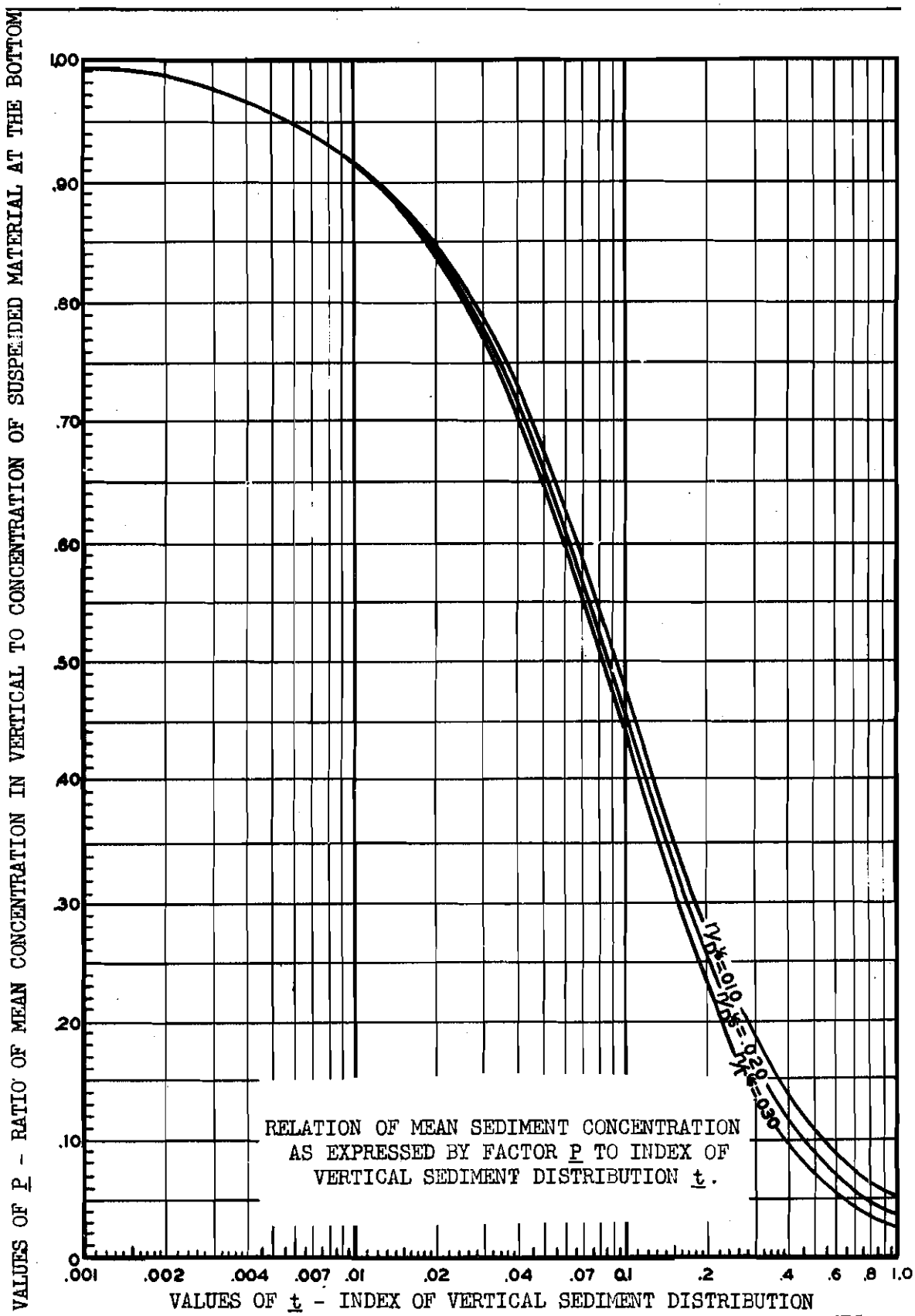

VALUES OF P - RATIO OF MEAN CONCENTRATION IN VERTICAL TO CONCENTRATION OF SUSPENDED MATERIAL AT THE BOTTOM
1.00
.90
.80
.70
.60
.50
.40
.30
.20
.10
0
.001
.002
.004
.007
.01
.02
.04
.07
0.1
.2
.4
.6
.8
1.0
VALUES OF t - INDEX OF VERTICAL SEDIMENT DISTRIBUTION
RELATION OF MEAN SEDIMENT CONCENTRATION AS EXPRESSED BY FACTOR P TO INDEX OF VERTICAL SEDIMENT DISTRIBUTION t.

FIG. 4

Sediment in suspension is in a constant state of motion in a vertical direction due to two factors (a) fluctuations of the vertical components of velocity and (b) the tendency for particles to settle due to the force of gravity. In equation form this may be stated as

$$\overline{Nu\uparrow} = \overline{Nu\downarrow} + \overline{N}c \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad 4$$

where $u\uparrow$ = instantaneous vertical velocity upward

and $u\downarrow$ = instantaneous vertical velocity downward

If signs are taken into account, this equation may be expressed as

$$\overline{Nu} = \overline{N}c \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad 5$$

Combining equations 3 and 5

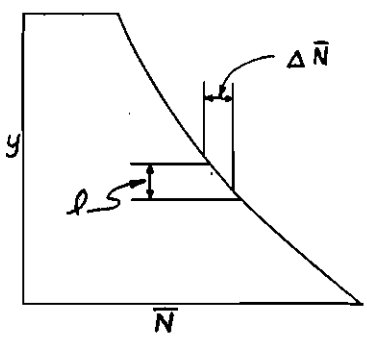
$$\overline{(N \pm \Delta N)u} = \overline{N}c$$

$$\overline{Nu \pm \Delta Nu} = \overline{N}c \qquad \text{but } \overline{u} = 0$$

and $\therefore$ $\pm\overline{\Delta Nu} = \overline{N}c$

If $\underline{\Delta N}$ and $\underline{u}$ are considered as always being of like sign, either (+) or (-), then

$$\Delta\overline{Nu} = \overline{N}c \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad 6$$



The accompanying sketch shows a typical vertical distribution of suspended sediment. From this curve

$$\Delta\overline{N} = \ell\frac{\overline{dN}}{dy} \quad \ldots\ldots\ldots \quad 7$$

where $\underline{y}$ is the distance from the stream bed to a point on the curve and $\underline{\ell}$ is the Prandtl "mixing length," which may

be thought of as the distance an eddy travels before it loses its identity. Combining equations 6 and 7

$$\overline{u\ell}\,\frac{\overline{dN}}{dy} = \overline{N}c \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad 8$$

In this equation the quantity $\overline{u\ell}$, commonly designated by ϵ, is known as the "momentum transfer coefficient," and is a measure of the diffusion power of the turbulence. If equation 8 is integrated it becomes

$$\log_e \overline{N}/N_a = -c\int_a^y dy/\epsilon \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad 9$$

This is the general equation for sediment distribution in the vertical in terms of a known concentration at some point, a. If ϵ can be expressed or evaluated in terms of some known quantity, this equation can be used to represent the shape of the sediment distribution curve. Prandtl used this term in expressing the relationship of the

unit shear τ between two layers of a fluid at any point, the unit density, ρ, and the slope of the vertical velocity curve $d\overline{V}/dy$.

$$\tau = \rho\,\epsilon\, d\overline{V}/dy \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad 10$$

The shear τ, and the slope of the vertical velocity curve $d\overline{V}/dy$, can now be evaluated. The shear at the stream bed may be expressed as the component of weight in the direction of stream slope. If the shear is assumed to vary linearly with depth, as it probably does in wide rivers where side effects are negligible, it may be expressed as

$$\tau = (D - y)\, wS \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad 11$$

By combining equations 10 and 11

$$\epsilon = \frac{(D - y)\ wS}{\rho\, d\bar{V}/dy} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad 12$$

When the shape of the vertical velocity curve has been determined this value of ϵ can be substituted in equation 9.

10. Vertical velocity distribution and its relation to roughness--An evaluation of $d\bar{V}/dy$, characterizing the shape of the vertical velocity curve, is necessary both in solving equation 9 and later in computing the mean sediment concentration in a vertical section of the stream. Many useful theoretical equations are available for expressing the vertical distribution of velocity. The following equation developed by Von Karman for velocity distribution in rough conduits is used in this analysis because of its physical significance:

$$x = 1 + \frac{\sqrt{\tau_o/\rho}}{0.4V_m}\ (1 + \log_e z) \quad \ldots\ldots\ldots\ldots\ldots \quad 13$$

where the factor 0.4 is a universal constant determined experimentally by Von Karman, equation 13 may also be written

$$\bar{V}/V_m = 1 + \frac{\sqrt{\tau_o/\rho}}{0.4V_m}\ (1 + \log_e y/D) \quad \ldots\ldots\ldots\ldots \quad 14$$

By substituting equation 11 for τ_o, and the Manning or Chezy equation for V_m, assuming the hydraulic radius to be equal to the depth, equation 13 may be simplified to

$$x = 1 + 9.50\ n/D^{\ 1/6}\ (1 + \log_e z)$$

$$= 1 + 14.2/C\ \ (1 + \log_e z) \quad \ldots\ldots\ldots\ldots\ldots \quad 15$$

Curves A of Fig. 2 show vertical velocities computed by equation 15 for various values of $n/D^{1/6}$. These curves do not turn back near the water surface as has often been found to be the case in actual streams, because the theory of turbulence exchange is not applicable to a negative slope. Equation 15 gives a zero velocity slightly above the bed, and negative velocities below that point. In this study the equation is applied only to the point at which the velocity is zero, all negative velocities being considered zero. The point at which the velocity equalled zero was practically at the bottom for the smallest relative roughness value used, 0.010, and was about one per cent of the depth above the bottom for the largest relative roughness value, 0.030. A computation was made also for a relative roughness factor of 0.025 to determine the error introduced due this deviation from the theoretical curve. In Fig. 5 is shown an adjusted vertical velocity curve, for a relative roughness value of 0.025 ($C = 60$), which has been arbitrarily adjusted so that it turns back near the water surface and so that the velocity equals zero at the stream bed. However, the area under the adjusted curve is equal to that under the theoretical curve. This adjusted curve approaches the actual vertical velocity distribution found in most streams. The theoretical sediment distribution curve was combined graphically with the adjusted velocity distribution curve for various values of t, then the mean sediment concentration was obtained by determining the area under this combined sediment-velocity curve. A comparison between these and similar values obtained by using the theoretical velocity curve revealed less than one per cent deviation for all values of t, which indicates that the effect of the adjustment of the theoretical

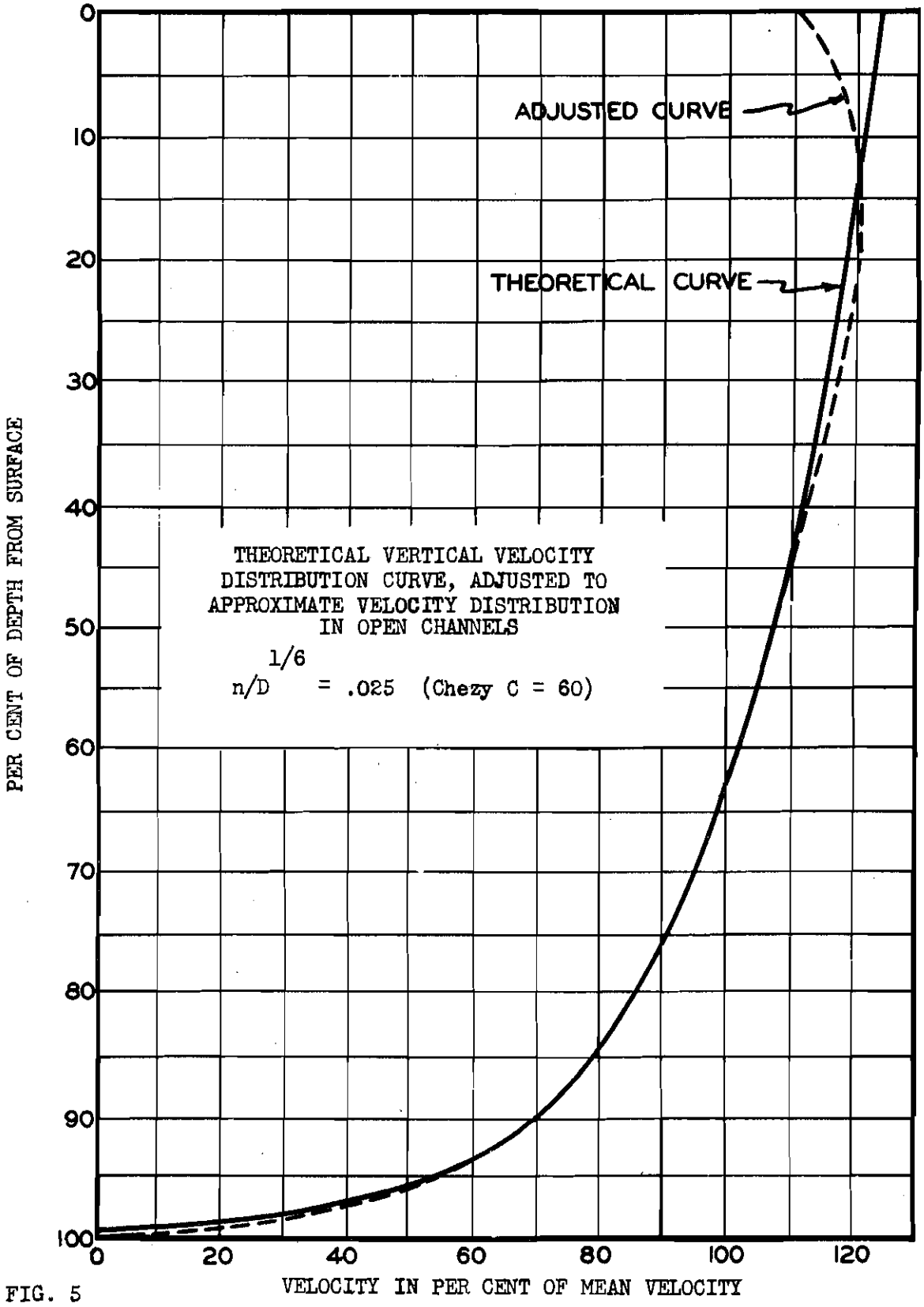

ADJUSTED CURVE
THEORETICAL CURVE
THEORETICAL VERTICAL VELOCITY DISTRIBUTION CURVE, ADJUSTED TO APPROXIMATE VELOCITY DISTRIBUTION IN OPEN CHANNELS
$n/D^{1/6} = .025$ (Chezy C = 60)
PER CENT OF DEPTH FROM SURFACE
0
10
20
30
40
50
60
70
80
90
100
0
20
40
60
80
100
120
VELOCITY IN PER CENT OF MEAN VELOCITY

FIG. 5

velocity curve is negligible.

Further analysis revealed that the theoretical vertical velocity curves agreed fairly well with an average curve which had been developed from velocity observations in streams. The 0.6 depth velocity or the mean of the velocities at the 0.2 and 0.8 depths, either of which in ordinary discharge measurements, is assumed to be the mean velocity in the vertical, checked the mean for all the curves within one per cent.

11. Simplified equation for vertical distribution of suspended sediment--By differentiating equation 14, the value of $d\bar{V}/dy$ is found to equal $\frac{\sqrt{\tau_o/\rho}}{0.4\ y}$. Substituting this value of dV/dy in equation 12 it becomes

$$\epsilon = 0.4D\sqrt{\tau_o/\rho}\,(1 - z)z \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad 16$$

When this value for ϵ is substituted in equation 9 the following expression for vertical distribution of suspended sediment is obtained:

$$\log_e N/N_a = \frac{c}{0.4\sqrt{\tau_o/\rho}} \int_a^z dz/z\,(1-z) \quad \ldots\ldots\ldots \quad 17$$

Plotting the expression, $\int_a^z dz/z\,(1-z)$, with respect to depth gives a curve which, for all practical purposes, may be treated as a straight line. This procedure simplifies equation 17 in which may now be written

$$\bar{N} = \bar{N}_a\ e^{-16t(z-a)} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad 18$$

where $t = c/\sqrt{\tau_o/\rho}$

Substituting for τ_o its equivalent value wDS and multiplying the members in the above equation for t by the respective members of the

Manning formula for mean velocity, this expression may be simplified to

$$t = \frac{0.0086\ c}{(n/D^{1/6})\ V_m} \quad \text{or} \quad \frac{0.0057\ c\ C}{V_m} \quad \ldots\ldots 19$$

where c = fall velocity of particle, cm./sec.

(If this term is expressed in ft./sec. the constants in equation 19 become 0.2619 and 0.1746, respectively.)

n = Manning roughness coefficient

D = Depth of stream, ft.

V_m = mean velocity in vertical ft./sec.

C = Chezy roughness coefficient ($1.5\ n/D^{1/6}$)

A nomograph for solution of equation 19 for various stream and sediment conditions has been presented in Fig. 3.

Equation 18 plots on semi-logarithmic paper as a straight line. If the sediment concentration at one point is known, the sediment distribution curve for the vertical can be constructed on this type of paper by simply drawing a straight line with a slope of $-16t$ through the known point. Several sediment distribution curves for different values of t are shown on rectangular coordinates in Fig. 1.

12. Determination of true mean sediment concentration in the vertical--The true mean sediment concentration in a vertical is a quantity such that when multiplied by the mean velocity in the vertical, the value of the actual suspended sediment discharge in a unit width of the stream is obtained. Such quantities were determined in this analysis by dividing the total suspended load in a unit width by the total water discharge in the corresponding unit width of stream. The total suspended load can be

computed by combining the concentrations obtained for individual points using equation 18, with the velocity at the corresponding points as expressed by equation 15. Integrating with respect to depth gives

$$W = P\bar{V}_m D\bar{N}_a e^{16ta} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad 20$$

where W = total load of suspended particles having a fall velocity $\underline{c}$ per unit width of stream.

P = ratio of true mean concentration in the vertical to sediment concentration in suspension at the bottom.

$$P = \left[(1 + 9.50\ n/D^{1/6})\int_0^1 e^{-16tz}\,dz\right] + \left[9.50\ n/D^{1/6}\int_0^1 e^{-16tz}\log_e z\,dz\right]$$

$$= \left[(1 + 14.2/C\int_0^1 e^{-16tz}dz\right] + \left[14.2/C\int_0^1 e^{-16tz}\log_e z\,dz\right] \quad \ldots\ldots \quad 21$$

$\underline{P}$ is a function only of $\underline{t}$ and the relative roughness, $N/D^{1/6}$. Although equation 21 is complicated, once the integrals have been evaluated for the range of conditions to be studied, they need not be computed again. In Fig. 4 are drawn curves which will facilitate evaluation of $\underline{P}$ for various values of $\underline{t}$. The curves shown cover a range of relative roughness from 0.010 to 0.030 and should be found adequate for nearly all practical conditions of natural streams and channels.

It should be remembered that equation 20 determines the sediment load for one particular size only. In order to determine the total load transported in suspension it will be necessary to calculate from the known sediment concentration and the size distribution at a single point, the quantity of materials of each individual size, considering that each of these values applies to a narrow gradation of material. The total load

can be found by summing the amounts in these narrow gradations.

The true mean concentration of a certain particle size can be determined by dividing the total load for that size by the water discharge, $V_m D$, in the vertical section of unit width. This results in the equation

$$\bar{N}_m = P\bar{N}_a e^{16ta} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad 22$$

where $\bar{N}_m$ = true mean concentration in a vertical of unit width.

IV. CONDITIONS OF ANALYSIS OF SAMPLING METHODS

13. Computation procedure--The various methods of sampling suspended sediment in a vertical were analyzed by comparing the mean sediment concentration as determined by each method with the true mean concentration in the vertical section. The sediment concentration at a point or combination of points, as defined by the particular method analyzed, was determined from the equation $\overline{N} = \overline{N}_o e^{-16tz}$, where $\overline{N}$ and $\overline{N}_o$ represent the mean sediment concentrations of a given particle size range at the point z and at the bottom, respectively. The true mean sediment concentration was determined from $\overline{N}_m = \overline{N}_o P$, P being determined for various values of t as shown in Fig. 4. The difference between the mean obtained by any method studied and this true mean is considered as a measure of the error in that method and is expressed in per cent of the true mean concentration.

14. Particle sizes and stream conditions considered--The errors in sampling due to gradation of particle size are of fundamental importance. These errors were calculated for individual particle sizes ranging from 0.010 to 0.65 mm. The particles were treated as quartz spheres and their fall velocities in water at 20° C., were used in all calculations. The particle sizes and corresponding fall velocities considered in this analysis are as follows:

Particle Diameter mm.	Fall Velocity cm./sec.
0.010	0.0089
0.025	0.056
0.050	0.215
0.115	1.00
0.35	5.00
0.65	10.00

To illustrate the errors of the various sampling methods upon total concentration where a gradation of particle size exists, as in natural streams, three size classifications, fine, medium, and coarse, were analyzed. The classifications used and the assumed conditions for analysis are presented in Section 27.

The errors due to particle size variations which would occur in sampling were determined by the turbulence-suspension theory of sediment distribution for the following conditions:

a.	Mean velocity in vertical, V_m,	0.5 to 15 ft./sec.
b.	Relative roughness $n/D^{1/6}$	0.010 to 0.030
	or corresponding Chezy C	150 to 50
c.	Particle size	0.010 to 0.65 mm.

15. Validity of results--In evaluating the errors in any individual method it was assumed that the sample collected is a true average of the fluctuating sediment concentration at the sampling point. However, a study of sampler entrance conditions, presented in Report No. 5 of this series, revealed that a sampler does not always collect a true sample, and also disclosed that as the size of the sediment increases the opportunity for error increases. In this analysis of sampling methods errors inherent in the sampler itself are not taken into account. Such errors will be superimposed and will either increase or decrease the total sampling error according to their algebraic signs. The effect of erroneous sampler action upon the validity of calculated sampling error is demonstrated by the depth-integration method of sampling a vertical when performed by opening the sampler at the bottom and allowing it to fill as it is raised at a uniform rate. The theoretical analysis indicates that

a high positive error will result because a relatively large portion of the sample is collected near the bed due to the pressure differential. However, the initial inrush would result in a significant loss in the larger sizes of sediment and a lower unit concentration than that representative of the true sediment concentration of the stream at that point.

If the inaccuracy of the sampler itself is insignificant, the errors in sampling methods determined in this analysis are comparable for the sediment and velocity variations assumed. Thus, for any case where the vertical velocity and sediment distributions are the same as those assumed, the comparisons will be valid, regardless of whether the sediment distribution is due to suspension, saltation, or other causes. The vertical velocity and suspended sediment distributions assumed in this analysis closely approximate those usually found in streams and it is believed, therefore, that, in the majority of cases, the results indicated are reasonably accurate.

As more data on the vertical distribution of sediment in streams is accumulated, and the physical laws governing sediment transportation are more completely understood, the conditions for which this method of analysis applies will become more evident. There is already some evidence that it does not apply where the stream bed or banks are covered with large rocks or other immovable objects, because, in such cases, the sediment distribution apparently is more uniform than the turbulence-suspension theory would indicate.

16. Manner of presenting results--In Figs. 6 to 8 are shown graphically the per cent deviation from the true mean concentration, or the errors in individual particle size determination in relation to the

sediment distribution as expressed in the index $\underline{t}$ for all point sample methods and for one depth-integration method. As previously emphasized, the validity of the results presented in this manner is dependent upon the existence of the sediment distribution expressed by $\underline{t}$ and upon the accuracy of the sampler. The auxiliary ordinate scales in these figures showing the ratio of particle fall velocity to mean stream velocity in the vertical, $\underline{c/V_m}$, are conditioned upon the reliability of the sediment distribution theory for the particular stream conditions being analyzed. If the actual sediment distributions conform to the theoretical, the $\underline{c/V_m}$ scale can be used in estimating errors in sampling methods for a rather wide range of sediment and stream conditions.

The values of the errors arising in using the point sample methods over representative ranges of particle size, stream roughness, and velocity are given in Table 1. In the interpretation of these results it should be recalled that they are dependent upon the conformity of the actual with the theoretical velocity and sediment distributions. Fig. 9 illustrates the relation of the errors in the various methods to the mean stream velocity in the vertical for various sizes of particles and an average relative roughness.

The errors which would occur in using the various depth-integration methods with various particle sizes and different types of sediment distributions, as determined by the stream conditions, are presented in Figs. 10, 11, and 14 for an average relative roughness condition. Tables 2 to 5 show the same results in terms of stream and sediment conditions for all depth-integration methods analyzed in this study.

The results of an analysis of methods used in determining total

concentration of graded materials, approximating gradations of particle size that exist in natural streams, are presented in relation to stream velocity in Figs. 16 and 17 for point sample and depth-integration methods, respectively.

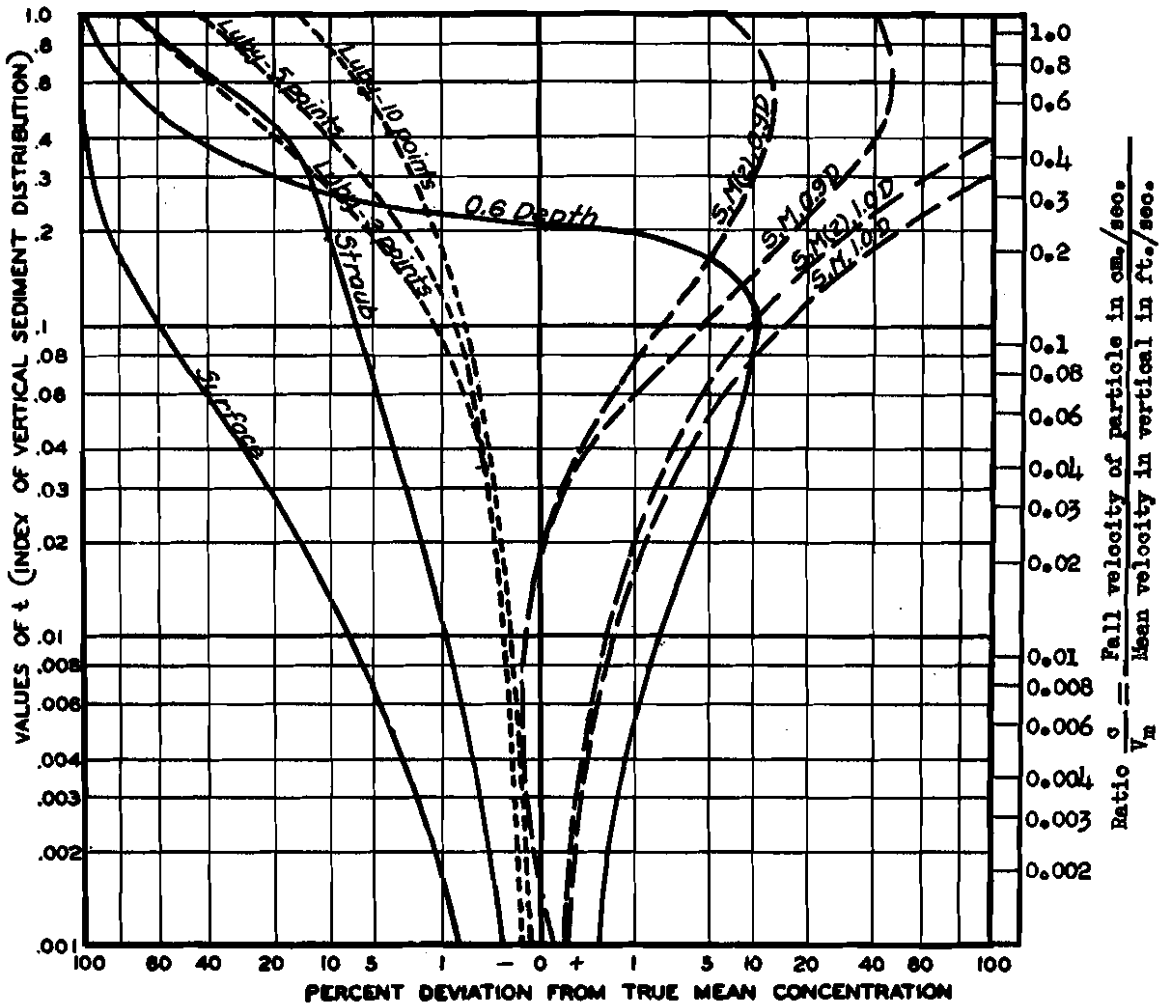
V. ANALYSIS OF POINT SAMPLE METHODS

17. Single point methods--Only the two single point methods of sampling that are commonly employed in routine sediment investigations, the surface and 0.6 depth methods, will be analyzed in this study. In the analysis of the surface method the sediment concentration at the surface was compared directly with the true mean sediment concentration without the application of a coefficient. Under all conditions the analysis indicates negative deviations from the true mean concentration that increase, as the curvature of the sediment distribution curve increases. The errors are appreciable for all particle sizes except the extremely fine material.

In the 0.6 depth method as it is commonly used, it is assumed that a sample collected at the 0.6 depth point is representative of the mean sediment concentration in the vertical. An analysis conducted on this basis indicated that this method generally results in less error than the surface method but the errors are less consistent. The deviations from the true mean concentration are positive for distributions expressed by values of about $t = 0.3$ or less, and are negative for distributions of greater curvature.

In Figs. 6 to 8 are shown the plotted values of deviation from the true mean for various values of t for this method, along with that of the other methods which will be described in the following sections of this chapter, for relative roughness factors of 0.010, 0.020, and 0.030, respectively.

18. Surface, mid-depth, and bottom methods--In past sediment studies it has been the practice to determine the mean sediment concentration



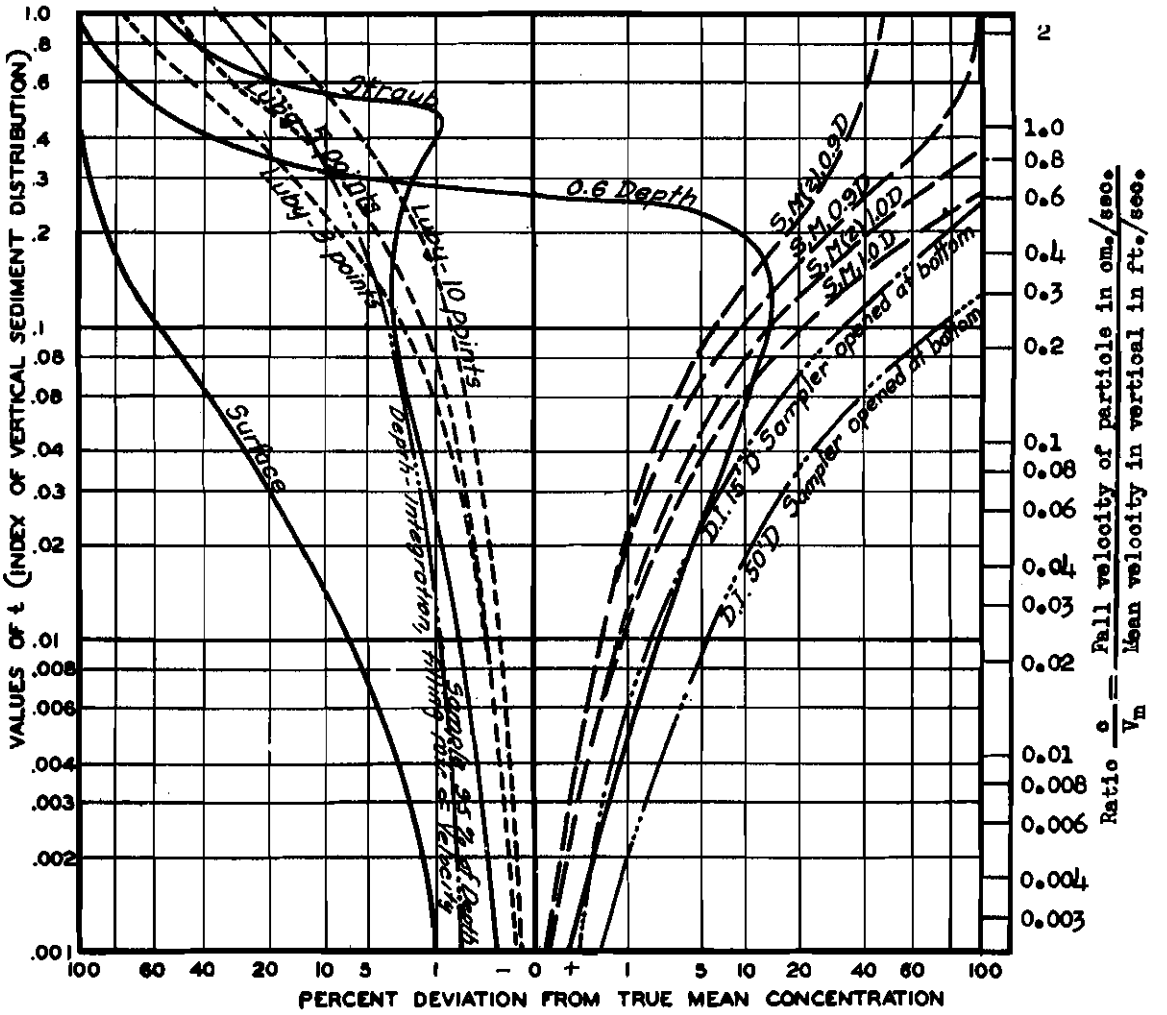
$$t = \frac{0.00858\ c}{V_m \frac{n}{D^{1/6}}}$$

where c = fall velocity of particle in cm./sec.
V_m = mean velocity in vertical in ft./sec.
n = Manning roughness coefficient.
D = depth of stream in ft.

For a relative roughness $\frac{n}{D^{1/6}} = 0.010$

$$t = 0.858 \frac{c}{V_m} \quad \text{or} \quad \frac{c}{V_m} = 1.166\ t$$

Fig. 6 - Accuracy of methods of sampling suspended sediment for various types of sediment distribution and a relative roughness 0.010.



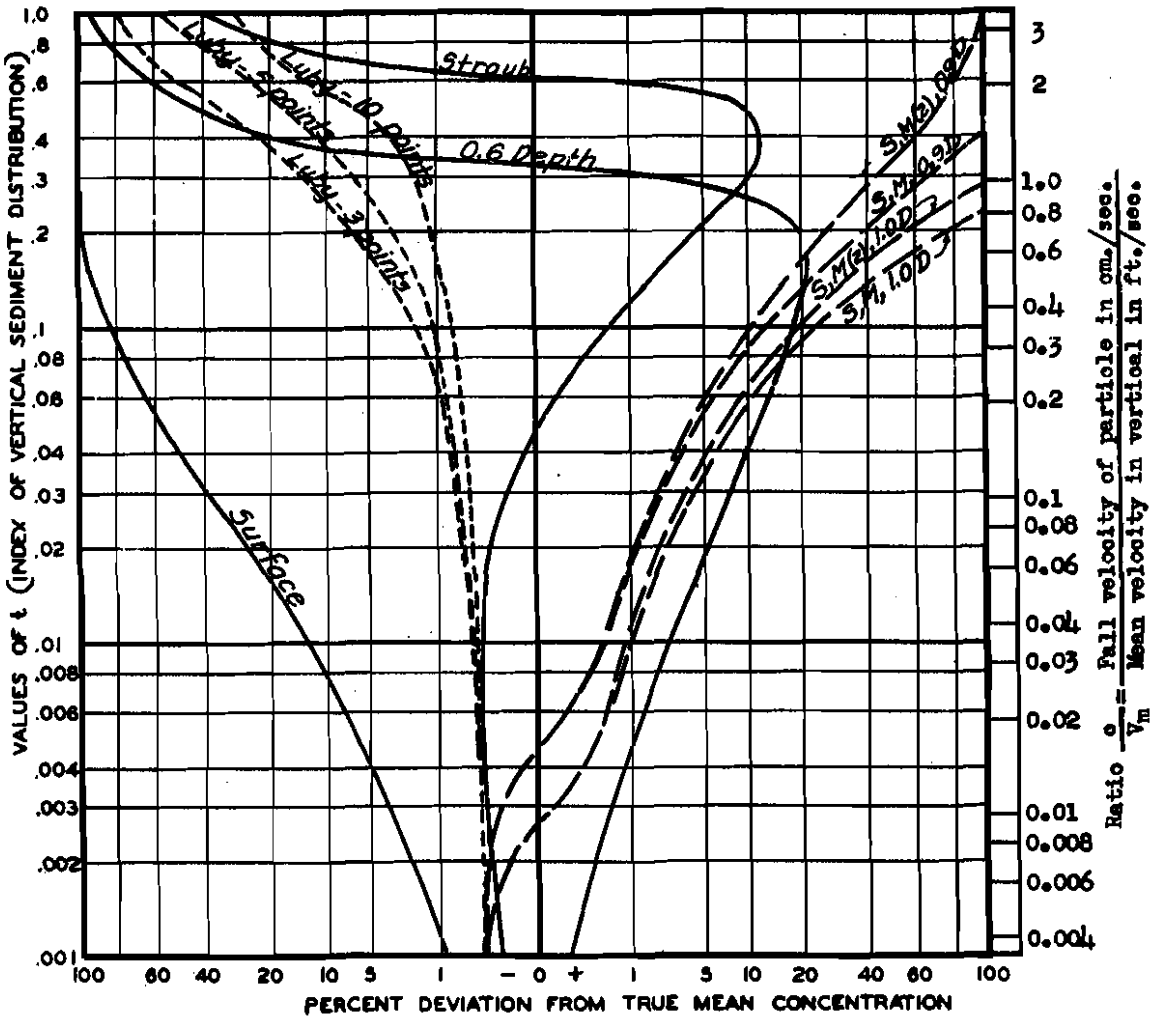
$$t = \frac{0.00858\ c}{V_m \frac{n}{D^{1/6}}}$$

where c = fall velocity of particle in cm./sec.
V_m = mean velocity in vertical in ft./sec.
n = Manning roughness coefficient.
D = depth of stream in ft.

For a relative roughness $\frac{n}{D^{1/6}} = 0.020$

$$t = 0.429 \frac{c}{V_m} \quad \text{or} \quad \frac{c}{V_m} = 2.33\ t$$

Fig. 7 - Accuracy of methods of sampling suspended sediment for various types of sediment distribution and a relative roughness 0.020.



$$t = \frac{0.00858\ c}{V_m \frac{n}{D^{1/6}}}$$

where c = fall velocity of particle in cm./sec.
V_m = mean velocity in vertical in ft./sec.
n = Manning roughness coefficient.
D = depth of stream in ft.

For a relative roughness $\frac{n}{D^{1/6}} = 0.030$

$$t = 0.286 \frac{c}{V_m} \quad \text{or} \quad \frac{c}{V_m} = 3.50\ t$$

Fig. 8 - Accuracy of methods of sampling suspended sediment for various types of sediment distribution and a relative roughness 0.030.

in a vertical either by averaging the concentrations in samples of equal volume which were collected at the surface, mid-depth, and bottom or by giving the mid-depth concentration double weight before averaging it with the concentrations in the other two samples. Although it is impossible to collect a suspended sediment sample exactly at the bottom of a stream, for purpose of this analysis the bottom sampling point was arbitrarily located at 90, 95, 98, and 100 per cent of the stream depth at the vertical.

The errors, or deviations from the true mean, are positive for practically all conditions because the bottom sample is taken at a point which is below that of mean concentration for the section of discharge it represents. Further analysis reveals that when double weight is given to the concentration of the sample from the mid-depth point, the error is reduced for all values of $\underline{t}$, index of vertical sediment distribution, greater than 0.10. In considering the point at which the bottom sample is collected it appears that the concentration in a sample from the 0.9 depth approaches the true mean closer than does a sample collected at the bottom. However, in any case the error is insignificant for $\underline{t}$ values below 0.02. Generally, the errors are less for these methods than for the surface or 0.6 depth methods for $\underline{t}$ values of 0.1 or less.

19. Straub method--Dr. Straub developed a method of sampling in the Missouri River (11) wherein the sum of 5/8 of the concentration in the sample from the 0.2 depth and 3/8 of the concentration in the sample from the 0.8 depth is considered to be the mean concentration in the vertical sampled. The sediment distribution in the vertical was considered to be linear. In this analysis the concentration determined in this manner was compared with the true mean.

The results indicate that this is one of the more accurate methods adapted to routine use. For sediment distributions of less curvature than expressed by $t = 0.03$ the errors for the three stream conditions shown are less than 3 per cent. For streams with relative roughness factors of 0.020 or 0.030 this method appears to be more accurate for a wider range of particle sizes and velocities than any other method analyzed, while for smooth channels, relative roughness of 0.010, it is not much more accurate than the 0.6 depth method. It is probable, however, that the favorable showing of this method may be offset in the field because some methods in which more samples are collected would probably better represent the actual sediment distribution in the vertical and reduce errors which are introduced due to momentary fluctuations of the sediment concentration.

20. Luby method--In the Luby method of sampling, a vertical section is divided into a number of increments of equal discharge and a sample is taken at the midpoint of each increment. By taking samples in this manner, each sample represents an equal discharge, therefore the concentrations at each point are properly weighted. This is accomplished by dividing the vertical velocity curve into equal areas and collecting a sample at the centroid of each area. A more complete description of this method is given in Report No. 1 of this series.

In these computations, a water-discharge summation curve for a unit stream width computed from the vertical velocity curve facilitated in the location of the sampling points. Curves B of Fig. 2 show the per cent of total discharge below each depth for the velocity distributions used in

this study. The cumulative per cent of total discharge from the surface to the midpoint of each increment of discharge indicates the location of the sampling points. Computations were made by using either 3, 5, or 10 points; in each case the concentrations were averaged and the composite sample thus obtained compared with the true mean.

Analysis revealed that the Luby method, using either 3, 5, or 10 points, is reasonably accurate for all sediment distributions of $t = 0.2$ or less. As might be expected, the analysis also indicated that in using the Luby method the deviations from the true mean, or error, decreased when the number of sampling points was increased. The errors are always negative because the samples are taken at the point of mean discharge in each section, while the actual point of mean concentration occurs at a lower point which has a higher concentration than that of the sampling point. Since the errors are consistently of one sign, and do not change rapidly for different conditions of velocity and sediment distribution, they could be reduced by applying a coefficient. However, if this is to be done the size distribution of the sediment must be known.

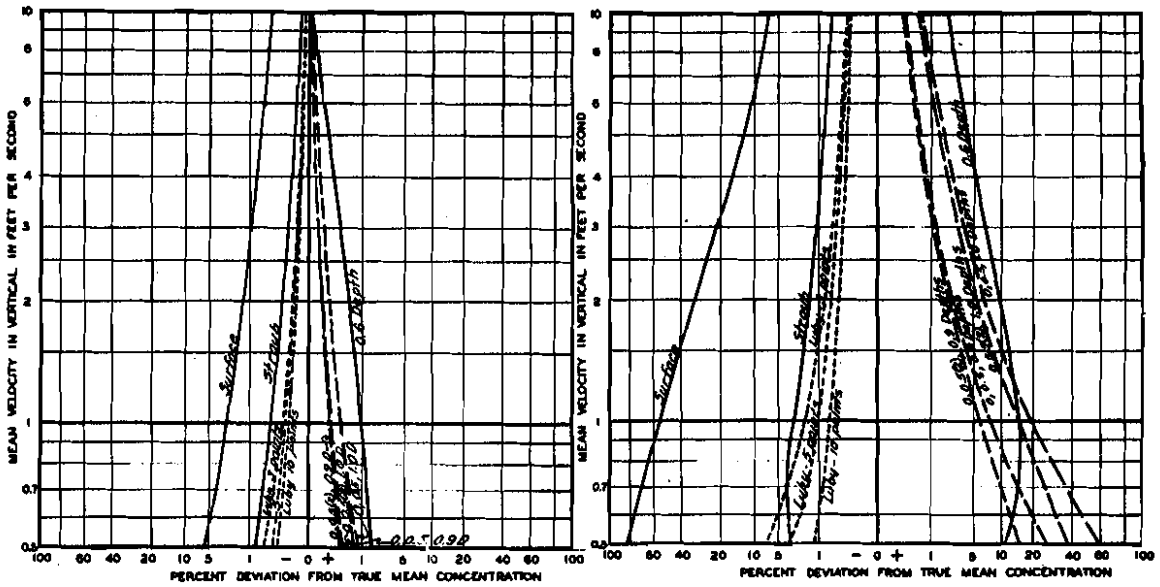
In the Luby method the velocity distribution at the time of sampling must be known although an average velocity distribution determined from a number of measurements can be used without introducing a large error. Although this method is somewhat more complicated than others it can be simplified by taking samples of equal volume. Then all the samples from one vertical can be combined into one composite sample for the analysis. The Luby is probably the best of the present methods for use in streams whose depths exceed 20 ft., and it is satisfactory also for shallower streams.

21. Sampling errors incident to stream and sediment conditions--The errors introduced in point sample methods due to variations in the vertical distribution of sediment have been discussed. Errors in sampling by various methods due to variations in sediment size and stream velocity with a relative roughness factor of 0.020 are shown graphically in Fig. 9. The validity of these results is naturally dependent upon the existence of a sediment distribution which conforms to that indicated by the turbulence-suspension theory for each given stream condition. The results by all the methods studied with three degrees of relative roughness and various particle sizes and stream velocities are summarized in Table 1. Concerning the individual methods, the following general conclusions can be drawn:

a. Single point method: The true mean concentration cannot be determined from single point samples except for a few limited cases, because the depth at which the mean sediment concentration occurs varies with particle size and with stream conditions. The surface method gives large negative errors for all except very small particle sizes. The results obtained in the 0.6 depth method are better; however, this method also gives large errors unless the particle size of the sediment is small.

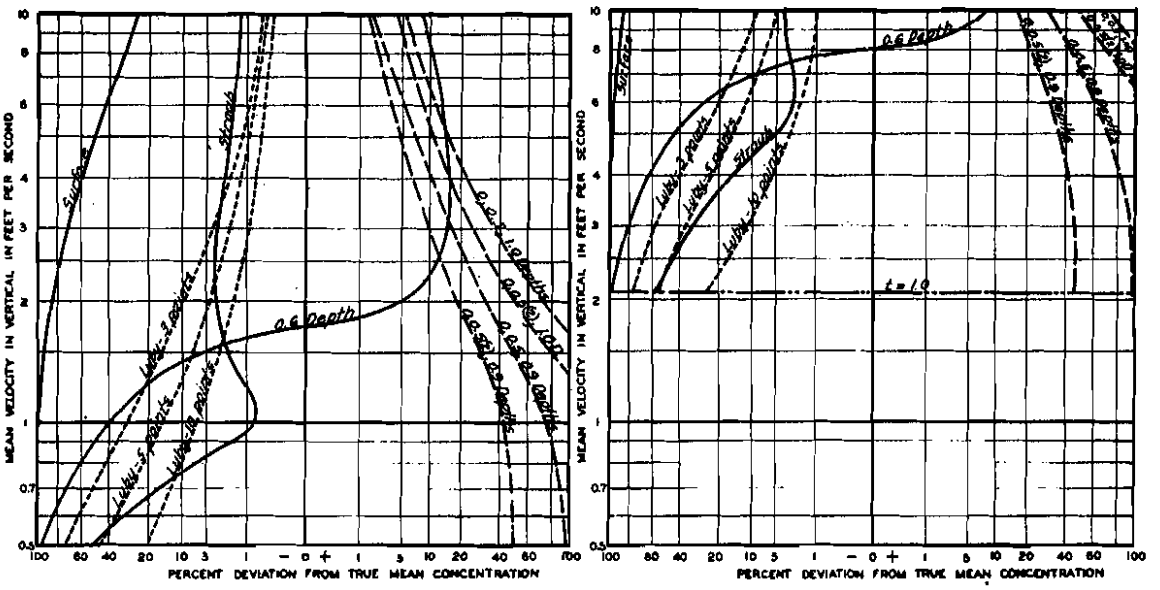
b. Surface, mid-depth, and bottom method: This method is reliable only for relatively fine particles or for high velocities. In most cases the error was positive for all stream conditions and particle sizes, because the bottom sample is collected at a point which is lower than the point of mean concentration of the section of discharge it represents. By assigning double weight to the mid-depth concentration the error is reduced somewhat; similarly, the error is usually less when the sample is collected at a point some distance above the bottom instead of at the bottom. However, the errors are still excessive for the larger particle sizes.

c. Straub method: This method is one of the more accurate of those adapted for routine use. The errors are negative for all particle sizes with relative roughness coefficients of 0.010 and 0.020, changing to positive for the larger particle sizes with a relative roughness coefficient of 0.030. For relative



(a) Particle diameter 0.010 mm.

(b) Particle diameter 0.050 mm.



(c) Particle diameter 0.115 mm.

(d) Particle diameter 0.35 mm.

Fig. 9 - Accuracy of point sample methods for various stream and sediment conditions. Relative roughness 0.020.

TABLE 1

ACCURACY OF POINT SAMPLING METHODS

Mean velocity in vertical ft./sec.	Sampling method*	Percentage error for various relative roughness conditions														
		Relative roughness $n/D^{1/6}$ = 0.010					Relative roughness $n/D^{1/6}$ = 0.020					Relative roughness $n/D^{1/6}$ = 0.030				
		Particle size, mm.					Particle size, mm.					Particle size, mm.				
		0.010	0.025	0.05	0.115	0.35	0.010	0.025	0.05	0.115	0.35	0.010	0.025	0.05	0.115	0.35
0.5	Surface	-12	-58	- 99	----	----	- 5.6	-32	-42	-100	----	- 3.9	-21	-44	- 99	----
	0.6 depth	+ 8.8	+10	- 51	----	----	+ 1.7	+10	+12	- 91	----	+ 1.2	+ 8.1	+21	- 58	----
	0, 0.5, 0.9 D	0.0	+ 4.5	+ 41	----	----	+ 0.2	+ 3.5	+27	----	----	+ 0.1	+ 2.3	+12	+120	----
	0, 0.5, 0.95 D	+ 0.8	+ 5.3	+ 52	----	----	----	----	----	----	----	+ 0.2	+ 3.6	+28	+260	----
	0, 0.5, 0.98 D	+ 0.6	+11	+ 81	----	----	----	----	----	----	----	+ 0.2	+ 4.2	+34	+370	----
	0, 0.5, 1.0 D	+ 0.9	+14	+180	----	----	+ 0.7	+ 7.6	+40	----	----	+ 0.4	+ 6.2	+40	+460	----
	0, 0.5 (2), 0.9 D	+ 0.2	+ 2.6	+ 14	----	----	+ 0.2	+ 2.6	+ 6.0	----	----	+ 0.1	+ 2.1	+14	+ 94	----
	0, 0.5 (2), 0.95D	+ 0.4	+ 6.0	+ 49	----	----	----	----	----	----	----	+ 0.2	+ 2.1	+20	+120	----
	0, 0.5 (2), 0.98D	+ 0.6	+ 6.2	+ 74	----	----	----	----	----	----	----	+ 0.2	+ 2.6	+24	+220	----
	0, 0.5 (2), 1.0 D	+ 0.7	+ 9.0	+ 96	----	----	+ 0.6	+ 5.6	+58	----	----	+ 0.2	+ 4.2	+27	+240	----
	Stremb	- 1.2	- 6.2	- 12	----	----	- 0.6	- 1.7	- 8.5	- 67	----	- 0.2	- 0.2	+ 1.7	+ 6.0	----
	Luby - 3 point	- 0.1	- 1.0	- 22	----	----	- 0.2	- 0.6	- 7.0	- 70	----	- 0.2	- 0.2	- 2.2	- 48	----
	Luby - 5 point	- 0.1	- 0.7	- 10	----	----	- 0.1	- 0.5	- 3.2	- 44	----	- 0.2	- 0.4	- 1.5	- 28	----
	Luby - 10 point	- 0.1	- 0.5	- 3.8	----	----	- 0.1	- 0.4	- 1.2	- 20	----	- 0.2	- 0.3	- 0.6	- 12	----
1.0	Surface	- 6.0	-33	- 94	-100	----	- 6.9	-17	-54	- 99	----	- 2.1	-11	-32	- 93	----
	0.6 depth	+ 1.5	+ 7.8	+ 1.6	- 93	----	+ 0.2	+ 5.1	+15	- 40	----	+ 0.4	+ 2.2	+14	+ 5.0	----
	0, 0.5, 0.9 D	0.0	+ 0.8	+ 12	+ 44	----	+ 0.1	+ 1.1	+ 7.5	+ 71	----	- 0.1	+ 0.7	+ 6.6	+ 84	----
	0, 0.5, 0.95 D	+ 0.2	+ 2.6	+ 29	+190	----	----	----	----	----	----	- 0.1	+ 1.0	+ 9	+100	----
	0, 0.5, 0.98 D	+ 0.2	+ 3.5	+ 32	+320	----	----	----	----	----	----	0.0	+ 1.2	+11	+120	----
	0, 0.5, 1.0 D	+ 0.4	+ 4.2	+ 46	+420	----	+ 0.3	+ 3.0	+19	+260	----	0.0	+ 1.5	+15	+120	----
	0, 0.5 (2), 0.9 D	+ 0.1	+ 1.0	+ 4.9	+ 10	----	+ 0.2	+ 1.0	+ 6.0	+ 40	----	0.0	+ 1.1	+ 5.7	+ 40	----
	0, 0.5 (2), 0.95D	+ 0.2	+ 2.3	+ 15	+120	----	----	----	----	+ 90	----	0.0	+ 1.1	+ 7.7	+ 67	----
	0, 0.5 (2), 0.98D	+ 0.2	+ 1.7	+ 23	+220	----	----	----	----	+120	----	0.0	+ 1.2	+ 9.1	+ 88	----
	0, 0.5 (2), 1.0 D	+ 0.6	+ 3.4	+ 26	+340	----	+ 0.3	+ 2.2	+15	+160	----	0.0	+ 1.2	+10	+100	----
	Stremb	- 0.6	- 3.5	- 11	- 89	----	- 0.2	- 0.8	- 2.9	- 0.7	----	- 0.3	- 0.5	0.0	+ 10	----
	Luby - 3 point	- 0.1	- 0.4	- 5.2	- 60	----	- 0.1	- 0.4	- 2.1	- 30	----	- 0.3	- 0.3	- 1.0	- 14	----
	Luby - 5 point	- 0.1	- 0.3	- 2.7	- 36	----	- 0.1	- 0.3	- 1.2	- 16	----	- 0.2	- 0.2	- 0.6	- 6.6	----
	Luby - 10 point	0.0	- 0.2	- 1.5	- 12	----	- 0.1	- 0.2	- 0.7	- 7.6	----	- 0.2	- 0.2	- 0.5	- 2.4	----
2	Surface	- 3.1	-16	- 68	- 99	----	- 1.5	- 7.4	-31	- 67	----	- 1.2	- 5.7	-21	- 69	-100
	0.6 depth	+ 0.7	+ 4.2	+ 10	- 49	----	+ 0.4	+ 2.5	+ 5.4	+ 5.4	----	+ 0.1	+ 1.9	+ 7.2	+ 22	- 77
	0, 0.5, 0.9 D	0.0	+ 0.2	+ 4.4	+ 44	----	+ 0.1	+ 0.9	+ 3.0	+ 69	----	0.0	+ 0.1	+ 2.4	+ 24	+170
	0, 0.5, 0.95 D	0.0	+ 0.6	+ 9.3	+ 99	----	----	----	----	+ 49	----	0.0	+ 0.1	+ 3.6	+ 33	+270
	0, 0.5, 0.98 D	+ 0.1	+ 1.0	+ 11	+140	----	----	----	----	+ 63	----	0.0	+ 0.2	+ 4.2	+ 40	+360
	0, 0.5, 1.0 D	+ 0.2	+ 1.5	+ 14	+180	----	+ 0.1	+ 1.1	+ 5.5	+ 72	----	0.0	+ 0.3	+ 4.7	+ 46	+760
	0, 0.5 (2), 0.9 D	0.0	+ 0.1	+ 1.9	+ 16	----	+ 0.1	+ 0.8	+ 2.2	+ 17	----	0.0	+ 0.1	+ 2.4	+ 17	+ 84
	0, 0.5 (2), 0.95D	+ 0.1	+ 0.6	+ 6.3	+ 56	----	----	----	----	+ 32	----	0.0	+ 0.2	+ 3.6	+ 28	+220
	0, 0.5 (2), 0.98D	+ 0.1	+ 0.9	+ 7.2	+ 92	----	----	----	----	+ 41	----	0.0	+ 0.2	+ 2.8	+ 31	+350
	0, 0.5 (2), 1.0 D	+ 0.2	+ 1.2	+ 6.7	+120	----	+ 0.1	+ 1.0	+ 5.5	+ 46	----	0.0	+ 0.3	+ 4.2	+ 33	+490
	Stremb	- 0.4	- 2.0	- 6.6	- 22	----	- 0.1	- 0.6	- 1.8	- 3.8	----	- 0.3	- 0.2	- 0.3	+ 2.7	- 12
	Luby - 3 point	- 0.1	- 0.1	- 1.2	- 22	----	- 0.1	- 0.2	- 0.7	- 2.5	----	- 0.2	- 0.3	- 0.5	- 3.6	- 61
	Luby - 5 point	0.0	- 0.1	- 0.8	- 10	----	- 0.1	- 0.2	- 0.6	- 4.2	----	- 0.2	- 0.2	- 0.4	- 1.9	- 37
	Luby - 10 point	0.0	- 0.1	- 0.5	- 2.6	----	0.0	- 0.2	- 0.2	- 2.4	----	- 0.2	- 0.2	- 0.3	- 1.0	- 16
3	Surface	- 3.1	-12	- 41	- 95	----	- 1.0	- 5.0	-22	- 71	-100	- 0.8	- 4.0	-14	- 53	-100
	0.6 depth	+ 0.5	+ 2.5	+ 2.9	- 16	----	+ 0.2	+ 1.9	+ 6.7	+ 15	- 82	0.0	+ 1.0	+ 5.1	+ 19	- 28
	0, 0.5, 0.9 D	0.0	+ 0.1	+ 1.5	+ 29	----	+ 0.1	+ 0.7	+ 1.4	+ 16	+ 88	- 0.1	0.1	+ 1.2	+ 12	+120
	0, 0.5, 0.95 D	0.0	+ 0.4	+ 4.1	+ 88	----	----	----	----	+ 26	----	- 0.1	+ 0.1	+ 1.9	+ 18	+210
	0, 0.5, 0.98 D	0.0	+ 0.6	+ 8.4	+ 80	----	----	----	----	+ 31	----	- 0.1	+ 0.1	+ 2.3	+ 20	+290
	0, 0.5, 1.0 D	+ 0.1	+ 0.7	+ 7.2	+ 94	----	+ 0.1	+ 0.6	+ 4.0	+ 37	+510	- 0.1	+ 0.1	+ 2.6	+ 22	+560
	0, 0.5 (2), 0.9 D	0.0	+ 0.1	+ 0.7	+ 10	----	+ 0.1	+ 0.7	+ 1.1	+ 9.8	+ 44	- 0.1	0.0	+ 1.3	+ 10	+ 66
	0, 0.5 (2), 0.95D	0.0	+ 0.3	+ 2.7	+ 31	----	----	----	----	+ 17	----	- 0.1	0.0	+ 1.2	+ 14	+180
	0, 0.5 (2), 0.98D	+ 0.1	+ 0.5	+ 4.0	+ 47	----	----	----	----	+ 23	----	- 0.1	0.0	+ 2.1	+ 16	+200
	0, 0.5 (2), 1.0 D	+ 0.1	+ 0.6	+ 4.9	+ 58	----	+ 0.1	+ 0.6	+ 3.1	+ 26	+380	- 0.1	0.0	+ 2.6	+ 18	+260
	Stremb	- 0.3	- 1.4	- 4.7	- 14	----	- 0.1	- 0.4	- 1.2	- 3.6	- 32	- 0.2	- 0.5	- 0.6	- 0.6	+ 10
	Luby - 3 point	0.0	- 0.1	- 0.5	- 11	----	- 0.1	- 0.2	- 0.5	- 3.2	- 24	- 0.2	- 0.2	- 0.2	- 1.2	- 31
	Luby - 5 point	0.0	- 0.1	- 0.3	- 4.7	----	0.0	- 0.2	- 0.3	- 2.4	- 32	- 0.2	- 0.2	- 0.3	- 1.0	- 17
	Luby - 10 point	0.0	- 0.1	- 0.3	- 2.1	----	0.0	- 0.2	- 0.2	- 1.3	- 17	- 0.2	- 0.2	- 0.2	- 0.6	- 7.4
4	Surface	- 1.6	- 9.0	- 32	- 88	----	- 0.9	- 4.5	-16	- 68	-100	- 0.7	- 3.1	-11	- 48	- 98
	0.6 depth	+ 0.4	+ 2.2	+ 7.4	- 3.2	----	+ 0.1	+ 1.3	+ 5.2	+ 16	- 62	0.0	+ 0.7	+ 3.9	+ 16	- 11
	0, 0.5, 0.9 D	0.0	0.0	+ 0.7	+ 18	----	0.0	----	+ 0.9	+ 11	+ 88	- 0.1	0.0	+ 0.9	+ 7.7	+ 98
	0, 0.5, 0.95 D	0.0	+ 0.2	+ 2.5	+ 35	----	----	----	----	+ 15	----	- 0.1	0.0	+ 1.4	+ 11	+140
	0, 0.5, 0.98 D	0.0	+ 0.3	+ 3.5	+ 48	----	----	----	----	+ 19	----	- 0.1	0.0	+ 1.5	+ 14	+185
	0, 0.5, 1.0 D	0.0	+ 0.4	+ 4.2	+ 56	----	0.0	----	+ 2.5	+ 22	+330	- 0.1	0.0	+ 1.8	+ 15	+220
	0, 0.5 (2), 0.9 D	0.0	0.0	+ 0.4	+ 6.4	----	0.0	----	+ 0.9	+ 7	+ 41	- 0.1	- 0.1	+ 1.0	+ 5.6	+ 53
	0, 0.5 (2), 0.95D	0.0	+ 0.1	+ 1.6	+ 20	----	----	----	----	+ 12	----	- 0.1	- 0.1	+ 1.3	+ 9.2	+ 97
	0, 0.5 (2), 0.98D	0.0	+ 0.3	+ 2.5	+ 28	----	----	----	----	+ 14	----	- 0.1	- 0.1	+ 1.6	+ 11	+130
	0, 0.5 (2), 1.0 D	0.0	+ 0.4	+ 3.2	+ 35	----	0.0	----	+ 2.2	+ 18	+230	- 0.1	- 0.1	+ 1.4	+ 12	+160
	Stremb	- 0.2	- 1.0	- 3.9	- 12	----	0.0	- 0.3	- 0.9	- 3.2	- 15	- 0.3	- 0.4	- 0.5	+ 0.1	+ 12
	Luby - 3 point	0.0	- 0.1	- 0.3	- 6.5	----	0.0	- 0.2	- 0.4	- 2.1	- 38	- 0.2	- 0.2	- 0.3	- 1.0	- 19
	Luby - 5 point	0.0	0.0	- 0.2	- 2.8	----	0.0	- 0.1	- 0.2	- 1.6	- 25	- 0.2	- 0.1	- 0.2	- 0.8	- 10
	Luby - 10 point	0.0	0.0	- 0.2	- 1.3	----	0.0	- 0.1	- 0.2	- 0.8	- 6.4	- 0.2	- 0.1	- 0.2	- 0.4	- 3.0
6	Surface	- 1.0	- 6.1	- 22	- 73	-100	- 0.5	- 3.0	-11	- 44	- 98	- 0.5	- 2.2	- 7.4	- 20	- 88
	0.6 depth	+ 0.3	+ 1.5	+ 5.3	+ 7.9	- 86	+ 0.1	+ 0.6	+ 3.6	+ 13	- 23	0.0	+ 0.4	+ 2.6	+ 11	+ 14
	0, 0.5, 0.9 D	0.0	0.0	+ 0.3	+ 9.5	+ 51	0.0	----	+ 0.6	+ 5.0	+ 41	- 0.1	0.0	+ 0.4	+ 4.4	+ 50
	0, 0.5, 0.95 D	0.0	+ 0.1	+ 1.0	+ 16	+160	----	----	----	+ 8.4	+110	- 0.1	0.0	+ 0.7	+ 8.2	+ 78
	0, 0.5, 0.98 D	0.0	+ 0.2	+ 1.7	+ 24	+270	----	----	----	+ 11	+150	- 0.1	0.0	+ 1.0	+ 7.3	+100
	0, 0.5, 1.0 D	0.0	+ 0.2	+ 2.1	+ 29	+360	0.0	----	+ 2.0	+ 12	+180	- 0.1	0.0	+ 1.2	+ 9.2	+110
	0, 0.5 (2), 0.9 D	0.0	0.0	+ 0.1	+ 3.2	+ 12	0.0	----	+ 0.6	+ 3.9	+ 32	- 0.1	- 0.1	+ 0.2	+ 4.1	+ 34
	0, 0.5 (2), 0.95D	0.0	0.0	+ 0.5	+ 11	+ 97	----	----	----	+ 6.4	+ 71	- 0.1	- 0.1	+ 0.5	+ 5.4	+ 54
	0, 0.5 (2), 0.98D	0.0	+ 0.2	+ 1.4	+ 18	+160	----	----	----	+ 7.9	+ 96	- 0.1	- 0.1	+ 0.7	+ 6.2	+ 64
	0, 0.5 (2), 1.0 D	0.0	+ 0.2	+ 1.7	+ 20	+260	0.0	----	+ 1.4	+ 9.1	+120	- 0.1	- 0.1	+ 0.9	+ 6.8	+ 76
	Stremb	- 0.1	- 0.7	- 2.6	- 9.3	- 47	0.0	- 0.3	- 0.7	- 2.3	- 3.1	- 0.2	- 0.4	- 0.4	- 0.1	+ 9.6
	Luby - 3 point	0.0	0.0	- 0.2	- 3.2	- 49	0.0	- 0.1	- 0.3	- 1.0	- 20	- 0.5	- 0.1	- 0.3	- 0.5	- 9.4
	Luby - 5 point	0.0	0.0	- 0.1	- 1.5	- 28	0.0	- 0.1	- 0.2	- 0.7	- 12	- 0.2	- 0.1	- 0.2	- 0.3	- 3.7
	Luby - 10 point	0.0	0.0	- 0.1	- 0.6	- 10	0.0	- 0.1	- 0.1	- 0.4	- 3.8	- 0.1	- 0.1	- 0.2	- 0.2	- 1.8
10	Surface	- 0.9	- 3.5	- 14	- 52	-100	- 0.3	- 1.9	- 6.9	- 28	- 97	- 0.3	- 1.2	- 4.6	- 16	- 89
	0.6 depth	+ 0.1	+ 0.8	+ 3.6	+ 10	- 48	0.0	+ 0.4	+ 2.1	+ 6.8	+ 7.3	0.0	+ 0.2	+ 1.4	+ 7.0	+ 21
	0, 0.5, 0.9 D	0.0	0.0	+ 0.1	+ 3.3	+ 44	0.0	----	+ 0.2	+ 2.0	+ 29	- 0.1	0.0	+ 0.2	+ 1.9	+ 24
	0, 0.5, 0.95 D	0.0	0.0	+ 0.6	+ 7.1	+100	----	----	----	+ 3.7	+ 49	- 0.1	0.0	+ 0.3	+ 2.1	+ 33
	0, 0.5, 0.98 D	0.0	+ 0.1	+ 0.9	+ 9.5	+140	----	----	----	+ 4.6	+ 63	- 0.1	0.0	+ 0.4	+ 3.7	+ 40
	0, 0.5, 1.0 D	0.0	+ 0.2	+ 1.0	+ 12	+180	0.0	----	+ 0.9	+ 5.4	+ 72	- 0.1	0.0	+ 0.5	+ 4.2	+ 45
	0, 0.5 (2), 0.9 D	0.0	0.0	0.0	+ 1.4	+ 15	0.0	----	+ 0.2	+ 2.0	+ 17	- 0.1	- 0.1	+ 0.2	+ 2.1	+ 17
	0, 0.5 (2), 0.95D	0.0	0.0	+ 0.4	+ 4.6	+ 58	----	----	----	+ 3.2	----	- 0.1	- 0.1	+ 0.4	+ 2.8	+ 28
	0, 0.5 (2), 0.98D	0.0	+ 0.1	+ 0.7	+ 6.1	+ 92	----	----	----	+ 3.7	----	- 0.1	- 0.1	+ 0.4	+ 3.1	+ 31
	0, 0.5 (2), 1.0 D	0.0	+ 0.2	+ 0.8	+ 7.6	+120	0.0	----	+ 0.7	+ 4.2	+ 48	- 0.1	- 0.1	+ 0.5	+ 3.7	+ 33
	Stremb	- 0.1	- 0.4	- 1.7	- 5.2	- 28	0.0	- 0.2	- 0.8	- 1.9	- 3.6	- 0.1	- 0.3	- 0.2	- 0.2	+ 2.7
	Luby - 3 point	0.0	0.0	- 0.1	- 1.3	- 22	0.0	- 0.1	- 0.2	- 0.6	- 9.6	- 0.1	- 0.1	- 0.2	- 0.6	- 3.5
	Luby - 5 point	0.0	0.0	- 0.1	- 0.7	- 10	0.0	- 0.1	- 0.1	- 0.4	- 4.2	- 0.1	- 0.1	- 0.2	- 0.2	- 1.6
	Luby - 10 point	0.0	0.0	- 0.1	- 0.5	- 4.0	0.0	- 0.1	- 0.1	- 0.2	- 2.2	- 0.1	- 0.1	- 0.2	- 0.1	- 1.0
15	Surface	- 0.4	- 2.8	- 10	- 38	- 96	- 0.2	- 1.2	- 4.6	- 20	- 71	- 0.2	- 0.9	- 3.2	- 13	- 82
	0.6 depth	+ 0.1	+ 0.6	+ 2.4	+ 6.4	- 17	0.0	+ 0.2	+ 1.4	+ 5.2	+ 13	0.0	+ 0.1	+ 0.9	+ 4.5	+ 19
	0, 0.5, 0.9 D	0.0	0.0	+ 0.1	+ 1.1	+ 29	0.0	----	+ 0.2	+ 1.0	+ 16	- 0.1	0.0	+ 0.1	+ 1.2	+ 12
	0, 0.5, 0.95 D	0.0	0.0	+ 0.2	+ 3.2	+ 58	----	----	----	+ 2.2	+ 25	- 0.1	0.0	+ 0.1	+ 1.9	+ 17
	0, 0.5, 0.98 D	0.0	+ 0.1	+ 0.8	+ 4.9	+ 80	----	----	----	+ 2.8	+ 31	- 0.1	0.0	+ 0.1	+ 2.0	+ 20
	0, 0.5, 1.0 D	0.0	+ 0.1	+ 0.6	+ 5.1	+ 94	0.0	----	+ 0.2	+ 3.3	+ 37	- 0.1	0.0	+ 0.2	+ 2.4	+ 22
	0, 0.5 (2), 0.9 D	0.0	0.0	0.0	+ 0.6	+ 10	0.0	----	+ 0.2	+ 1.1	+ 9.6	- 0.1	- 0.1	+ 0.1	+ 1.3	+ 10
	0, 0.5 (2), 0.95D	0.0	0.0	+ 0.2	+ 2.3	+ 31	----	----	----	+ 1.9	----	- 0.1	- 0.1	+ 0.2	+ 1.7	+ 14
	0, 0.5 (2), 0.98D	0.0	+ 0.1	+ 0.5	+ 3.8	+ 47	----	----	----	+ 2.2	----	- 0.1	- 0.1	+ 0.2	+ 1.9	+ 16
	0, 0.5 (2), 1.0 D	0.0	+ 0.1	+ 0.5	+ 4.2	+ 58	0.0	----	+ 0.5	+ 2.3	+ 26	- 0.1	- 0.1	+ 0.2	+ 2.1	+ 18
	Stremb	0.0	- 0.2	- 1.1	- 4.6	- 14	0.0	- 0.1	- 0.3	- 1.2	- 3.9	- 0.1	- 0.2	- 0.2	- 0.4	+ 0.3
	Luby - 3 point	0.0	0.0	- 0.1	- 0.8	- 11	0.0	- 0.1	- 0.1	- 0.3	- 4.0	- 0.1	- 0.1	- 0.2	- 0.4	- 1.9
	Luby - 5 point	0.0	0.0	0.0	- 0.5	- 4.7	0.0	- 0.1	- 0.1	- 0.2	- 2.4	- 0.1	- 0.1	- 0.1	- 0.2	- 1.2
	Luby - 10 point	0.0	0.0	0.0	- 0.4	- 2.1	0.0	- 0.1	- 0.1	- 0.2	- 1.5	- 0.1	- 0.1	- 0.1	- 0.1	- 0.8

*Nomenclature: D refers to depth of stream; (2) means that the concentration of the 0.6 depth sample is given double weight.

roughness values of 0.020 and 0.030 this method is accurate for a wide range of particle sizes and velocities, while for a relative roughness factor of 0.010 the errors increase indicating that the method is less accurate in this range.

d. Luby method: This method is accurate for a wide range of particle sizes and stream conditions, and the degree of accuracy increases with an increase in the number of sampling point. However, the errors are excessive for very large particle sizes at low velocities.

In general, the errors increase as the particle size increases and they decrease as the velocity and relative roughness increase.

VI. ANALYSIS OF DEPTH-INTEGRATION METHODS

22. Filling rate equal to stream velocity--A slow filling sampler which will consistently draw a sample of inflowing water without disturbing either the magnitude or the direction of the stream velocity ahead of its mouth, will collect a true integrated sample. If such a sampler is held stationary, a time-integrated point sample will be obtained, whereas, if the sampler fills while traversing some distance in a vertical, it will take a so-called depth-integrated sample. In the discussion which follows this type of sampler will be described as filling at a rate equal to the stream velocity.

Obviously none of the samplers investigated in the study were capable of taking a true integrated sample in accordance with the above definition. Only the Rock Island sampler indicated a filling rate approximately proportional to the stream velocity, but the filling rate was considerably augmented, particularly at stream velocities below 2 ft./sec., by the exchange of air and water as in the normal filling process in other samplers. However, the analysis of the depth-integration method is made on the assumption that the sampler does fill at a rate equal to the stream velocity and that it does not disturb either the flow lines or velocity ahead of its intake. Since it is impossible to sample immediately at the stream bed, because of the physical shape of the sampler, and since such a practice is undesirable, because of the probability of disturbing the bed material, all computations were made assuming that the sampler traversed 80, 90, 95 and 98 per cent of the depth at the vertical.

The mean sediment concentration was determined from the area under

the combined velocity and sediment distribution curve above the maximum depth to which the sampler was lowered. A comparison between the mean concentration of this area and the mean concentration of the area under the curve from the surface to the stream bed indicated the error, or deviation from the true mean sediment concentration.

TABLE 2

ACCURACY OF DEPTH-INTEGRATION METHOD WHEN SAMPLING DIFFERENT PROPORTIONS OF THE STREAM DEPTH AT A FILLING RATE EQUAL TO VELOCITY

Mean velocity in vertical ft./sec.	Extent of sampling vertical percent of stream depth	Percentage error											
		Relative roughness $n/D^{1/6} = 0.010$						Relative roughness $n/D^{1/6} = 0.030$					
		Particle size in mm.						Particle size in mm.					
		0.010	0.025	0.05	0.115	0.35	0.65	0.010	0.025	0.05	0.115	0.35	0.65
0.5	80	- 2.2	-14	-59	----	----	----	- 1.0	- 4.5	-14	-64	----	----
	90	- 1.4	- 8	-37	----	----	----	- 0.7	- 2.5	- 7	-37	----	----
	95	- 0.9	- 3.9	-19	----	----	----	- 0.5	- 1.7	- 3.5	-14	----	----
	98	- 0.5	- 1.8	- 6	----	----	----	- 0.4	- 1.0	- 2.0	- 5	----	----
1.0	80	- 1.2	- 7	-29	-90	----	----	- 0.6	- 2.5	- 8	-39	----	----
	90	- 0.7	- 3.8	-17	-66	----	----	- 0.4	- 1.6	- 4.2	-18	----	----
	95	- 0.5	- 2.1	- 9	-40	----	----	- 0.4	- 1.0	- 2.2	- 7	----	----
	98	- 0.3	- 0.9	- 3.6	-18	----	----	- 0.3	- 0.7	- 1.3	- 3.2	----	----
2	80	- 0.6	- 3.5	-14	-66	----	----	- 0.4	- 1.5	- 4.2	-18	-75	----
	90	- 0.4	- 2.1	- 8	-41	----	----	- 0.3	- 1.0	- 2.5	- 9	-44	----
	95	- 0.3	- 1.2	- 4.0	-21	----	----	- 0.3	- 0.7	- 1.5	- 4.0	-19	----
	98	- 0.2	- 0.6	- 1.7	- 9	----	----	- 0.2	- 0.5	- 0.9	- 2.2	- 6	----
3	80	- 0.4	- 2.3	- 9	-45	----	----	- 0.3	- 1.0	- 3.0	-11	-58	-84
	90	- 0.3	- 1.4	- 5	-26	----	----	- 0.2	- 0.8	- 1.9	- 6	-30	-56
	95	- 0.2	- 0.8	- 2.7	-13	----	----	- 0.2	- 0.5	- 1.1	- 3.0	-12	-26
	98	- 0.1	- 0.5	- 1.1	- 6	----	----	- 0.2	- 0.4	- 0.8	- 1.8	- 4.3	- 7
4	80	-----	- 1.8	- 7	-34	----	----	-----	- 0.8	- 2.4	- 8	-46	-75
	90	-----	- 1.0	- 3.6	-19	----	----	-----	- 0.7	- 1.5	- 4.8	-23	-44
	95	-----	- 0.7	- 2.2	-10	----	----	-----	- 0.5	- 0.9	- 2.5	- 9	-19
	98	-----	- 0.4	- 0.8	- 4.4	----	----	-----	- 0.4	- 0.7	- 1.6	- 3.7	- 6
6	80	- 0.2	- 1.2	- 4.5	-22	-85	----	- 0.2	- 0.7	- 1.7	- 6	-32	-58
	90	- 0.2	- 0.8	- 2.6	-13	-62	----	- 0.2	- 0.6	- 1.2	- 3.3	-15	-30
	95	- 0.1	- 0.6	- 1.6	- 7	-36	----	- 0.1	- 0.4	- 0.8	- 1.8	- 6	-12
	98	- 0.1	- 0.3	- 0.7	- 2.8	-16	----	- 0.1	- 0.3	- 0.6	- 1.3	- 2.9	- 4.2
10	80	-----	- 0.7	- 2.7	-13	-64	-90	-----	- 0.5	- 1.2	- 3.5	-18	-39
	90	-----	- 0.5	- 1.6	- 7	-41	-68	-----	- 0.4	- 0.8	- 2.3	- 8	-18
	95	-----	- 0.4	- 0.9	- 3.8	-22	-41	-----	- 0.3	- 0.6	- 1.3	- 4.0	- 7
	98	-----	- 0.3	- 0.5	- 1.7	-10	-18	-----	- 0.3	- 0.5	- 1.0	- 2.2	- 3.2
15	80	- 0.1	- 0.5	- 1.8	- 8	-48	-76	- 0.1	- 0.4	- 0.8	- 2.9	-11	-26
	90	- 0.1	- 0.4	- 1.0	- 4.8	-27	-52	- 0.1	- 0.3	- 0.7	- 1.8	- 6	-12
	95	0	- 0.3	- 0.7	- 2.6	-14	-29	- 0.1	- 0.3	- 0.5	- 1.0	- 3.0	- 5
	98	0	- 0.2	- 0.4	- 1.2	- 6	-13	- 0.1	- 0.2	- 0.4	- 0.9	- 1.8	- 2.6

The results of this study are shown in Table 2 and Fig. 10. In all cases the errors are negative, increasing with increasing values of t and decreasing with increasing values of relative roughness. For the 0.010-mm. particle size sediment, this method was extremely accurate for all depths sampled. The results were fairly good for the 0.025-mm. sediment; the maximum error of 14 per cent occurred at the lowest velocity and relative roughness value and the shallowest depths sampled. In general, the errors appear to decrease when greater percentages of the depth are sampled and when the relative roughness coefficient increases but increase with the larger particle sizes of sediment.

23. Filling rate constant--A number of slow filling samplers used in the depth-integration method are not appreciably affected by stream velocity because they do not face into the stream flow and they tend to fill at a constant rate when lowered or raised at a rate of 1 to 2 ft./sec. The characteristics of slow filling samplers are discussed in Chapter VI of Report No. 5 of this series. In analyzing the depth-integration method it is assumed that the sampler fills at a constant rate while being lowered or raised. Computations were made assuming that 80, 90, 95 and 98 per cent of the depth was sampled. Since the intake rate was constant at all depths, the mean concentration in the sample was equal to the mean of the area under the sediment distribution curve from the surface to the maximum depth sampled.

The deviations from the true mean sediment concentration indicated for the various depths sampled and for various values of relative roughness are shown in Table 3 and Fig. 11. This method is very accurate for all percentages of stream depth sampled for the 0.010-mm. particle size

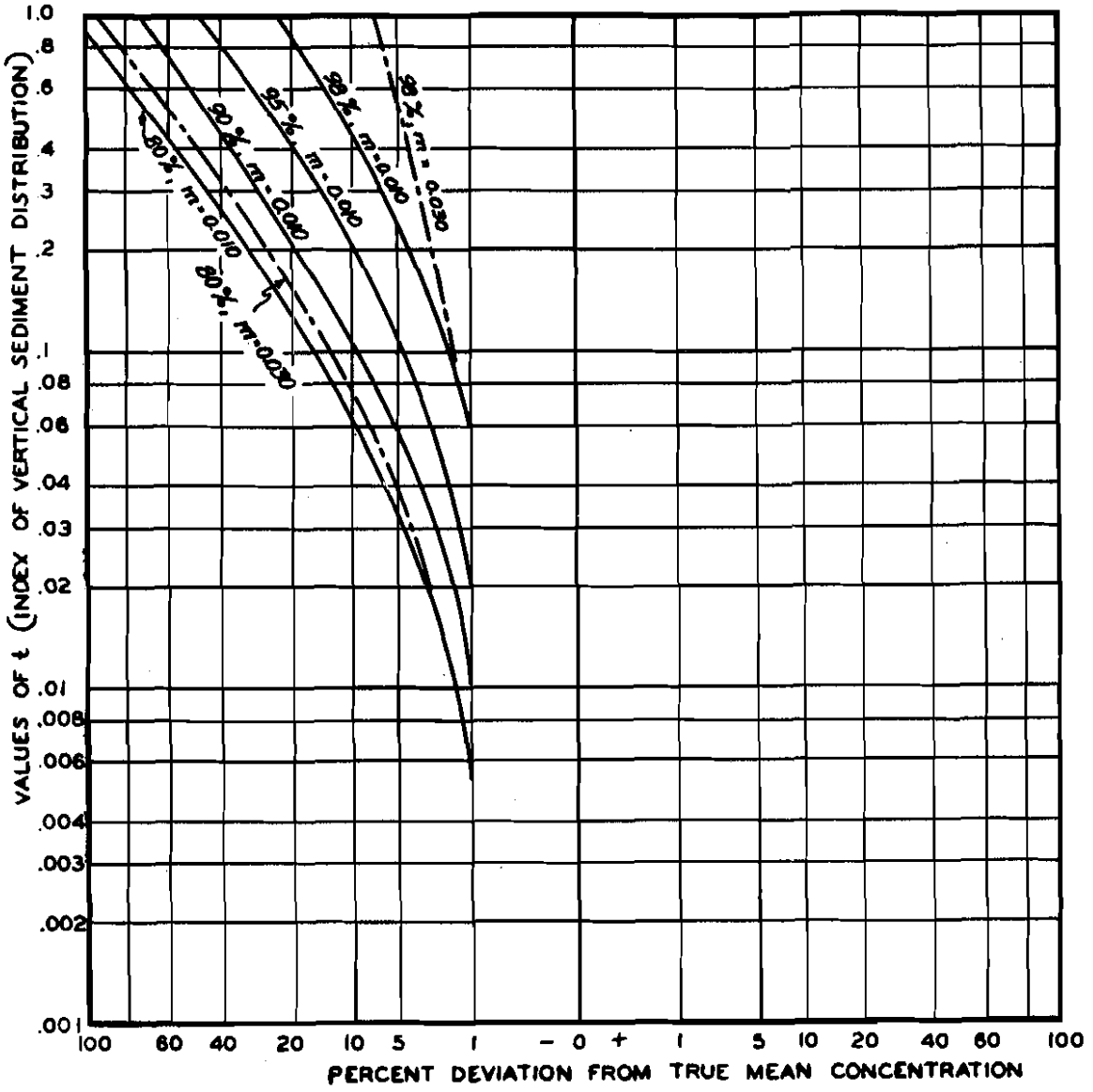


Fig. 10 - Accuracy of depth-integration method when sampling different portions of the stream depth at a filling rate equal to velocity.

sediment and is fairly accurate for the 0.025-mm. size, there being a deviation of 12 per cent at the minimum velocity for a relative roughness value of 0.010. The errors decrease with an increase in percentage of stream depth sampled and as the velocity decreases.

For a limited range of depth and for some of the relative roughness

TABLE 3

ACCURACY OF DEPTH-INTEGRATION METHOD WHEN SAMPLING DIFFERENT PROPORTIONS OF THE STREAM DEPTH WITH FILLING RATE CONSTANT

Mean velocity in vertical ft./sec.	Extent of sampling vertical percent of stream depth	Percentage error											
		Relative roughness $n/D^{1/6} = 0.010$						Relative roughness $n/D^{1/6} = 0.020$					
		Particle size in mm.						Particle size in mm.					
		0.010	0.025	0.05	0.115	0.35	0.65	0.010	0.025	0.05	0.115	0.35	0.65
0.5	80	- 1.8	-12	-56	----	----	----	- 0.7	- 4.3	-24	-80	----	----
	90	- 0.6	- 4.1	-31	----	----	----	- 0.1	- 0.5	- 6	-46	----	----
	95	- 0.1	- 0.2	-11	----	----	----	+ 0.3	+ 1.6	+ 6	- 3.7	----	----
	98	+ 0.4	+ 2.5	+ 5	----	----	----	+ 0.4	+ 2.2	+14	+52	----	----
1.0	80	- 0.9	- 5.6	-28	-69	----	----	- 0.4	- 2.1	-10	-55	----	----
	90	- 0.5	- 2.0	-13	-63	----	----	0	- 0.2	- 1.6	-22	----	----
	95	- 0.1	0	- 3.0	-31	----	----	+ 0.2	+ 0.8	+ 3.2	+ 2.2	----	----
	98	+ 0.2	+ 1.1	+ 4.2	- 0.8	----	----	+ 0.1	+ 1.5	+ 7	+30	----	----
2	80	- 0.6	- 2.7	-13	-61	----	----	- 0.2	- 1.0	- 4.5	-27	----	----
	90	- 0.3	- 1.0	- 4.8	-36	----	----	0	- 0.1	- 0.5	- 7	----	----
	95	- 0.1	0	- 0.3	-13	----	----	+ 0.1	+ 0.4	+ 1.7	+ 6	----	----
	98	0	+ 0.6	+ 2.6	+ 5	----	----	0	+ 0.7	+ 3.4	+15	----	----
3	80	- 0.4	- 1.8	- 8	-42	----	----	- 0.2	- 0.7	- 3.0	-17	-74	----
	90	- 0.2	- 0.8	- 2.8	-22	----	----	0	- 0.1	- 0.3	- 3.3	-39	----
	95	- 0.1	0	0	- 7	----	----	0	+ 0.2	+ 1.1	+ 4.5	- 1.3	----
	98	0	+ 0.4	+ 1.7	+ 5	----	----	0	+ 0.4	+ 2.2	+10	+45	----
4	80	- 0.4	- 1.3	- 6	-32	----	----	- 0.2	- 0.5	- 2.3	-12	-64	----
	90	- 0.2	- 0.6	- 2.2	-14	----	----	0	0	- 0.2	- 1.9	-29	----
	95	- 0.1	- 0.1	0	- 4.0	----	----	0	+ 0.2	+ 0.8	+ 3.5	+ 0.3	----
	98	0	+ 0.3	+ 1.3	+ 4.5	----	----	0	+ 0.3	+ 1.6	+ 8	+36	----
6	80	- 0.3	- 0.9	- 4.0	-21	-85	----	- 0.1	- 0.4	- 1.4	- 7	-46	-74
	90	- 0.1	- 0.5	- 1.5	- 8	-66	----	0	0	- 0.1	- 1.0	-17	-39
	95	0	- 0.1	0	- 1.3	-26	----	0	+ 0.1	+ 0.5	+ 2.5	+ 5	- 1.3
	98	0	+ 0.2	+ 0.6	+ 3.6	+ 0.1	----	0	+ 0.2	+ 1.0	+ 5	+25	+45
10	80	- 0.1	- 0.6	- 2.3	-12	-61	-89	- 0.1	- 0.3	- 0.9	- 4.0	-27	-55
	90	- 0.1	- 0.3	- 0.6	- 4.0	-35	-63	0	0	- 0.1	- 0.5	- 7	-22
	95	0	- 0.1	0	- 0.1	-13	-31	0	0	+ 0.3	+ 1.4	+ 6	+ 2.2
	98	0	+ 0.1	+ 0.5	+ 3.4	+ 5	- 0.8	0	+ 0.1	+ 0.5	+ 3.1	+15	+30
15	80	- 0.1	- 0.5	- 1.5	- 7	-42	-77	- 0.1	- 0.2	- 0.6	- 2.6	-17	-38
	90	0	- 0.3	- 0.6	- 2.6	-22	-46	0	0	0	- 0.3	- 3.3	-12
	95	0	- 0.1	0	0	- 7	-19	0	0	+ 0.2	+ 1.0	+ 4.5	+ 6
	98	0	0	+ 0.3	+ 1.5	+ 5	+ 1.5	0	+ 0.1	+ 0.3	+ 2.0	+10	+21

factors which were assumed, it appears that a sampler which fills at a uniform rate is more efficient, i.e., shows less deviation from the true mean, than does a sampler that fills at a rate equal to the velocity. However, a more detailed examination of Fig. 11 reveals that for all other depths sampled and other relative roughness values, the deviations are greater in this method. For example, from Fig. 11, when 98 per cent of the depth is sampled and the relative roughness m is 0.020, the

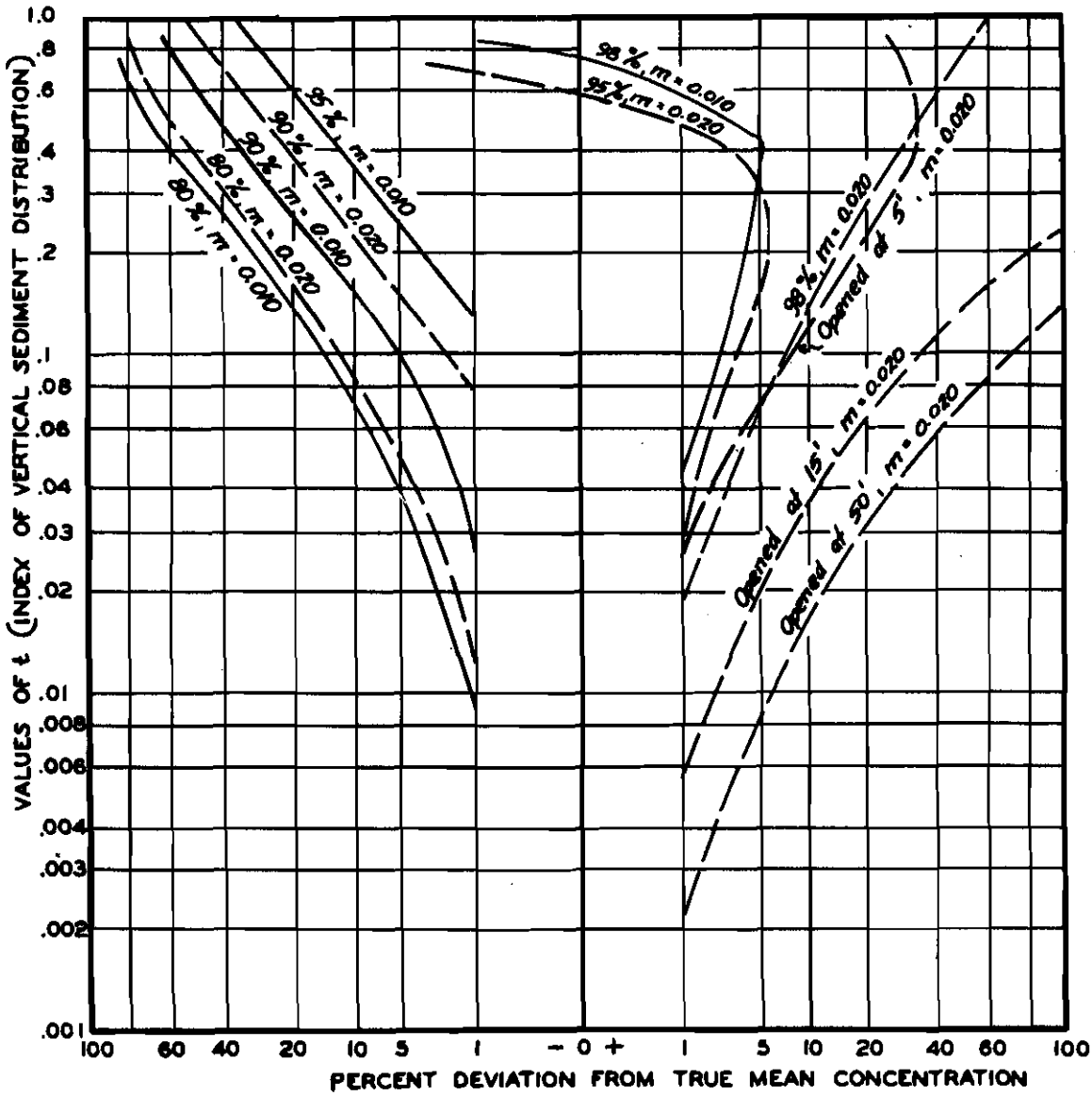


Fig. 11 - Accuracy of depth-integration method when sampling different portions of the stream depth at a constant filling rate and accuracy when sampler is opened at different depths.

error varies between 1 per cent for a value of t of 0.02 and 60 per cent for a value of t, of 1.0, whereas, from Fig. 10 for the same range of depth and for a relative roughness factor of 0.030, the maximum deviation for all values of t was less than 7 per cent.

Although it appears that sampling at a uniform rate would in certain

limited cases be better than the method in which the sampler is filled at a rate equal to the velocity, it may be further said that field experience and detailed laboratory analysis of the problem indicates that it is impossible for a sampler to fill at a uniform rate and at the same time collect a sample of a true composition. Consequently, the inherent error resulting from this type of filling action is necessarily superimposed upon these results and thereby greatly reduces the accuracy. A detailed analysis of this problem is presented in Report No. 5 of this series.

24. Sampler opened near stream bed--It has been a common practice to open a slow filling, rigid container sampler at the stream bed and to raise the sampler at a uniform rate, such that it will reach the surface before the container is filled completely. In such a procedure, part of the sample will be collected almost instantaneously from the initial rapid filling which is due to a necessary pressure equalization, and the remainder of the sample will be collected as the sampler is raised. The amount of the initial inrush to equalize the pressures is a function of the hydrostatic pressure and the volume of the sample container. In the various types of samples tested, this pressure equalization was shown, in the experimental study of sampler filling characteristics presented in Report No. 5 of this series, to occur within 1 sec. The theoretical relationship between depth at which the sampler is opened and the volume entering due to initial inrush is shown in Fig. 12. The results obtained with the samplers tested checked this theoretical relationship very closely.

After the initial inrush, the filling rate was considered constant since the error introduced by the volume of initial inrush is so large

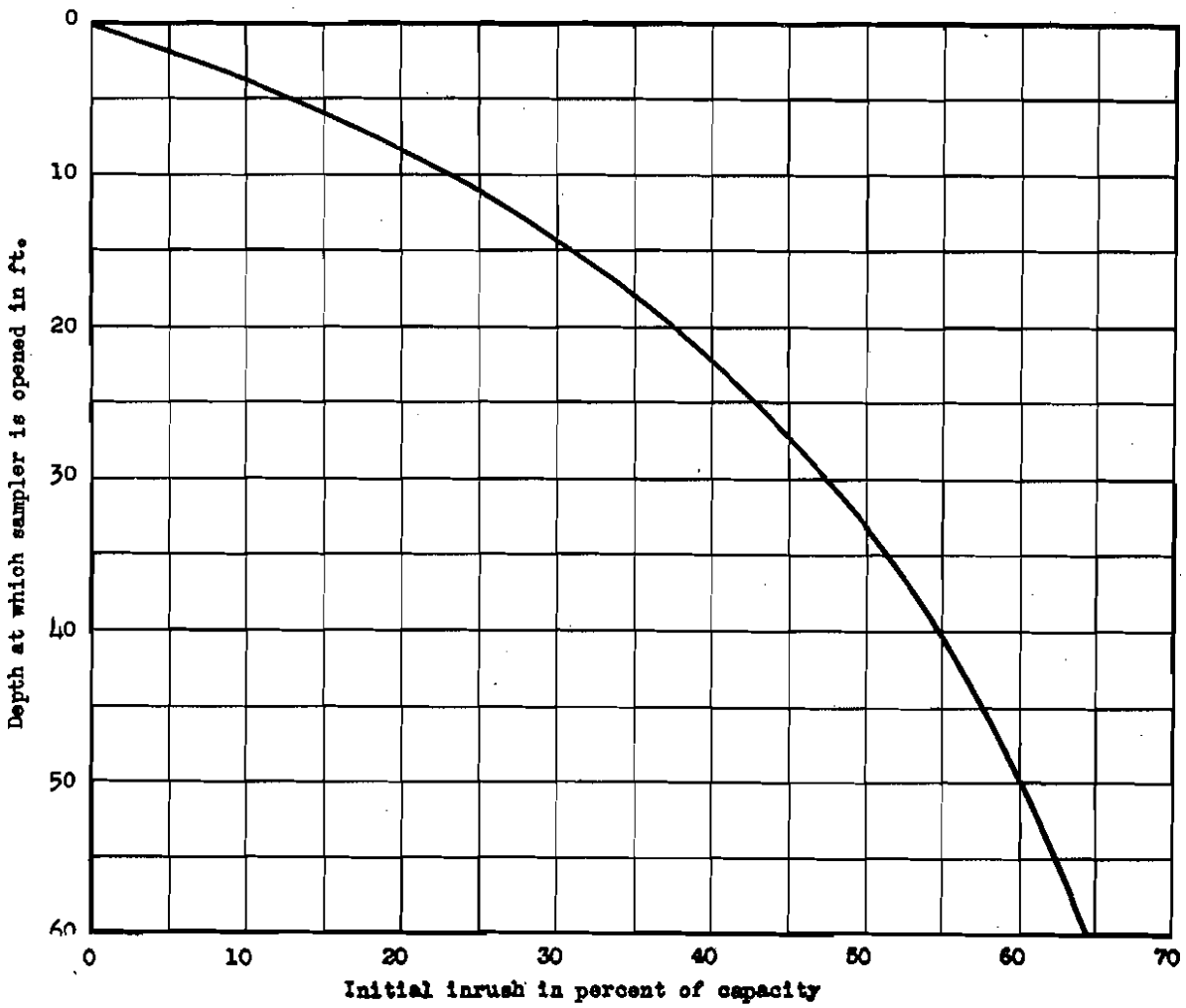


Fig. 12 - Relation of depth to per cent of sampler capacity filled in initial pressure equalization.

that the variations in the normal filling rates for the different types of slow filling samplers is insignificant.

For purposes of this analysis, it was also assumed that 90 per cent of the sampler capacity would be filled when it reached the water surface. This value was selected because usually when the sampler returns to the surface completely filled the sample is rejected. The mean concentration in the sample would then be:

$$N_m = \frac{N_o\, i + N_d\,(0.90 - i)}{0.90}$$

where N_o = concentration at the point of opening which varies according to the minimum distance from the intake to the stream bed for different samplers.

i = ratio of volume of initial inrush to capacity of sampler, determined from Fig. 12.

N_d = concentration in that part of the sample collected while being raised.

The errors in samples collected in a sampler which was opened at depths of 5, 15, and 50 ft. in a stream with a relative roughness factor of 0.020 are shown in Table 4 and Fig. 11. This method also is very

TABLE 4

ACCURACY OF DEPTH-INTEGRATION METHOD PERFORMED BY OPENING SAMPLER AT THE STREAM BED AND FILLING WHILE BEING RAISED

Relative roughness $n/D^{1/6} = 0.020$

Mean velocity in vertical ft./sec.	Depth of stream ft.	Percentage error					
		Particle size mm.					
		0.010	0.025	0.05	0.115	0.35	0.65
0.5	5	- 0.7	+ 2.5	+ 16	+ 27	----	----
	15	+ 1.5	+ 12	+ 70	+ 330	----	----
	50	+ 4.5	+ 29	+ 170	+ 1000	----	----
1.0	5	- 0.8	+ 0.7	+ 7	+ 34	----	----
	15	+ 0.6	+ 6	+ 30	+ 190	----	----
	50	+ 2.1	+ 14	+ 65	+ 470	----	----
2	5	- 0.8	- 0.5	+ 2.5	+ 16	----	----
	15	+ 0.2	+ 2.7	+ 13	+ 82	----	----
	50	+ 0.8	+ 6	+ 30	+ 190	----	----
3	5	- 0.7	- 0.7	+ 1.3	+ 11	+ 31	----
	15	+ 0.1	+ 1.5	+ 8	+ 50	+ 290	----
	50	+ 0.5	+ 4.5	+ 20	+ 120	+ 850	----
4	5	- 0.6	- 0.8	+ 0.8	+ 8	+ 34	----
	15	0	+ 0.9	+ 6	+ 36	+ 230	----
	50	+ 0.3	+ 3.0	+ 14	+ 80	+ 600	----
6	5	- 0.5	- 0.8	- 0.1	+ 4.8	+ 31	+ 31
	15	0	+ 0.6	+ 4.2	+ 22	+ 180	+ 290
	50	+ 0.1	+ 2.0	+ 9	+ 47	+ 360	+ 850
10	5	- 0.4	- 0.7	- 0.7	+ 2.2	+ 18	+ 34
	15	0	+ 0.3	+ 2.0	+ 11	+ 82	+ 190
	50	0	+ 1.0	+ 5	+ 21	+ 190	+ 470
15	5	- 0.3	- 0.7	- 0.8	+ 1.0	+ 11	+ 26
	15	0	+ 0.1	+ 1.2	+ 7	+ 50	+ 120
	50	0	+ 0.6	+ 3.5	+ 16	+ 120	+ 290

accurate at all velocities and depths for the 0.010-mm. particle size sediment; however, the errors increase to a maximum of 29 per cent for the 0.025-mm. particles. Even at a 5-ft. depth the errors exceed 30 per cent for value of t greater than 0.4. At 15 and 50 ft. the errors are excessive in all cases except those in which the sediment distribution is practically linear. In using this method the errors introduced are much greater than in any of the others. With larger size particles the deviations are so great that this method should not be used.

This analysis was made on the assumption that a true sample would be obtained with the sampler. However, from the study of sampler action presented in Report No. 5 of this series, it appears that a loss in sediment does occur with the initial inrush and that the error thus introduced varies with the particle size of the sediment.

25. Depth-integration with an existing sampler--The filling rates of present samplers are neither uniform nor equal to the stream velocity at any point in the vertical. Hence, the accuracy of the depth-integration method as performed with one of the more common samplers was analyzed. The one selected for study was the Rock Island slow filling sampler which is described in Reports Nos. 1 and 5 of this series.

Because in this case the errors do not vary directly with the parameter t the procedure of analysis was somewhat different. The method used consisted of combining the experimental data on the actual filling rate of this sampler, as presented in Report No. 5, with various sediment distribution curves. It was assumed that the filling rate varied with the stream velocity as indicated by the calibration curve in Fig. 51, Report No. 5, and that the sampler was lowered and raised at a constant

rate. The relation of filling rate to velocity shown by the calibration curve in this figure was applied to the vertical velocity distribution curve for a relative roughness of 0.020, with mean velocity varying from 1.0 to 6.0 ft./sec., and the variation of filling rate with depth for each mean velocity considered was found to be as shown in Fig. 13.

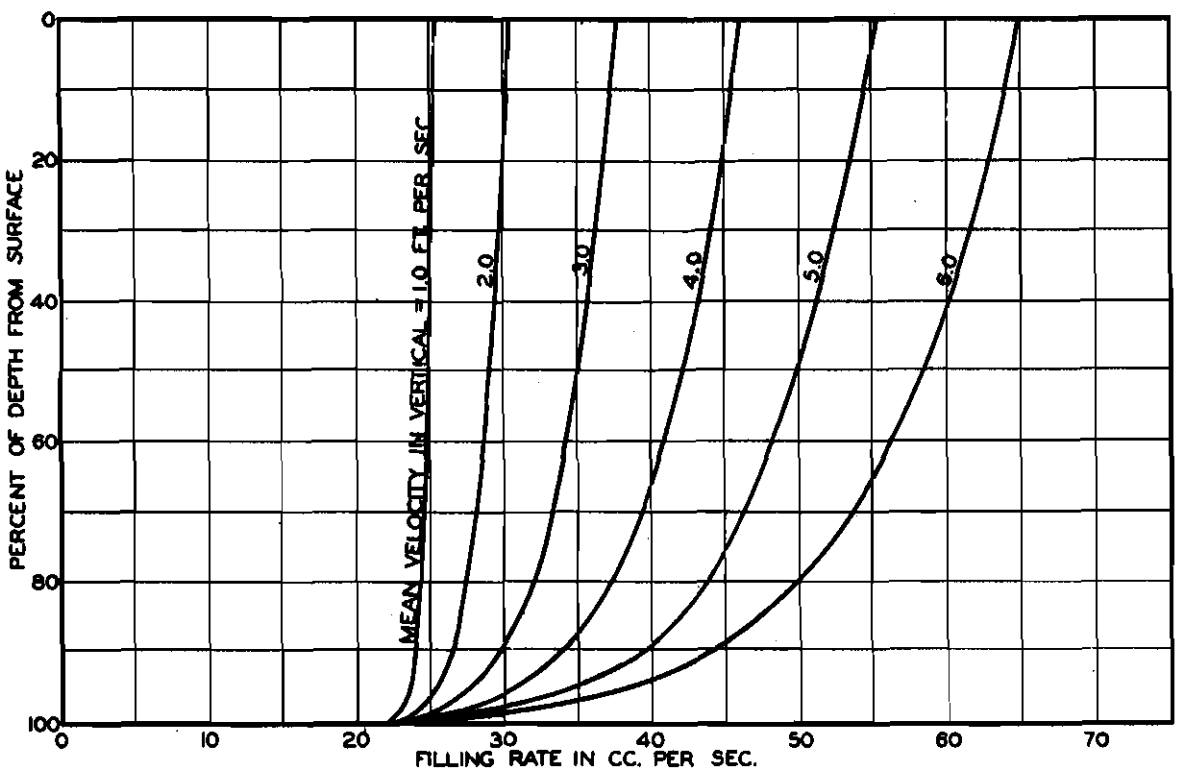


Fig. 13 - Variation of filling rate of Rock Island sampler with depth for various velocity distributions established by a relative roughness of $n/D^{1/6} = 0.020$.

Each filling rate curve was combined with the sediment distribution curve appropriate to the chosen mean velocity and sediment size, to form a series of sediment discharge curves. The areas under these curves are proportional to the weights of sediment in the respective samples. Similarly, the area under each filling rate-depth curve is proportional to

the weight of water-sediment in the sample. The ratio of the areas under two corresponding curves is the apparent mean concentration in the sample, and to determine the accuracy of the sampling method this mean concentration is compared with the true mean concentration in the vertical.

The Rock Island sampler cannot sample less than 3 in. from the stream bed. Therefore, in the assumed depths of 5 and 15 ft. this sampler could traverse 95 and 98.4 per cent of the depths, respectively. In Table 5 and Fig. 14 are presented the results of this analysis. It appears that for the percentages of depth sampled, 95 and 98.4, respectively, this sampler gives very accurate results for 0.025-mm. or smaller sediment at velocities between 1.0 and 6 ft./sec. and shows only 7 per cent error for the 0.05-mm. particles. For larger sizes of sediment the errors are appreciable. However, it should be noted that this analysis was based on the assumption that a true representative sample was collected, whereas, laboratory analysis, Report No. 5, has demonstrated that unless the sampler fills at a rate equal to the velocity at every point in the vertical, it is impossible to collect a true representative sample.

TABLE 5

ACCURACY OF DEPTH-INTEGRATION METHOD PERFORMED BY ROCK ISLAND SAMPLER, FILLING WHILE BEING LOWERED AND RAISED

Relative roughness $n/D^{1/6} = 0.020$

Mean Velocity in Vertical	Percentage Error									
	5-ft. stream depth 95% of depth sampled					15-ft. stream depth 98.4% of depth sampled				
	Particle size, in mm.					Particle size, in mm.				
ft./sec.	0.010	0.025	0.05	0.115	0.35	0.010	0.025	0.05	0.115	0.35
1.0	+0.2	+2.0	+2.6	+1.4	----	+0.3	+2.3	+7.0	+22	----
2	0.0	+0.3	+0.7	+1.0	----	0.0	+0.4	+2.0	+10	----
3	0.0	0.0	+0.3	+0.5	-19	0.0	+0.3	+1.2	+4.0	+20
4	-0.1	0.0	+0.1	0.0	-15	0.0	0.0	+0.5	+2.2	+7.0
6	-0.1	0.0	-0.1	-0.7	-11	0.0	0.0	+0.2	+1.0	+1.5

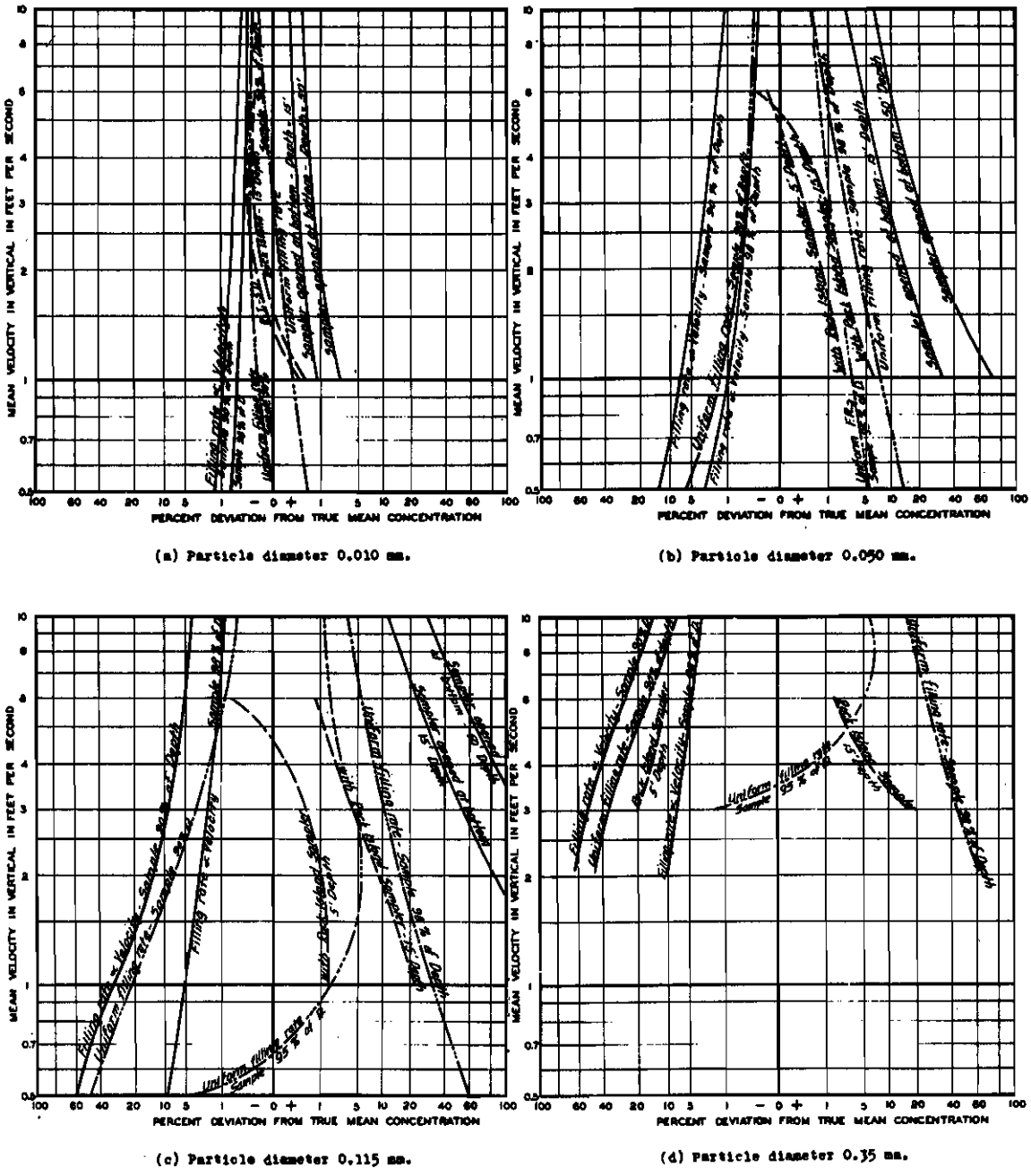


Fig. 14 - Accuracy of depth-integration method for various stream and sediment conditions. Relative roughness 0.020.

VII. METHOD FOR DETERMINING TOTAL CONCENTRATION

26. Sediment gradations and methods used in analysis--The various point sampling and depth-integration methods were applied to typical composite water sediment samples in order to determine their relative accuracy when used in actual stream conditions. The gradation curves of three types of sediment which were used in the analysis are shown in Fig. 15. These curves, designated A, B, and C, respectively, represent fine, medium, and coarse-grained sediment.

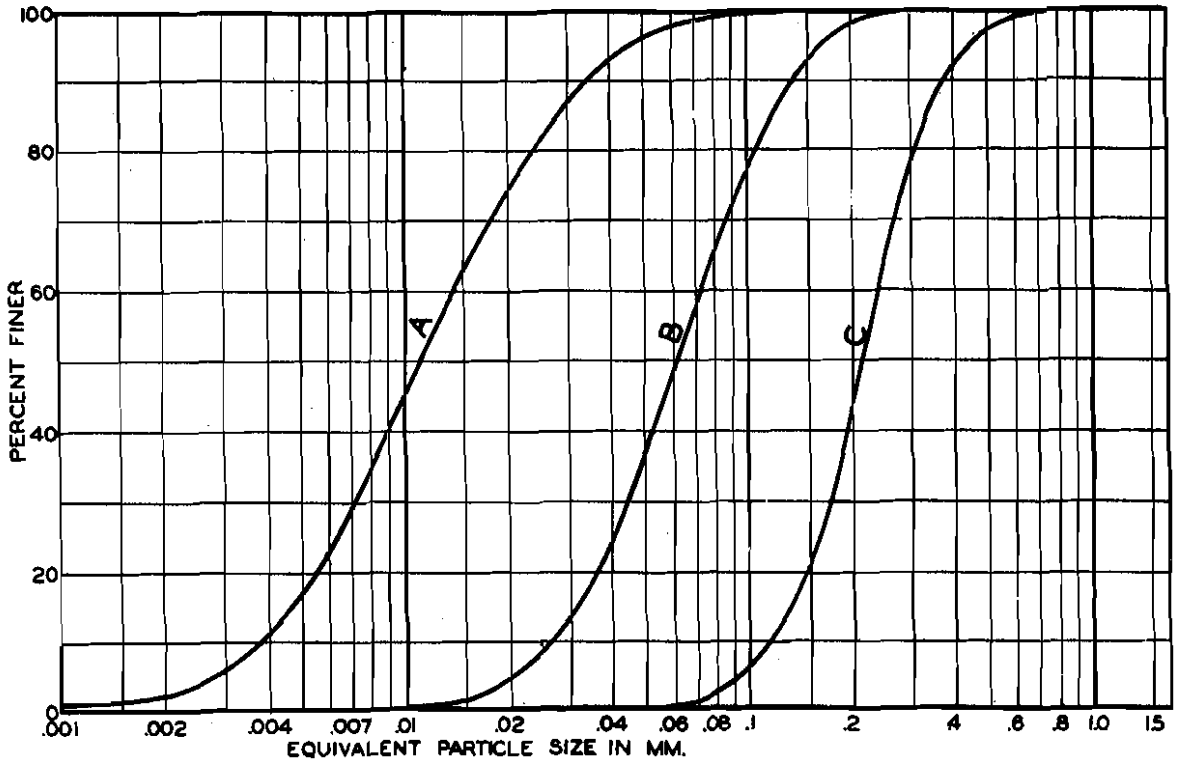
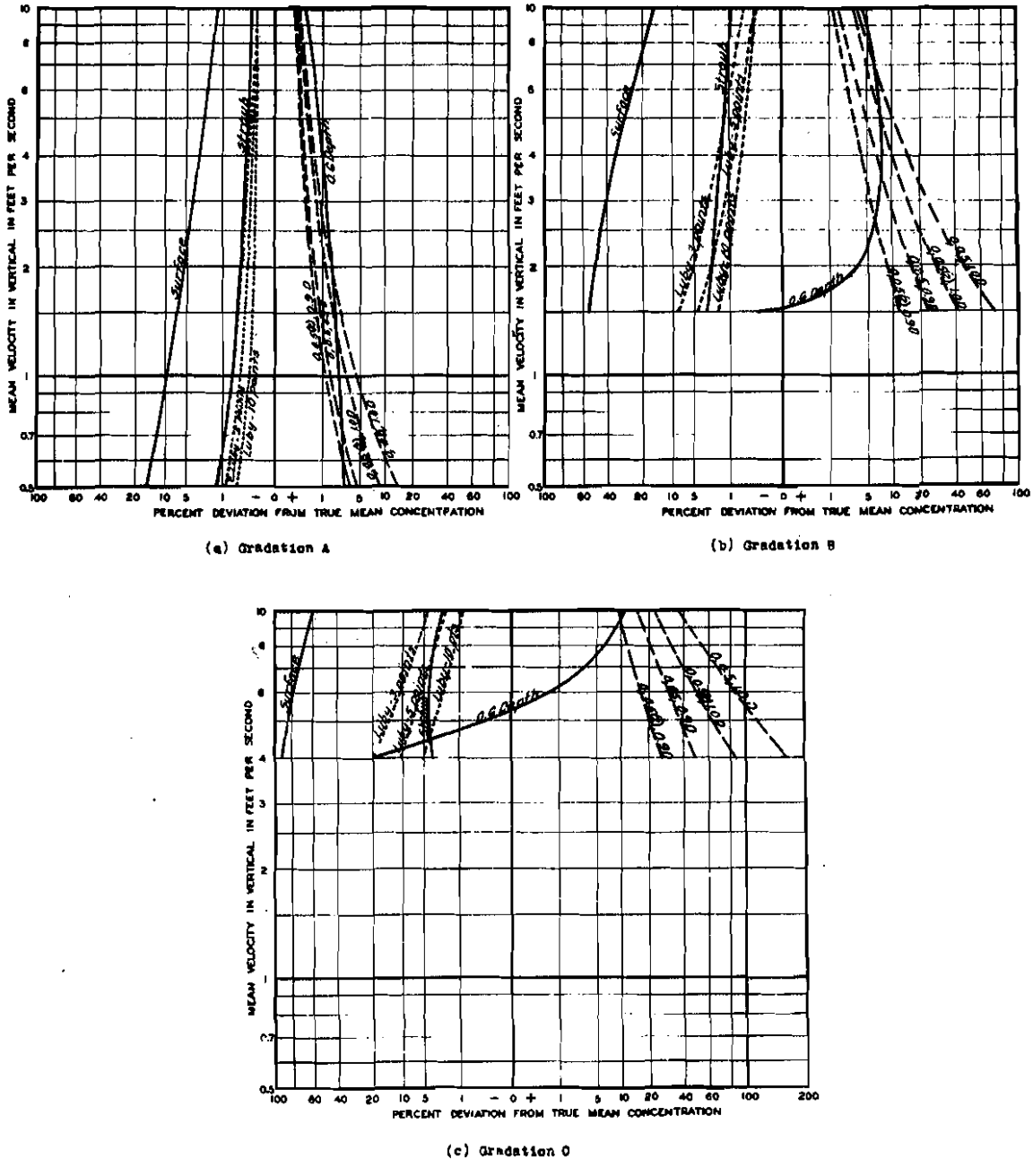


Fig. 15 - Sediment gradations used in analysis of accuracy of methods for total concentration determination.



(a) Gradation A

(b) Gradation B

(c) Gradation C

Fig. 16 - Accuracy of point sample methods for total concentration determination. Relative roughness 0.020.

Each gradation curve was divided into a number of small increments; then the error for each increment was computed by using the average particle size in the increment. The error for the composite sample was determined by weighting each increment error according to the percentage of sediment in that increment with respect to the total sample. A relative roughness factor of 0.020, corresponding to a Chezy coefficient of 75, was used throughout.

27. Errors in point sample methods for determining total concentration--The results of the analysis of point sample methods in sampling the three gradations of sediment are presented in Fig. 16 in terms of percentage error at various mean velocities in the vertical. The validity of these results depends upon the velocity and sediment distributions which were determined analytically. The accuracies of the methods are summarized according to gradation of sediment as follows:

a. Gradation A: Only the surface method shows an appreciable error for mean velocity greater than 1 ft./sec. but for mean velocities of 3 ft./sec. or greater even this method gives fair results. This indicates that for fine sediments, there is little choice among most of the point sample methods for velocities greater than 1 ft./sec.

b. Gradation B: The surface method shows a high negative error at all ordinary stream velocities. In the 0.6 depth method the error is not great, being less than 8 per cent for all velocities. The 3-point method, in which the middle sample is given double weight and the bottom sample taken at 0.9 depth, is fairly accurate for velocities greater than about 4 ft./sec. The other 3-point sampling methods show greater deviations. For velocities above 2 ft./sec. the Luby and Straub are the most accurate methods.

c. Gradation C: The surface and 3-point methods show an excessive negative and positive error even for relatively high stream velocities. The Straub and the Luby 5 and 10-point methods are fairly accurate. The error in the 0.6 depth method ranges from 20 per cent negative to 12 per cent positive as the velocity increases from 4 to 10 ft./sec.

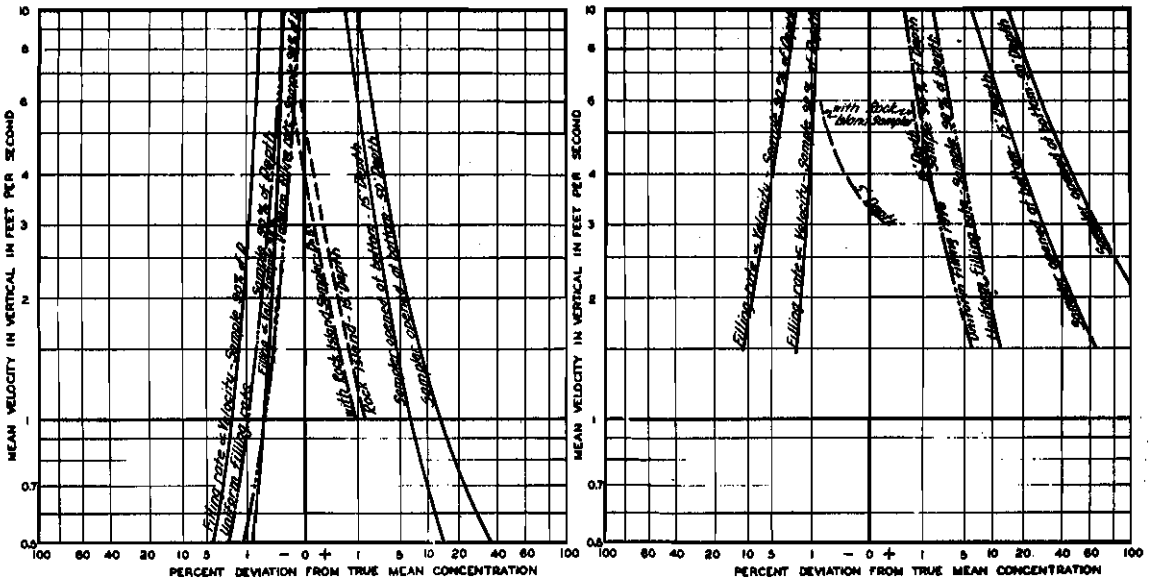
28. Errors in depth-integration method for determining total concentration--The deviations from the true mean concentration, when the depth-integration method is applied to the three types of graded sediment, are shown in Fig. 17. The results may be summed up as follows:

a. Gradation A: For filling rates which are either uniform, equal to velocity, or as with the Rock Island sampler at 5 and 15-ft. depths, the errors are inappreciable. The most serious errors are introduced when the sampler is opened near the bed. However, the errors in this method become appreciable with this gradation only at low velocities and great depths.

b. Gradation B: For velocities below 5 ft./sec. all methods except that in which the Rock Island sampler was used in a 5-ft. depth show deviations greater than 1 per cent. Next in accuracy to the Rock Island, is the method in which a sampler fills at a rate which is equal to the velocity for the 98 per cent depth. The procedure in which a sampler is opened at the bed shows a high positive error that decreases with velocity but increases with depth.

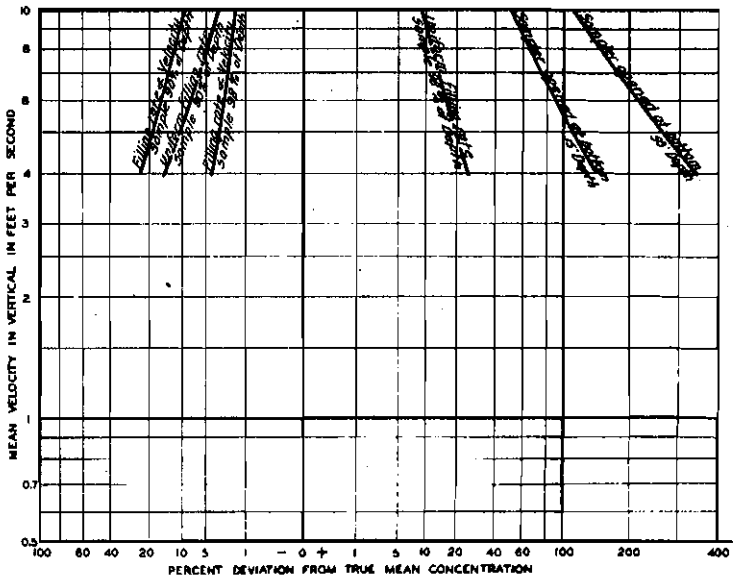
c. Gradation C: Only two procedures, namely, sampling 98 per cent of the depth at a filling rate equal to the stream velocity at every point in the vertical, and sampling 90 per cent of the depth at a uniform filling rate, show a fair degree of accuracy. Other procedures give large errors, particularly the one in which the sampler is opened near the bottom.

29. Coefficients of mean concentration--The comparisons presented in the previous sections show that no single method is accurate for all conditions, although several were found to be fairly reliable for most average velocity and sediment distributions and for ordinary stream conditions. In even the more accurate methods the errors exceed 50 per cent for extremely large particles and relatively low velocities. While it is realized that the computations in this analysis are not quantitatively correct, because they depend upon a theoretical relationship of sediment distribution and stream conditions which were based upon a relatively small amount of field data, and are further limited by the accuracy of



(a) Gradation A

(b) Gradation B



(c) Gradation C

Fig. 17 - Accuracy of depth-integration methods for total concentration determination. Relative roughness 0.020.

the sample collected, the analysis indicates that under certain conditions the sampling errors are excessive.

A study was made of the practicability of applying coefficients to the results obtained with the various sampling methods to correct these excessive errors. Obviously, such a procedure would be unsatisfactory for cases in which the coefficient varied rapidly for small changes in stream and sediment conditions because in such cases it is difficult to select the proper coefficient. For the best sampling conditions in all methods the value of the coefficient approaches the limit, 1.0, and changes least rapidly for any given change in stream and sediment conditions. Therefore, it is advisable to use a method in which the value of the coefficient is always close to 1.0. For the more accurate methods, the coefficient is rarely larger than 2.0 or less than 0.9 and it is fairly easy to select a good value.

By applying coefficients the results obtained over a period of time would probably be more accurate than if no coefficient were applied; however, an individual measurement would not necessarily be aided by such a procedure. In any case, it would be an unnecessary refinement to apply a coefficient if the error in the method was less than 10 per cent.

Figs. 18 to 20 show the coefficients which may be applied to a sample taken from a single given depth for a limited size range under various stream conditions. Each set of curves are for a given relative stream roughness. The coefficient to use in order to obtain the mean concentration in the vertical varies with parameter $\underline{t}$. As is evident from the nomograph, this parameter varies with the relative stream roughness, with the particle size of the sediment, and with the mean velocity in the

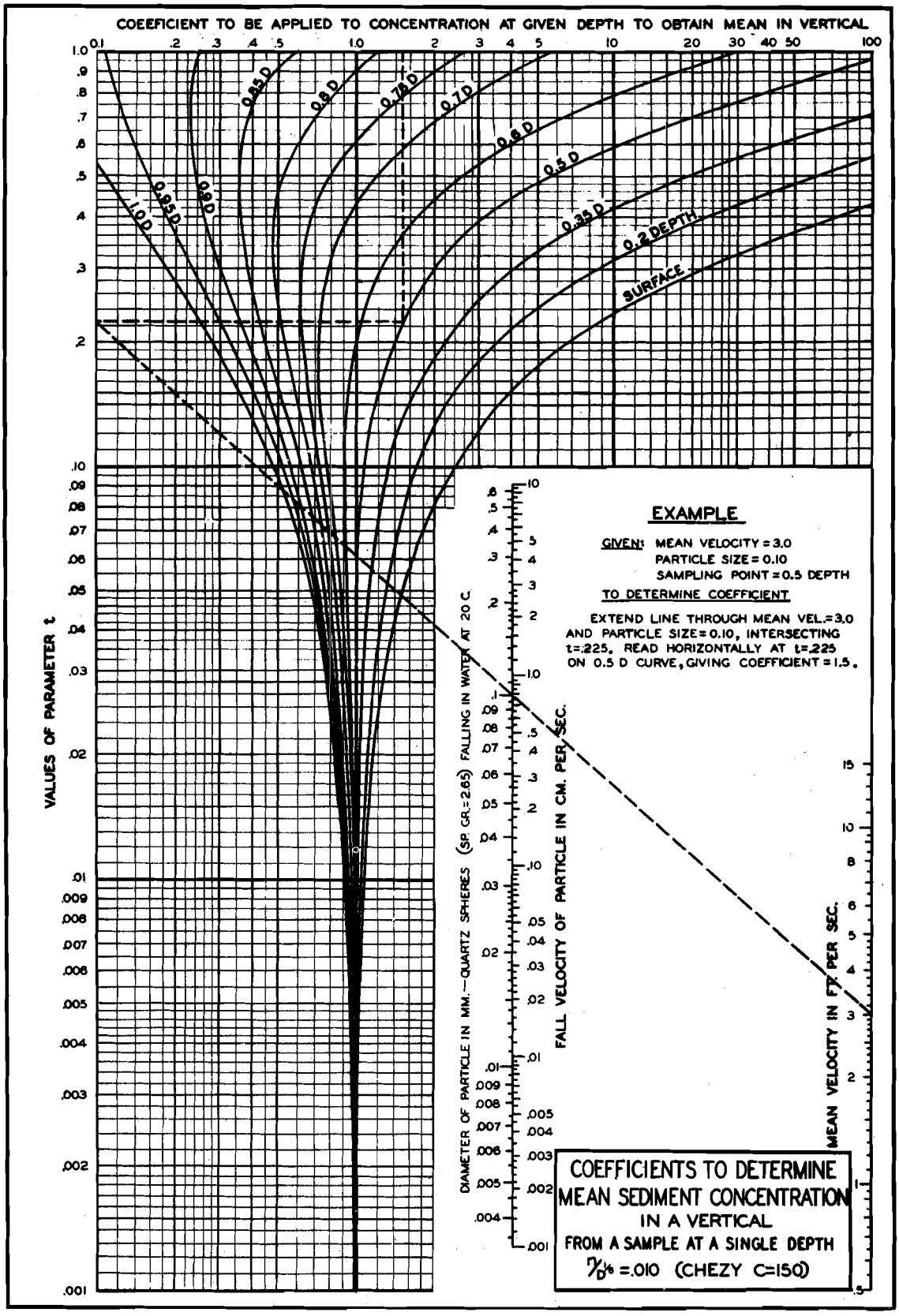

COEFFICIENT TO BE APPLIED TO CONCENTRATION AT GIVEN DEPTH TO OBTAIN MEAN IN VERTICAL
VALUES OF PARAMETER t
1.0 D
0.95 D
0.9 D
0.85 D
0.8 D
0.75 D
0.7 D
0.6 D
0.5 D
0.35 D
0.2 DEPTH
SURFACE
DIAMETER OF PARTICLE IN MM.—QUARTZ SPHERES (SP. GR.=2.65) FALLING IN WATER AT 20 C.
FALL VELOCITY OF PARTICLE IN CM. PER SEC.
MEAN VELOCITY IN FT. PER SEC.
EXAMPLE
GIVEN: MEAN VELOCITY = 3.0
PARTICLE SIZE = 0.10
SAMPLING POINT = 0.5 DEPTH
TO DETERMINE COEFFICIENT
EXTEND LINE THROUGH MEAN VEL.=3.0 AND PARTICLE SIZE=0.10, INTERSECTING t=.225. READ HORIZONTALLY AT t=.225 ON 0.5 D CURVE, GIVING COEFFICIENT = 1.5.
COEFFICIENTS TO DETERMINE MEAN SEDIMENT CONCENTRATION IN A VERTICAL FROM A SAMPLE AT A SINGLE DEPTH
7/D⅙ =.010 (CHEZY C=150)

FIG. 18

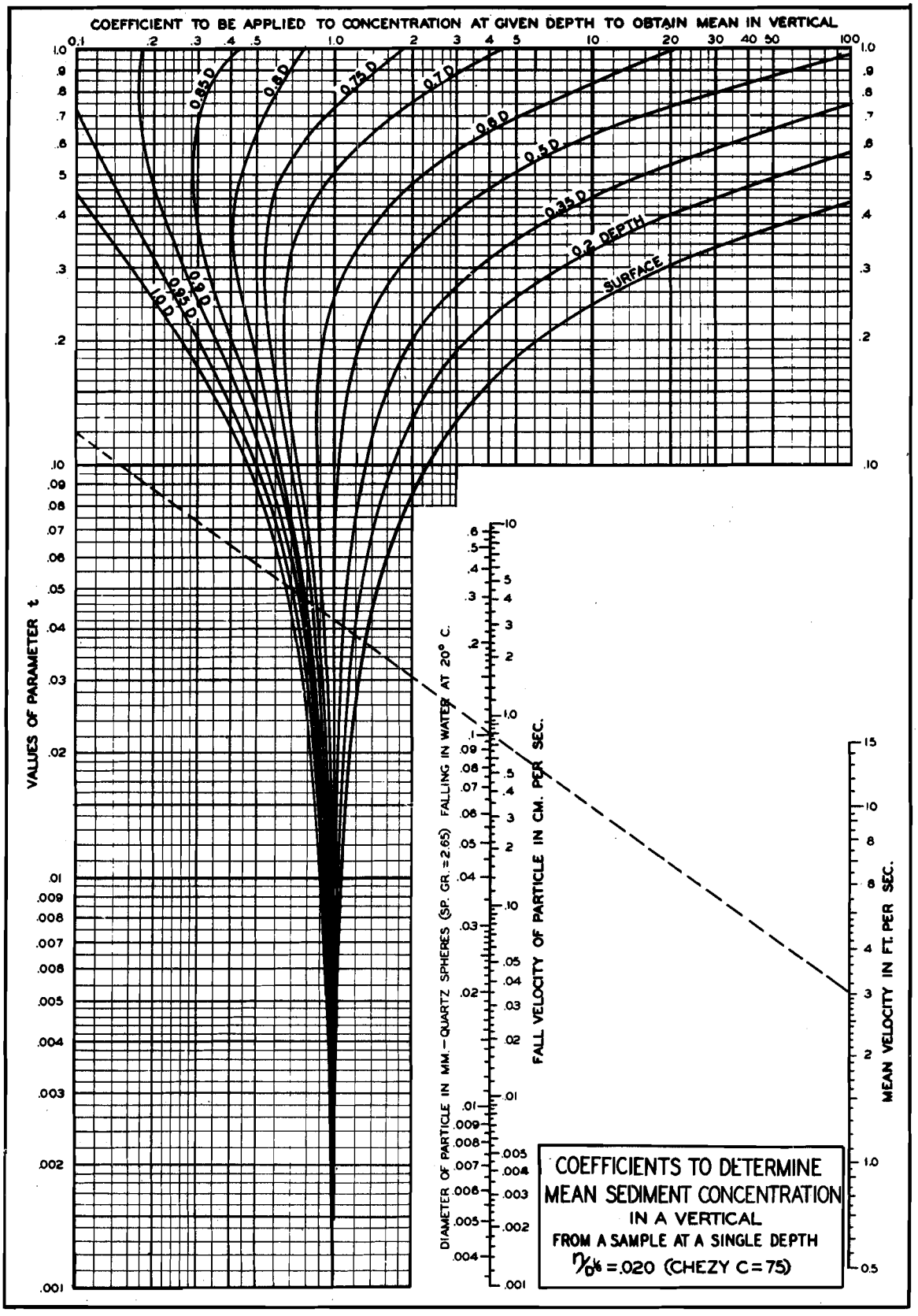

COEFFICIENT TO BE APPLIED TO CONCENTRATION AT GIVEN DEPTH TO OBTAIN MEAN IN VERTICAL
0.1 .2 .3 .4 .5 1.0 2 3 4 5 10 20 30 40 50 100
0.95 D
0.9 D
0.75 D
0.7 D
0.6 D
0.5 D
0.35 D
0.2 DEPTH
SURFACE
1.0 D
0.95 D
0.9 D
VALUES OF PARAMETER t
DIAMETER OF PARTICLE IN MM.—QUARTZ SPHERES (SP. GR. = 2.65) FALLING IN WATER AT 20° C.
FALL VELOCITY OF PARTICLE IN CM. PER SEC.
MEAN VELOCITY IN FT. PER SEC.
COEFFICIENTS TO DETERMINE
MEAN SEDIMENT CONCENTRATION
IN A VERTICAL
FROM A SAMPLE AT A SINGLE DEPTH
η/D^1/6 = .020 (CHEZY C = 75)

FIG. 19

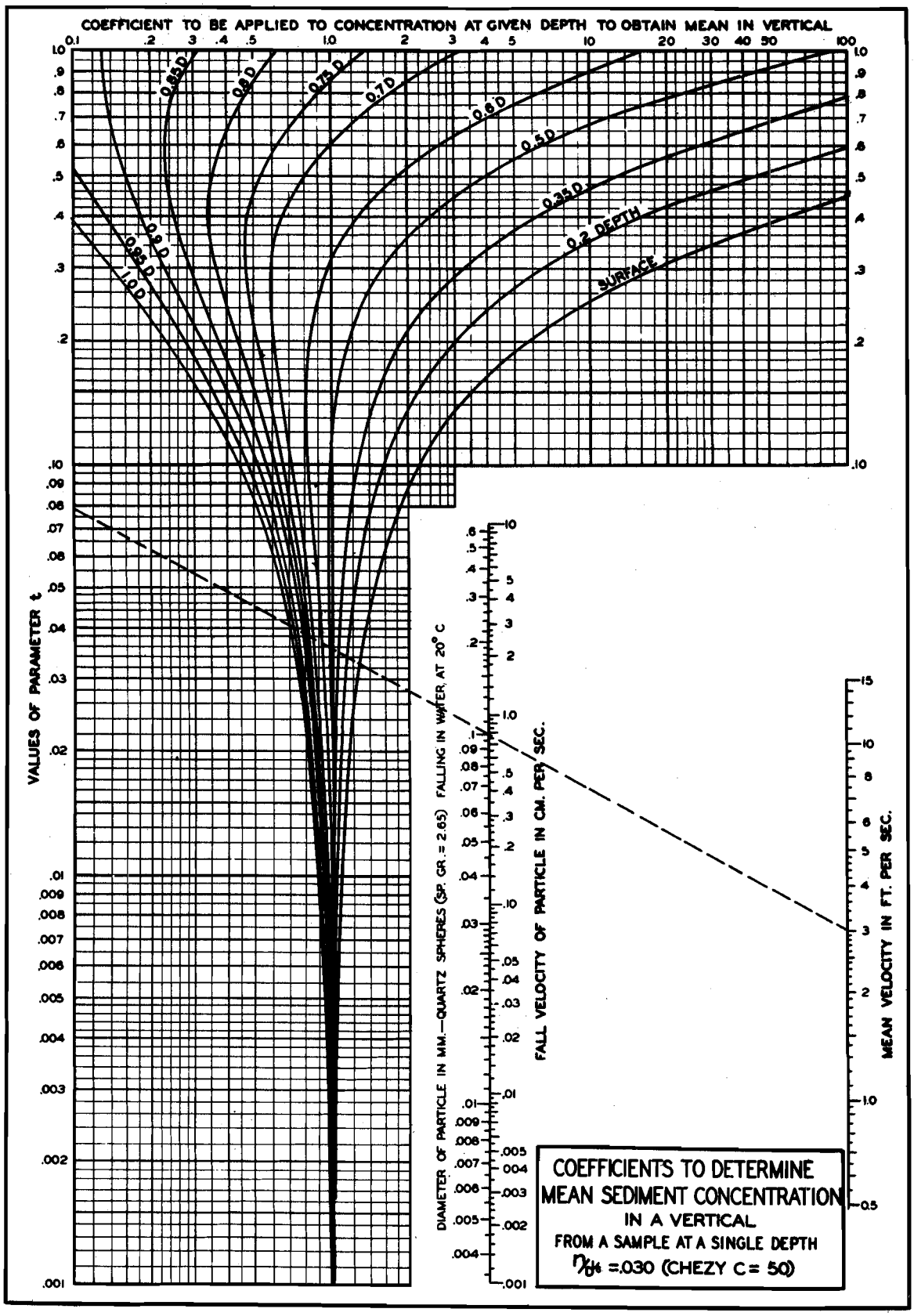

COEFFICIENT TO BE APPLIED TO CONCENTRATION AT GIVEN DEPTH TO OBTAIN MEAN IN VERTICAL
0.1 .2 .3 .4 .5 1.0 2 3 4 5 10 20 30 40 50 100
0.95D
0.9D
0.75D
0.7D
0.6D
0.5D
0.35D
0.2 DEPTH
SURFACE
0.99D
0.95D
1.0D
VALUES OF PARAMETER t
DIAMETER OF PARTICLE IN MM.—QUARTZ SPHERES (SP. GR. = 2.65) FALLING IN WATER AT 20° C
FALL VELOCITY OF PARTICLE IN CM. PER SEC.
MEAN VELOCITY IN FT. PER SEC.
COEFFICIENTS TO DETERMINE
MEAN SEDIMENT CONCENTRATION
IN A VERTICAL
FROM A SAMPLE AT A SINGLE DEPTH
n/y4 =.030 (CHEZY C= 50)

FIG. 20

vertical. In Fig. 18 an example is given to illustrate the procedure by which the proper coefficient for given conditions is determined.

As may be seen from these curves, if the sample is taken from points near the surface, the coefficient is large for a considerable range of conditions. This indicates that even though a coefficient may be applied to the concentration, it is inadvisable to collect a single sample from the upper portion of a vertical when the sediment and velocity distributions are of considerable curvature. Further inspection of the curves indicates that the best points to collect samples from would be between the 0.6 and 0.8 depths, as within this range the coefficients are, in most cases, fairly close to 1.0. Selection of the best sampling point would depend upon the particular conditions; for example, for large sizes of sediment, a low sampling depth would be best.

In Table 6 is presented a typical set of computations in which coefficients are applied to correct for the size distributions of a sample taken from a given depth under known conditions. These computations are for size gradation Curve B of Fig. 15 for the following assumed conditions:

a. Total concentration in sample = 1,000 p.p.m.

b. Sampling depth = 0.75 of total depth

c. Relative roughness, $n/D^{1/6}$ = .010, Chezy C = 150

d. Mean velocity in vertical = 5 ft./sec.

This method must be used cautiously because it is based upon a relatively small amount of field data. Furthermore, it is essential that a true representative sample be obtained from the single sampling point. At present, certain inherent errors which exist in most samplers tend

TABLE 6

EXAMPLE OF COMPUTATIONS TO CORRECT SIZE DISTRIBUTION FOR A SAMPLE FROM A SINGLE POINT BY THE APPLICATION OF A COEFFICIENT

Mean Diameter given size range mm.	Per Cent of sample in size range	Concentration of size range p.p.m.	Coefficient (Fig. 18)	Corrected concentration p.p.m.
0.32	1	10	1.40	14
.21	4	40	.67	27
.14	10	100	.61	61
.095	20	200	.68	136
.067	20	200	.79	158
.048	20	200	.87	174
.031	20	200	.96	192
.014	5	50	1.00	50

to further limit the use of this method. However, the use of these coefficients affords an approximate means for determining the mean sediment concentration in a vertical from a single sample.

VIII. SUMMARY

30. Basis of the analysis--The accuracies of the various point sample and depth-integration methods of sampling a vertical have been analyzed and the results are presented graphically and in tabular form. The errors inherent in the various methods are correlated to certain types of sediment distribution curves and to given stream conditions for a particular size of sediment. The first comparison depends only upon the existence of such sediment distribution curves and vertical velocity curves and upon the assumption that the sampler collects a true representative sample at the sampling point in the vertical. The errors shown for given stream and sediment characteristics are based entirely on an extension of the theory of turbulence exchange as applied to the vertical distributions of suspended sediment and upon the assumption that saltation load movement is insignificant in the sampling zone. The latter comparison, therefore, is only as accurate as the original theory and the assumptions made in using the theory to predict the distribution of sediment in the vertical. The relatively small amount of pertinent field data available tends to substantiate the theory.

A comparison was made also of the accuracy of the various methods of determining the total sediment concentration of a graded sediment. Three typical gradations in particle sizes of sediment were used, representing the probable range of sizes usually found in suspension. The error in total concentration was determined for each curve by summing up the errors for each size weighted by the percentage of that size in each sample.

The analyses are based on the assumption that each sample taken is a true representation of the water-sediment mixture at the sampling point.

Actually, this condition is not fulfilled in present field operations because as determined in the investigation of sampling action, presented in Report No. 5 of this series, existing samplers do not collect representative samples. It has been found that for particles larger than 0.03 mm. diameter significant errors in sampling occur if the pressure of the sampler changes the normal flow lines upstream from the sampling point. Changes in direction of the flow lines, as with the bottle type sampler, or changes in the pattern due to a difference between the intake velocity and the normal stream velocity at the sampling point are common to most existing samplers. Therefore, in applying this analysis, the type and action of the sampler used must be considered because the inherent error of the sampler must necessarily be superimposed upon the errors which were indicated by the various methods used in this report.

31. Comparison of methods--No point sample or depth-integration method was found to be accurate for the entire range of stream and sediment conditions assumed. The surface and 0.6 depth methods, in which no coefficients are used, and the method in which a sample is collected at the surface, mid-depth, and bottom, generally show appreciable error. The depth-integration method in which the sampler is opened at some point below the surface and fills while ascending yields very large errors if this initial point is at a great depth. These errors may be compensated for, somewhat, by the loss in sediment that occurs due to the initial inrush; however, this method is not recommended for large sizes of sediment and deep streams. Of all the methods analyzed, the three which appear to be reasonably accurate for most conditions are classified and described briefly in the subparagraphs which follow:

a. Straub method: Samples are obtained at the 0.2 and 0.8 depth and their sediment concentrations are weighted 5/8 and 3/8, respectively; a summation of these weighted concentrations is considered the mean concentration in the vertical. Since this method was developed for a linear distribution of sediment, it is most accurate for that particular distribution.

b. Luby method: Samples are obtained at the midpoint of sections of equal water discharge, the number of samples collected depending upon the accuracy desired. Usually, sampling 5 points in a vertical will yield fairly accurate results.

c. Depth-integration method: In this method the sampler fills while both descending and ascending. A sampler which fills at a rate equal to the velocity at every point in the vertical and which will traverse 95 per cent of the depths, or more, gives the most accurate results.

Table 7 presents a comparison between these three methods, considering the depth-integration method as performed with the Rock Island sampler. The theory indicates that, for a given particle size, the errors for all mean stream velocities above a certain minimum will fall within a given per cent. These minimum velocities are shown in the table for three different percentages of error and for four particle sizes.

TABLE 7

COMPARISON OF STRAUB, LUBY, AND DEPTH-INTEGRATION METHODS FOR VARIOUS SEDIMENT SIZES AND STREAM VELOCITIES

Maximum limit of error, per cent	Particle size, mm.	Minimum stream velocity in ft./sec. for errors not to exceed given limit		
		Depth-Integration method with Rock Island Sampler, 15-ft., 98.4 per cent of depth	Luby method with 5 points	Straub method 0.2 and 0.8 depth
5	.010	0.5	0.5	0.5
	.050	1.25	.5	.5
	.115	2.6	2.0	.85
	.35	4.5	10.0	5.0
10	.010	.5	.5	.5
	.050	.75	.5	.5
	.115	2.0	1.25	.75
	.35	3.5	6.5	4.5
20	.010	.5	.5	.5
	.050	.5	.5	.5
	.115	1.25	.9	.65
	.35	2.75	4.5	4.0

The data shown in this table are based on an average relative roughness factor. Other roughness factors would have some effect, but similar comparisons would yield the same relative results.

The table indicates that for particle sizes of 0.115 mm. or less, the Straub method allows the largest velocity range, while for larger sizes the depth-integration method has the widest range. The Luby method apparently falls between the others. Since this comparison is entirely analytical, it does not account for inherent errors in the sampling methods.

The Straub method indicates a greater accuracy for a wider range of conditions than the others. This apparent advantage is offset, in part, by the fact that in sampling only two points in a vertical, it is possible that a true representative sample may not be obtained due to fluctuations in the sediment concentration. The Straub method requires somewhat less

field work than does the Luby method but requires twice as much laboratory work because in the former each sample must be analyzed separately, while in the latter, usually, only one laboratory analysis is needed.

The Luby method in which at least five samples are collected seems to be reasonably accurate under ordinary conditions. Since a sample collected by this method is probably more representative of the true sediment distribution in the vertical, it is likely to yield better results than the others. This method requires more detailed control of the field work, but, as previously mentioned, the laboratory work is simple because if all the samples are of equal volume they may be combined for a single analysis. The Luby method is not limited as to sampling depth and is probably the most reliable method for deep streams except under conditions in which a linear distribution of sediment occurs; in such cases, the Straub method is equally as good or better.

By using the depth-integration method with a sample which is designed to fill at a rate equal to the stream velocity at every point in the vertical and which traverses 95 per cent of the stream depth at the vertical, results are obtained which are about as accurate as the Straub or Luby method. If a greater percentage of the stream depth is sampled in this manner, the error is even less. Therefore, for shallow streams, below 20 ft., this method is the most accurate and requires a minimum of field work and laboratory analysis. For deeper streams, a sampler which integrates at a point, i.e., fills at a rate equal to the stream velocity at the particular sampling point, offers a method which also always obtains a true representative sample and which, likewise, requires a minimum of field and laboratory work.

One phase of the future work in this project is to develop samplers which will fill at a rate equal to the stream velocity either at a point or while traversing a vertical in a stream, thereby, always collecting a true representative sample of the suspended sediment.

In using the depth-integration method in which the sampler fills at a uniform rate it appears that under certain conditions, for example, when 95 per cent of the depth is sampled for a relative roughness factor of 0.020, the per cent of error is lower than that of any of the other methods used; however, this is true only for a rather limited range of depths sampled. In general, this method is an inferior one because it is impossible to collect a true representative sample in a sampler which fills at a uniform rate.

Applying a coefficient to the concentration of a single point sample in a vertical in order to determine the mean concentration in the vertical has the obvious advantage of a single field operation and a single laboratory analysis. Curves have been presented from which the proper coefficient to apply to a sample taken from any point in the vertical may be determined. Since there is a sediment distribution, expressed by $\underline{t}$ in these curves, for each size of sediment present in the sample, theoretically the coefficients should be applied to each distribution. As a matter of fact, they may be applied to a narrow range of sizes which are treated as a single particle size. If these coefficients are used, the sample from the lower portion of the vertical, preferably between 0.6 and 0.8 of the stream depth. The use of this method is limited at present by an inadequacy of accurate sampling equipment, and by the need for further verifying the theory upon which these coefficients are based. However,

these coefficients afford a rapid method for determining the approximate mean concentration of suspended sediment in a stream vertical from a single sample collected at any point in the stream vertical.

BIBLIOGRAPHY

1. Bakhmeteff, B. A.; "Mechanics of Turbulent Flow," Princeton University Press, 1936.

2. Christiansen, J. E.; "Distribution of Silt in Open Channels," Trans., American Geophysical Union, p. 478, Part II, 1935.

3. Kalinske, A. A. and Van Driest, E. R.; "Applications of Statistical Theory of Turbulence of Hydraulic Problems," Proc. 5th International Congress of Applied Mechanics, 1939.

4. Kalinske, A. A.; "Relation of the Statistical Theory of Turbulence to Hydraulics," Proc. A.S.C.E., pp. 1387-1406, Vol. 65, No. 8, October, 1939.

5. Lane, E. W. and Kalinske, A. A.; "The Relation of Suspended to Bed Material in Rivers," Trans., American Geophysical Union, pp. 637-41, 1939.

6. Lane, E. W. and Kalinske, A. A.; "Engineering Calculations of Suspended Sediment," Trans., American Geophysical Union, 1941.

7. O'Brien, M. P.; "Review of Theory of Turbulent Flow and its Relation to Sediment Transportation," Trans., American Geophysical Union, p. 487, 1933.

8. Richardson, E. G.; "Suspension of Solids in a Turbulent Stream," Proc., Royal Society of London, p. 583, Vol. 162, 1937.

9. Rouse, Hunter; "Experiments on the Mechanics of Sediment Suspension," Proc., 5th International Congress of Applied Mechanics, 1939.

10. Rouse, Hunter; "An analysis of Sediment Transportation in the Light of Fluid Turbulence," U. S. Department of Agriculture, Soil Conservation Service, Sedimentation Division, July, 1939.

11. Missouri River Report, House Document No. 238, 73d Congress, 2d Session, pp. 1189-1202, 1935.